Test of Independence of Errors Across Equations in Systems of Equations
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In Chap. 6 (Section 6.4.3, page 202) of "Statistical Analysis of Management Data" (Gatignon 2014), Bartlett's test and Lawley's approximation test are proposed (Morison 1976). In addition and similar to these tests, STATA provides an option for the SUR estimation (STATA command "sureg") that displays the correlation matrix of errors across equations and the Breusch-Pagan test of independence of the errors (Breusch and Pagan 1980).

This Lagrange Multiplier test is based on the statistic *LM*:

$$LM = T\sum_{i}\sum_{j>i} r_{ij}^2 \qquad (1)$$

where r_{ij} is the estimated correlation of the cross-equation error terms.

LM is distributed as a chi-square with ν degrees of freedom $(\nu = N(N-1)/2)$.

For a large sample size (*T*), the Lawley's approximation and the *LM* statistics converge.

The command example provided in Chap. 6 (Fig. 6.6 for the input and Fig. 6.9 for the output) is modified to include the Breusch-Pagan test using the "corr" STATA option command. The commands and results are displayed in Table 1.

Table 1 — Modified Example of STATA input file for SUR estimation (examp6-1_Mac.do)

```
. sureg (dadvl dtol dresl) ///
> (dradicl dco1l dtol dgrow0l ddemuncl dresl) ///
> (dcostl dtol ic1 dgrow0l ddemuncl dresl), corr

Seemingly unrelated regression
------------------------------------------------------------------------------
Equation           Obs  Parms        RMSE    "R-sq"       chi2        P
------------------------------------------------------------------------------
dadvl              360      2    .3856953    0.2210     102.14   0.0000
dradicl            360      5    .4186645    0.2840     144.32   0.0000
dcostl             360      5    .4988242    0.0834      33.20   0.0000
------------------------------------------------------------------------------

------------------------------------------------------------------------------
             |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
dadvl        |
        dtol |   .2715593   .0484446     5.61   0.000     .1766097     .366509
       dresl |   .2072549   .0380967     5.44   0.000     .1325868    .2819231
       _cons |  -.0117502    .020334    -0.58   0.563    -.0516042    .0281037
-------------+----------------------------------------------------------------
dradicl      |
       dco1l |  -.0974295   .0432495    -2.25   0.024    -.1821969   -.0126621
        dtol |   .6050885   .0565433    10.70   0.000     .4942657    .7159113
     dgrow0l |   .0343637   .0097226     3.53   0.000     .0153078    .0534197
    ddemuncl |  -.0944976   .0478577    -1.97   0.048    -.1882969   -.0006983
       dresl |  -.0726166   .0415731    -1.75   0.081    -.1540984    .0088651
       _cons |  -.1524931   .0443876    -3.44   0.001    -.2394912   -.0654951
-------------+----------------------------------------------------------------
dcostl       |
        dtol |   .1748069   .0672157     2.60   0.009     .0430665    .3065473
         ic1 |  -.1594804   .0390791    -4.08   0.000     -.236074   -.0828869
     dgrow0l |  -.0194596   .0115465    -1.69   0.092    -.0420902     .003171
    ddemuncl |  -.1342377   .0568209    -2.36   0.018    -.2456045   -.0228709
       dresl |   .1162543   .0494856     2.35   0.019     .0192644    .2132442
       _cons |   .0697575   .0526953     1.32   0.186    -.0335234    .1730385
------------------------------------------------------------------------------

Correlation matrix of residuals:

           dadvl  dradicl   dcostl
  dadvl   1.0000
dradicl   0.1155   1.0000
 dcostl  -0.1296  -0.0019   1.0000

Breusch-Pagan test of independence: chi2(3) =    10.852, Pr = 0.0126
```

The results indicate that the three correlation coefficients are jointly significant at the 0.05 level but not at the 0.01 level ($\chi^2_{v=3} = 10.852$).

References

Breusch, T. S. and A. R. Pagan (1980), "The Lagrange multiplier test and its applications to model specification tests in econometrics," *Review of Economic Studies*, 47, 146, 239-53.

Gatignon, Hubert (2010), *Statistical Analysis of Management Data*, Springer Science+Business Media, LLC, New York, NY.

Morrison, Donald F. (1976), *Multivariate Statistical Methods,* New York, NY: McGraw-Hill Book Company.