Confirmatory Factor Analysis: Internal Consistency and Average Variance Extracted (AVE)

HG Note

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Internal Consistency or Reliability for a Construct[1]

The measurement model is given by Equation (1)

$$y_i = \lambda_i \xi + \delta_i \tag{1}$$

The variance of the observed item, which is equal to unity if the items are standardized, is then given by Equation (2)

$$\mathrm{V}[y_i] = \lambda_i^2 + \mathrm{V}[\delta_i] = 1 \tag{2}$$

Consequently we can express the variance of the measurement error as a function of the factor loading as in Equation (3):

$$\mathrm{V}[\delta_i] = 1 - \lambda_i^2 \tag{3}$$

The internal consistency measure is defined as per Equation (4)

$$\frac{\left(\sum_{i=1}^{K} \lambda_i\right)^2}{\left(\sum_{i=1}^{K} \lambda_i\right)^2 + \sum_{i=1}^{K}(1 - \lambda_i^2)} \tag{4}$$

where the denominator is simply the variance of the sum of the items. This can be shown noting Equation (5)

$$\begin{aligned} X = \sum_{i=1}^{K} y_i = \sum_{i=1}^{K} (\lambda_i \xi + \delta_i) &= \sum_{i=1}^{K} [(\lambda_i \xi) + \delta_i] \\ &= \sum_{i=1}^{K} (\lambda_i \xi) + \sum_{i=1}^{K} \delta_i = \xi \sum_{i=1}^{K} \lambda_i + \sum_{i=1}^{K} \delta_i \end{aligned} \tag{5}$$

from which it follows Equation (6) that

$$\mathrm{V}[X] = \left(\sum_{i=1}^{K} \lambda_i\right)^2 + \sum_{i=1}^{K} \mathrm{V}[\delta_i] = \left(\sum_{i=1}^{K} \lambda_i\right)^2 + \sum_{i=1}^{K} (1 - \lambda_i^2) \tag{6}$$

[1] Fornell, Claes, and David F. Larcker (1981), "Evaluating Structural Equation Models with Unobservable Variables and Measurement Error," *Journal of Marketing Research*, 18(1), 39–50.

Average Variance Extracted (AVE)

The average variance extracted is, as its name indicates the average of the variances "explained" by the common factor. For each item, the variance explained by the factor ξ is the square of the corresponding factor loading, i.e., λ_i^2. When the items are standardized, the factor loading is the correlation between the factor ξ and the item y_i (the factor being standardized with mean 0 and variance equal to 1). The AVE is therefore the sum over all items of the variance explained divided by the number of items, as shown in equation (7)

$$\frac{\sum_{i=1}^{K} \lambda_i^2}{K} \tag{7}$$

If the items are not standardized, the denominator is the sum of the variances of all the items, which, according to Equation (2) leads to Equation (8)

$$\frac{\sum_{i=1}^{K} \lambda_i^2}{\sum_{i=1}^{K} \lambda_i^2 + \sum_{i=1}^{K} \mathrm{V}[\delta_i]} \tag{8}$$

If the items are standardized, following Equation (3), the denominator of Equation (7) becomes Equation (9)

$$\sum_{i=1}^{K} \lambda_i^2 + \sum_{i=1}^{K} \mathrm{V}[\delta_i] = \sum_{i=1}^{K} \lambda_i^2 + \sum_{i=1}^{K} (1 - \lambda_i^2) = K \tag{9}$$

These two measures (internal consistency and AVE) can easily be computed from a spreadsheet using the template provided on the Excel file accessible from my personal page

http://faculty.insead.edu/hubert-gatignon/publications/books

by clicking on

"Click here to download Excel Spreadsheet template to compute Internal Consistency and AVE measures"