INTERNATIONAL ECONOMIC REVIEW
Vol. 49, No. 2, May 2008

ASPIRATION LEVEL, PROBABILITY OF SUCCESS AND FAILURE, AND EXPECTED UTILITY*

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Aspiration levels are a relevant aspect of decision making. We develop a model that includes the overall probabilities of success and failure relative to the aspiration level into an expected utility representation. This turns out to be equivalent to expected utility with a discontinuous utility function. We give a behavioral foundation to the proposed model and provide conditions to determine the relative weights of the overall probabilities of success and failure. An aspiration level reinforces loss aversion, can account for simultaneous risk-averse and risk-seeking behavior, and can explain choices violating the mean-variance approach.

1. INTRODUCTION

Aspiration levels play an important role in everyday decision making. New York cab drivers are motivated to earn a daily target return (Camerer et al., 1997). On rainy, busy days, their earnings per hour are high and they go home early. On hot summer days, when many New Yorkers prefer to take a walk, they earn less per hour and make long hours to reach their target. When managers have to decide which projects or portfolios to invest in, they are keen on achieving a target rate of return as well. They disregard investment possibilities that are likely to result in a rate of return below their target (Payne et al., 1980, 1981). Some farmers also appear to have a minimal level of revenues that they want to achieve. Up to the subsistence level, they choose to cultivate "safe" crops with a stable return. The remainder of their land is allocated to "risky" crops (Lopes, 1987).

Common to all these examples is the presence of an aspiration level. The cab driver, the manager, and the farmer are confronted with risky choices on a daily basis. Some of them are willing to take risks, but often they also want to "win at least something," earn a daily target income, or prevent themselves from falling below the subsistence level. Therefore, in making their risky choices, subjects focus on reaching a special outcome, the aspiration level.

In this article, we present a simple model of choice under risk that includes an aspiration level. Once an aspiration level is included, the probabilities of success

* Manuscript received July 2005; revised September 2006.

[1] We wish to thank Marco LiCalzi, R. Duncan Luce, Fabio Maccheroni, Rakesh Sarin, George Wu, and especially Peter P. Wakker, Charles Horioka (the Editor), and three anonymous referees for very helpful suggestions. Jeroen van de Ven gratefully acknowledges support from the Netherlands Organization for Scientific Research (NWO) grant 457-05-005. Please address correspondence to: Enrico Diecidue, INSEAD, Bd. de Constance, 77305 Fontainebleau Cedex, France. E-mail: *enrico.diecidue@insead.edu*.

and failure are naturally identified: these are the overall probabilities that the aspiration will be reached or will fall short of. Our model entails that subjects are sensitive to the overall probabilities of success and failure. Payne (2005) tests several traditional models of decision making, including reference point based theories. His results provide clear support for the use of probability of success in decision making. He concludes that traditional models such as expected utility and prospect theory cannot account for his evidence, and if a model is to match the results from his experiments, such a feature is even indispensable.

We combine an aspiration level with expected utility (von Neumann and Morgenstern, 1944). Given that it is intuitive to think about an aspiration level in terms of the overall probabilities of success and failure, it is remarkable that this model turns out to be mathematically equivalent to expected utility with discontinuities. Despite its simplicity it can accommodate and predict several behavioral regularities. The presence of an aspiration level reinforces loss aversion, it explains the simultaneous purchase of lottery tickets and insurance policies, and it clarifies choices violating the mean-variance approach in finance.

The contribution of the article is as follows. We give a behavioral foundation for our model. We present necessary and sufficient conditions under which the overall probability of success is more important than the overall probability of failure. Finally, we show that the focus on the overall probabilities of success and failure can induce risk-seeking behavior for prospects close to the aspiration level. This is true, for instance, for prospects with two outcomes, one at the aspiration level and one below the aspiration level. Subjects become risk-seeking if the lower outcome gets close to the aspiration level. We call this risk-seeking from below.

We believe that our results are relevant to understand the impact of the overall probabilities of success and failure on risk-taking behavior close to the aspiration level. Although not exclusively, this might be of particular importance for the interpretation of subjects' choices made in laboratory experiments. We therefore propose a way to elicit experimentally the relative weights given to the overall probabilities of success and failure.

2. ASPIRATION LEVELS

2.1. *Motivations.* Imagine a subject having to choose among risky prospects. We interpret an aspiration level as an outcome that takes a special position in the decision process. Subjects code outcomes above the aspiration level as successes and outcomes below the aspiration level as failures. They place value on the overall probability of success and the overall probability of failure. Whereas in decision theory and psychology the focus is on the probability of success, in finance the emphasis is on avoiding falling short of a target rate of return on investments (Roy, 1952), hence on the overall probability of failure. In the modeling section, we focus on the overall probabilities of both success and failure.

When subjects focus exclusively on the overall probabilities of success and failure, they clearly ignore information and might ignore dominance issues. The natural question is why subjects would, deliberately or not, do so. One important reason might be to simplify decision problems. Because cognitive abilities of subjects are limited, focusing on an aspiration level can help to reduce the complexity of a problem (Roy, 1952; Simon, 1955; Langer and Weber, 2001, p. 731; Mezias et al., 2002; Brandstätter et al., 2006). Particularly in complex decision problems, the overall probabilities of success and failure can be helpful as a screening device (Manski, 1988). The overall probability of failure is an important element in the Value-at-Risk (VaR) method (Dowd, 1998), used to measure risk in a single summary statistic to simplify decision making. First, an overall probability of failure is set (also called the shortfall probability). The VaR is then calculated as the amount of money such that the probability of losing more than the VaR equals the shortfall probability.

An aspiration level is linked to probabilities and it is different from a reference point. A reference point is not related to probabilities. It is used in prospect theory as the point where the value function changes curvature and slope (kink), and divides outcomes into gains and losses (Kahneman and Tversky, 1979). We show that the presence of an aspiration level, while inducing the overall probabilities of success and failure, creates a discontinuity of the utility function at the point of the aspiration level, which may or may not coincide with the reference point. If they coincide, the overall probabilities of success and failure can be interpreted as the overall probabilities of gain and loss, respectively. Below we discuss experiments that discriminate between the use of an aspiration level and prospect theory.

2.2. *Evidence.* Payne (2005) provides clear experimental evidence supporting a focus on the overall probability of success. This evidence is in contrast with expected utility and prospect theory (Tversky and Kahneman, 1992). We discuss this experiment in detail because its results explicitly call for the inclusion of an aspiration level in a model. Subjects were shown a base mixed prospect, with some positive outcomes, a zero outcome, and some negative outcomes. One of the base mixed prospects gave outcomes \$100, \$50, \$0, −\$25, −\$50 with equal probability. Thus, $X = (0.2, 100; 0.2, 50; 0.2, 0; 0.2, -25; 0.2, -50)$. The subjects were then given several choices. In each case, they were told that they could add a sum of money (\$38) to either one of two possible outcomes and were asked which one they preferred to add it to.

In the first case, they could add the money either to the outcome that paid \$100 or to the one that paid \$0. The majority of the subjects indicated a preference to add it to the zero outcome. In the second case the money could again be added to the outcome that paid \$100 or to the outcome that paid \$50. Now there was no longer a majority for either possibility. What is interesting is that in the former case there was the possibility to increase the overall probability of a gain and the majority preferred to do so. In the latter case there was no such a possibility and there was no longer a clear pattern in choices.

There is a wealth of other evidence supportive of the relevance of the overall probabilities of success and failure. Closely related evidence is from a series of experiments by Payne et al. (1980, 1981). In these experiments, subjects had to choose between pairs of prospects, for example between $X' = (0.5, 74; 0.1, 30; 0.4, -25)$ and $Y' = (0.3, 40; 0.5, 30; 0.2, 15)$. Note that both prospects have an identical expected value of 30 but that prospect X' entails a higher risk because the spread of the prizes is larger. Naturally, many subjects prefer Y' to X'. The same decision problem was given but with all outcomes reduced by 60. This gives

$X'' = (0.5, 14; 0.1, -30; 0.4, -85)$ and $Y'' = (0.3, -20; 0.5, -30; 0.2, -45)$. Out of these two prospects, many subjects prefer X'', the one with the larger spread. These preferences are not easily compatible with expected utility and they may seem surprising, but note that both Y' and X'' have a higher overall probability of a gain than their alternatives X' and Y''. The behavior is therefore easily explained by an aspiration level.

Some recent experimental evidence is also provided by Lopes and Oden (1999). Their experiment has similar features as that of Payne et al. (1980, 1981). Subjects are asked to choose between pairs of prospects. Two operations were performed on the base prospects: a shift that added the same payoff to all outcomes and a scaling that multiplied all outcomes by the same factor. Interestingly, the prospects that before the shift had a high overall probability of a zero outcome became relatively much more attractive to the subjects after the shift than the prospects with a small probability on a zero outcome. Prospects that before the shift had a high probability of a zero outcome later had a high overall probability of a gain. The prospects with a small probability on a zero outcome did not become much more attractive because they already had a high overall probability of a gain. The scaling did not influence choices much, which is understandable because the overall probability of a gain is unaffected by this operation. In their study, they also report protocols from their experiments (Lopes and Oden, 1999, p. 304). Subjects, asked to comment on their choices, motivate them by statements such as "Have a greater chance of winning at least some money" and "There is a better chance to win money." Very similar reactions by subjects are reported in Schneider and Lopes (1986).

Other experimental studies lead to similar conclusions. Baucells and Heukamp (2006) attribute violations of loss aversion to subjects focusing on the overall probability of success. Edwards (1954, p. 396) shows that subjects do not like to lose, thus prefer low probabilities of losing large amounts of money to high probabilities of losing small amounts. Siegel (1957) presents early psychological evidence for paying attention to the probability of being above aspiration (success) when choosing between prospects. Langer and Weber (2001) present empirical evidence that subjects, when choosing between prospects, pay special attention to the overall probabilities of gains and losses.

Next, we turn to evidence from finance and risk management. The focus is on the probability of *not* reaching the aspiration level. The response of one manager makes it very clear that an aspiration level plays a role when he says that

> Risk is the prospect of not meeting the target of return (Mao, 1970, p. 353).

In like manner, another manager puts it like this:

> I never worry about the project going above the return. Risk is what might happen when the return is going to be less (Mao, 1970, p. 354).

Both statements unambiguously point to the focus on an aspiration level. Overall, Mao concludes from his study that "risk is primarily considered to be the prospect of not meeting some target rate of return" (Mao, 1970, p. 354), i.e., the probability of failure. And the managers he interviewed are no exception in that respect: Petty and Scott (1980) found this idea of a target return among *most* managers in a more extensive study. Furthermore March and Shapira (1987, p. 1043) conclude that "...the primary focus is on avoiding actions that might place one below [the target level]. The dangers of falling below the target dominate attention; the opportunities for gains are less salient... since it is the dangers that are noticed, the opportunities are less important." Roy (1952, p. 432) argues that agents, when holding financial assets, will seek to reduce the overall probability of being below a level, i.e., that the gross return is not less than some predetermined quantity. Browne (1995) shows the equivalence between the use of an exponential utility function and the minimization of the probability of failure. Relatedly, Stutzer (2003) gives a behavioral foundation for the use of a power utility function, based on the idea that investors minimize the probability of failure. However, also in the domain of finance, there are motivations to study the overall probabilities of success and failure. The behavior of paying particular attention to accomplishing the aspiration level is observed, especially among decision makers in investments (Fishburn, 1977 and the references thereafter; Markowitz, 1959). Davies (2006) considers the probability of success as a measure of risk.

In practice, however, decisions are not based on an aspiration level alone. The aspiration level and the overall probabilities of success and failure may receive special attention, but subjects will not be completely insensitive to difference within the classes of gains and losses (Libby and Fishburn, 1977, p. 289; Payne et al. 1980, p. 1047 and p. 1053; Lopes, 1996; Luce, 1996; Lopes and Oden, 1999). Aspiration levels alone capture a significant part of experimental data, and do so surprisingly well, but are too restricted to fit all the data (Lopes and Oden, 1999). Libby and Fishburn (1977, pp. 285–6) suggest that the probability of meeting a target return is traded off with expected value. Hence, a model solely based on an aspiration level is too crude to be normatively or descriptively relevant. This brings us to the next section, where we integrate the aspiration level into a more refined model of choice behavior.

3. A MODEL WITH AN ASPIRATION LEVEL

Imagine a subject having to choose among risky lotteries. These objects will be called *prospects* and denoted $X = (p_1, x_1; \ldots; p_n, x_n)$. The prospect X yields $\$x_j$ with probability $p_j, j = 1, \ldots, n$. Probabilities are nonnegative and sum to one. Without loss of generality assume that outcomes are rank ordered from best to worst, i.e., prospect $X = (p_1, x_1; \ldots; p_n, x_n)$ satisfies $x_1 \geq \cdots \geq x_n$. The set of outcomes of a prospect strictly above (below) the aspiration level is indicated by $x^+(x^-)$. The aspiration level is exogenously fixed and, without loss of generality, set to zero. Preferences over prospects are denoted by $\succcurlyeq$, with $\succ$ (strict preference) and $\sim$ (indifference) as usual.

For the prospect X, $P(x^+)$ is the *overall probability of success*, and $P(x^-)$ is the *overall probability of failure*. As people do not go exclusively by the overall probabilities of success and failure, we introduce a refined model in which these probabilities are inserted into expected utility. We then proceed by showing that the functional is mathematically equivalent to discontinuous expected utility and provide a set of necessary and sufficient conditions to determine the relative importance of the overall probabilities.

3.1. *Background on Expected Utility.* Expected utility constitutes a key model of individual decision making under risk. This should come as no surprise, since its assumptions are normatively appealing for a wide range of choice problems (von Neumann and Morgenstern, 1944). Descriptively, however, expected utility proved to have its limitations (Allais, 1953; Ellsberg, 1961). This has stimulated researchers to formulate alternative decision theories that can account for the observed behavior. In the last decades several models have been proposed that explain the descriptive violations. All these models have been generally labeled as *nonexpected utility* (for surveys, see Camerer and Weber, 1992; Starmer, 2000; Schmidt, 2004).

Let u denote the *utility* function. u is a real valued function defined on the set of outcomes. In this article outcomes are monetary. Then, the *expected utility* (EU) representation states that an agent evaluates a prospect X through the following formula:

$$X \mapsto \sum_{i=1}^{n} p_i u(x_i), \tag{1}$$

which says that the value of a prospect $X = (p_1, x_1; \ldots; p_n, x_n)$ is equal to the sum of the utilities weighted by the probabilities $p_1, \ldots, p_n$ attached to the respective outcomes.

3.2. *An EU Functional with Aspiration Level.* We propose the following real-valued functional V to evaluate the prospect X:

$$X \mapsto V(X) = \sum_{j=1}^{n} p_j u(x_j) + \mu P(x^+) - \lambda P(x^-), \quad \mu, \lambda \in \mathbb{R}^+. \tag{2}$$

That is, we propose a sum of expected utility, overall probability of success, and overall probability of failure. The importance of the overall probability of success is weighted by μ, and that of the overall probability of failure is weighted by λ. Both evaluations play an important role in the psychology of risk. We propose to combine them in an additive way. u, μ, and λ in (2) are unique up to a joint scaling factor. We may normalize $u(0) = 0, u(1) = 1$, after which μ and λ are uniquely determined. Alternatively, if $\lambda > 0$, we can always set $\lambda = 1$, by replacing u by u/λ and μ by μ/λ. More details are provided in Theorem 1.

We do not see the aspiration level as primarily part of the intrinsic values of outcomes, but as a consequence of a decision heuristic of decision makers to simplify decisions. The latter is more naturally modeled as the probability of a chosen target event. These two notions are distinct. It is therefore remarkable that Equation (2) turns out to be mathematically equivalent to discontinuous expected utility. To see this, rearrange Equation (2) and w.l.o.g. use $u(0) = 0$ to get

$$V(X) = \sum_{j:x_j \in x^+} [p_j u(x_j) + \mu p_j] + \sum_{j:x_j \in x^-} [p_j u(x_j) - \lambda p_j] \tag{3}$$
$$= \sum_{j:x_j \in x^+} p_j [u(x_j) + \mu] + \sum_{j:x_j \in x^-} p_j [u(x_j) - \lambda]$$

and define $v(x_j) = u(x_j) + \mu$ for $x_j \in x^+$ and $v(x_j) = u(x_j) - \lambda$ for $x_j \in x^-$. This gives

$$X \mapsto \sum_{j=1}^{n} p_j v(x_j). \tag{4}$$

Thus, we are back to expected utility but with a utility function v that is discontinuous around the aspiration level zero: $\lim_{x_j \uparrow 0} v(x_j) \neq \lim_{x_j \downarrow 0} v(x_j)$, provided μ or λ (or both) is strictly positive. This result raises three points. First, the evidence by Payne (2005) can be accounted for by expected utility, provided that the utility function is discontinuous. Second, it is not a kink in the utility function as in the value function of prospect theory, but a discontinuity that is closely related to loss aversion (see Section 4). Third, it shows that the existence of an aspiration level might be regarded as providing a behavioral foundation for discontinuous expected utility. The next section elaborates on this point.

3.3. *An Equivalence Theorem.* The aim is to characterize a jump at the aspiration level zero. First of all, expected utility is derived in a standard manner using Fishburn (1970) or Jensen (1967). For this, we use the following axioms. These are all standard and we present them for the sake of completeness.

(A1) *Weak Order*: The preference relation $\succcurlyeq$ is a weak order if it is complete (for all prospects X, Y, $X \succcurlyeq Y$ or $Y \succcurlyeq X$) and transitive (for all prospects X, Y, Z, if $X \succcurlyeq Y$ and $Y \succcurlyeq Z$, then $X \succcurlyeq Z$).

(A2) *Continuity in probability (Archimedean axiom)*: For all prospects X, Y, Z, if $X \succ Y \succ Z$, then there exist γ and $\gamma' \in (0, 1)$, such that $\gamma X + (1 - \gamma) Z \succ Y \succ \gamma' X + (1 - \gamma') Z$.

(A3) *(vN-M) Independence*: For all prospects X, Y, Z, and $\gamma \in (0, 1)$, if $X \succcurlyeq Y$ then $\gamma X + (1 - \gamma) Z \succcurlyeq \gamma Y + (1 - \gamma) Z$.

The following axiom on stochastic dominance will allow us to restrict the domains of the relative weights given to the overall probabilities of success and failure.

(A4) *Stochastic dominance*: For any two prospects $X = (p, x_1; 1 - p, x_2)$ and $Y = (q, x_1; 1 - q, x_2)$, if $x_1 \geq x_2$ and $p > q$, then $X \succcurlyeq Y$.

To characterize the discontinuity at the aspiration level, we introduce a weakened definition of continuity in outcomes. This is done in axioms A5 and A6. The following well-known observation serves as a preparation.

Observation 1. *If the utility function u is nondecreasing, then it can be discontinuous at countably many points at most.*

(A5) *Lower continuity outside zero*: The utility function u is said to be lower continuous at α if for every sequence $\{\alpha^j\}$ converging from below to $\alpha, u(\alpha^j)$ converges from below to $u(\alpha)$, with $\alpha \neq 0$.
(A6) *Upper continuity outside zero*: The utility function u is said to be upper continuous at α if for every sequence $\{\alpha^j\}$ converging from above to $\alpha, u(\alpha^j)$ converges from above to $u(\alpha)$, with $\alpha \neq 0$.

As an intermediate step, the following lemma shows that the latter two axioms characterize the discontinuity of utility at zero. We consider prospects of the following type: $(0.5, x; 0.5, \alpha)$. We often write it simply as $(x; \alpha)$.

Lemma 1. *Lower and upper continuity of utility. (1) For all $\alpha \neq 0$, $\beta \neq 0$, there exist sequences $\{\alpha^j\}$ and $\{\beta^j\}$ converging to α and β from below/above, respectively, with $(\alpha^j; \beta) \sim (\alpha; \beta^j)$ for all j if and only if utility is lower/upper continuous except possibly at zero. (2) Assume that lower/upper continuity holds everywhere outside zero.*

(a) Then, for $\alpha = 0$, and for all $\beta \neq 0$, there exist sequences $\{\alpha^j\}$, $\{\beta^j\} \uparrow$ and $\{\beta^j\} \downarrow$ converging to α and β from below/above, respectively, with $(\alpha^j; \beta) \sim (\alpha; \beta^j)$ for all j if and only if utility is lower/upper continuous at zero. Then utility is lower/upper continuous also at zero ($\mu = \lambda = 0$).
(b) If such a β and the sequences $\{\beta^j\} \uparrow$ or $\{\beta^j\} \downarrow$ do not exist s.t (a) above holds, then utility is discontinuous at zero (and the reverse also holds), and Equation (2) holds with $\mu, \lambda \geq 0$ and at least one strictly larger than zero.

Proof. See the Appendix.

Axioms (A1)–(A6) will be used to characterize the shape of the function V in Equation (2). In addition, we can characterize the relative importance of the overall probabilities of success and failure. The following two conditions will prove to provide necessary and sufficient conditions to determine the relative weights μ and λ. Consider the outcomes $x, y > 0$.

Condition 1. $\exists\, x$ *s.t.* $\forall y (0.5, x; 0.5, -y) \preccurlyeq 0$.

Hence, under this condition there is at least one outcome x for which the subject will always prefer the sure outcome that gives zero to the risky prospect $(0.5, x; 0.5, -y)$. The following condition reverses the role of x and $-y$.

Condition 2. $\exists\, y$ *s.t.* $\forall x (0.5, x; 0.5, -y) \succcurlyeq 0$.

In the next theorem we formally state the central result. It essentially tells that Axioms A1 to A6 are equivalent to representing preferences by V, and it provides the conditions under which the weight μ given to the overall probability of success is greater, equal, or smaller than the weight λ given to the overall probability of failure.

THEOREM 1. *The preference relation $\succcurlyeq$ satisfies Axioms A1–A6 if and only if there exist a continuous nondecreasing $u: R \to R$ and $\mu, \lambda \in R^+$ such that $X \mapsto V(X) = \sum_{j=1}^{n} p_j u(x_j) + \mu P(x^+) - \lambda P(x^-)$ represents preferences. Furthermore:*

(a) if lower and upper continuity hold at zero, then $\mu = \lambda = 0$; if only upper continuity holds, then $\lambda > 0$ and $\mu = 0$; if only lower continuity holds, then $\mu > 0$ and $\lambda = 0$.
(b) if condition 1 holds, then $\lambda > \mu$, if condition 2 holds, then $\lambda < \mu$, if neither condition 1 nor 2 holds, then $\lambda = \mu$.
(c) u is unique up to location and a positive scaling factor, and μ, λ are unique up to the same scaling factor as u.

PROOF. See the Appendix.

This behavioral foundation can be extended to the case where the aspiration level is not necessarily zero. Furthermore, Theorem 1 allows us to characterize an endogenous aspiration level, i.e., to infer an endogenous aspiration level from preferences: Whenever there is a jump in the utility function, there may be an aspiration level. Besides the aspiration level, an exogenous reference point could be included without much complication. The utility function would be defined in terms of gains and losses from the reference point and then definitions and axioms for the aspiration and the overall probabilities will still apply. To keep the notation simple and the model focused, we do not elaborate on this and we leave it for further research. The prospects discussed in the empirical part fit the case where the reference point coincides with the aspiration level.

3.4. *Comments on Related Mathematical Results.* Theorem 1 characterizes a functional V representing preferences. V is equivalent to the mathematical expectation of a prospect with respect to a discontinuous von Neumann–Morgenstern utility function. Our model could also be characterized by imposing Foldes' (1972) and Grandmont's (1972) weak continuity at all but one outcome. We prefer an alternative approach to continuity, one that is simpler in not involving variation of probability, is more general and flexible, and has a higher intuitive content.

Other approaches are present in the mathematical literature. Arzac and Bawa (1977) and Markowitz (1959) characterized a decision maker maximizing preferences over expected value of wealth and probability of ruin. They derived equilibrium conditions in a capital asset price model. Their functional is somewhat more restrictive because of the assumption of linear utility (Shefrin and Statman, 2000) and can be considered a special case of Equation (2). Meginniss (1976) presented axioms to represent preferences through a combination of expected utility and entropy (a measure of uncertainty) for a prospect. Castagnoli and LiCalzi (1996) reinterpreted EU without a cardinal utility function: The EU of a lottery is the probability that the lottery outperforms a given independent benchmark-lottery. The probability of surpassing the benchmark replaces utility, whereas in our model it comes in addition to utility. Our approach has the advantage that we can, for example, account for the data of Payne (2005). Bordley and LiCalzi (2000) showed the equivalence between Savage's (1972) maximization of subjective expected utility and maximization of the probability of meeting an uncertain target, thus extending Castagnoli and LiCalzi (1996) to the domain of uncertainty.

3.5. *Estimating μ and λ.* An important advantage of our model is that the empirical elicitation of μ and λ is straightforward. The first step is to fix an outcome $x > 0$, and then to elicit the probability p that makes the subject indifferent between the prospect $(p, 1; 1 - p, 0)$ and the sure outcome x. If we normalize the utility such that $u(0) = 0, u(1) = 1$, then, as x converges to zero from above, p converges to $\mu/(1 + \mu)$.

We can use a similar procedure to estimate λ. We now elicit the probability p that makes the subject indifferent between the prospect $(p, -1; 1 - p, 0)$ and the sure outcome $-x$. Then, as $-x$ converges to zero from below, p converges to $\lambda/(\lambda - u(-1))$. It is necessary to elicit $u(-1)$, because we can normalize the utility of two outcomes only. To do so, one can elicit the probability q that makes the subject indifferent between the prospect $(q, 1; 1 - q, -1)$ and the sure outcome 0. Substituting for the resulting $u(-1)$ we then get that, as $-x$ converges to zero from below, p converges to $\frac{\lambda}{1+\mu}\frac{1-q}{q}$.

This elicitation works as long as the aspiration level is known, but we conjecture that in experimental settings it is safe to assume it equal to zero (Payne, 2005). Many mixed prospects have the property that $P(x^+) = 1 - P(x^-)$. In that case, the combined weight of μ and λ is observed. These can, however, possibly still be separated by the use of duplex prospects (also known in the literature as duplex gambles). *Duplex prospects* are compound prospects, consisting of two "pure prospects": one with a positive outcome and a zero outcome and another with a negative outcome and a zero outcome (Slovic and Lichtenstein, 1968; Payne and Braunstein, 1971). Slovic and Lichtenstein (1968) argue that the weights attached to success and failure that are derived from duplex prospects also predict choices in prospects with $P(x^+) = 1 - P(x^-)$. Some caution is necessary, as several authors find that subjects react differently to mixed prospects (like duplex prospects) than to pure prospects (see, for instance, Luce, 2000).

Although not exclusively, the elicitation of μ and λ might be of particular importance for the interpretation of subjects' choices made in laboratory experiments. The evidence and the results on risk attitude may be confused by the presence of a subjective aspiration level, the overall probabilities of success and failure, and their respective weights. The next section elaborates on this.

4. DISCUSSION AND PREDICTIONS

The prospect evaluation $X \mapsto \sum_{j=1}^{n} p_j u(x_j) + \mu P(x^+) - \lambda P(x^-)$ gives rise to a rich body of behavioral predictions. To illustrate this point, we now present several of them. Some implications of Equation (2) are confirmed by previous empirical and experimental studies; others lend themselves to be tested.

4.1. *A Discontinuity around the Aspiration Level.* The most notable feature of our model is the discontinuity in outcomes. Continuity is not satisfied

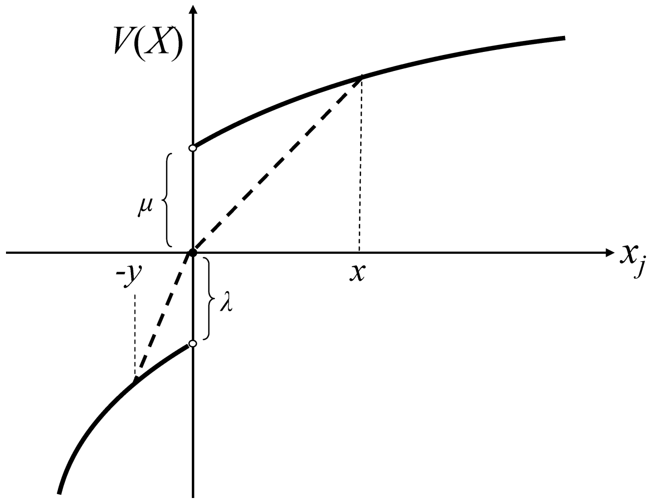

FIGURE 1

THE VALUATION OF PROSPECT X WITH DISCONTINUITIES AROUND THE ASPIRATION LEVEL

at the aspiration level. When an outcome that is at the aspiration level is increased/decreased to slightly above/below the aspiration level, it receives an extra weight μ (/$-$ λ). An arbitrarily small increase/decrease in an outcome x_j can therefore lead to a discrete jump in the valuation of a specific prospect (see the solid lines in Figure 1).[2]

The discontinuity is a feature that is still to be tested. Jeffrey and Larrick (2002) allude, nevertheless, to its presence in goal-related experiments. In the health domain, Stalmeier et al. (2005) provide empirical data showing evidence for discontinuity of utility around death. Fishburn (1977, p. 122) reports that the behavior of subjects exhibiting a particular appreciation for the outcomes above the aspiration level is often found in the literature and it can be represented by a "pronounced change in the shape of their utility function." Swalm (1966) reports similar evidence and shows that this point is approximately zero (break-even point on a project). Holthausen (1981), when reviewing the literature, reports similar findings. Mezias (1988) presents evidence of the effect of an aspiration level in the pricing of securities in the stock market that alludes to a jump in the valuation of securities in correspondence with a fixed and predetermined aspiration return. Further research in the finance domain is desirable to detect for which financial investments this discontinuity is more pronounced.

4.2. *Aspiration Levels as Extreme Loss Aversion.* In the case of EU the utility function for final wealth is commonly assumed to be differentiable. Under loss

[2]Empirical evidence suggests that utility for losses can be convex. We conform to the convention of a utility function that is concave everywhere, in the tradition of EU, but our model is not restricted to the latter case.

aversion the utility function for gains has a different slope than the utility for losses: Loss aversion, entailing a drastic change at zero, is characterized by a (non-differentiable) kink in the utility function. A standard definition of loss aversion (Wakker and Tversky, 1993) for a differentiable function is: $x > 0 : u'(x) < u'(-x)$. Köbberling and Wakker (2005) proposed a behaviorally founded index of loss aversion. The case becomes even more extreme if we assume a power utility, as in Kahneman and Tversky (1979) and Tversky and Kahneman (1992). Then the first derivative of utility at zero is infinite, so that utility is extremely steep at zero. The extreme version of loss aversion that we present in this article is a natural next step: The discontinuity in the utility function reinforces loss aversion. Our model is the first to characterize loss aversion as discontinuous utility. The psychology of loss aversion not only applies in the domain of failure, but also in the domain of success as we take the limit from above: $\lim_{x_j \downarrow 0} V(X)$. We still call it loss aversion because the underlying psychology of the phenomenon is the same as in the domain of failure.[3] Empirical evidence for loss aversion may be partly driven by a sensitivity to the overall probabilities of success and failure in addition to a bigger sensitivity to loss outcomes.

4.3. *Risk Attitude and μ and λ.* The functional V evaluating prospects is discontinuous at the aspiration level for strictly positive values of μ and λ. The corresponding jumps determine the risk-attitude that subjects have for prospects close to the aspiration level (taken to be the zero outcome here). For two-outcome prospects that involve the aspiration level, we now show that subjects are always *risk-averse from above* and *risk-seeking from below*. This result holds independent of the shape of the function $u(\cdot)$. The dashed lines in Figure 1 illustrate this fact. They represent the expected values of prospects of the form $(p, x; 1-p, 0)$ and $(p, -y; 1-p, 0)$.

Consider the outcomes $x, y > 0$ and let z_i be the degenerate prospect that gives the expected value of prospect X_i, $E(X_i)$, with certainty.

PROPOSITION 1. *Suppose preferences can be represented by V. Then, (a) if $\mu > 0$, subjects are risk-averse from above: $\forall\ p\ \exists\ \delta_1 > 0$ such that $X_1 = (p, x; 1-p, 0) \prec z_1\ \forall x < \delta_1$, and (b) if $\lambda > 0$, subjects are risk-seeking from below: $\forall p\ \exists\ \delta_2 > 0$ s.t. $X_2 = (p, -y; 1-p, 0) \succ z_2\ \forall y < \delta_2$.*

The type of prospects for which this result holds is often used in prospect theory to establish that subjects are risk-averse for gains and risk-seeking for losses. Provided that the aspiration level coincides with the reference points, these choices are also consistent with subjects paying attention to the overall probabilities of success and failure. Our model and prospect theory differ strongly in predictions however, for prospects not involving the zero outcome. For example, if prospect theory is correct in that subjects are risk-averse for gains, then subjects should also be risk-averse if a constant c is added to all outcomes in X_1 and $z_1 = E(X_1)$, i.e., $X'_1 = (p, x+c; 1-p, c) \prec z'_1 = z_1 + c$. If instead subjects pay attention to the

[3] An alternative definition of loss aversion (Kahneman and Tversky, 1979) is $0 \succcurlyeq (0.5, x; 0.5, -x)$. In that case, a larger λ reinforces risk aversion, but a larger μ reduces it.

overall probabilities of success and failure, then even if $u(\cdot)$ is convex everywhere, subjects might also prefer the riskless option z_1 to X_1, but they would prefer the risky prospect after the shift $X_1' \succ z_1'$. Lopes and Oden (1999) find indeed that the risky prospect in the gain domain (their prospect "LS") becomes more attractive to subjects after such a shift. A more direct test involving only two-outcome prospects is desirable. Relatedly, Fennema and van Assen (1999) claim that the results from several studies that find risk seeking in the domain of losses, do not necessarily imply a convex utility function.

Incidentally, the above result also implies that subjects may be simultaneously engaged in risk-averse behavior and risk-seeking behavior, for instance buying insurance and lottery tickets. Friedman and Savage (1948) explain such behavior by considering local convexities in the utility function. This turned out to be inconsistent with empirical data and raised criticism (Markowitz, 1952; Yaari, 1965; Thaler and Ziemba, 1988; Quiggin, 1991). The behavior can also be explained by subjects paying attention to the overall probabilities of success and failure. For instance, from Equation (2) it appears that a subject with a utility function $u(\cdot)$ that is concave everywhere might be willing to buy lottery tickets if this provides an opportunity to reach the aspiration levels and if μ and/or λ are sufficiently large. Indeed, Hirshleifer and Riley (1992, p. 28) advance that the behavior is caused by a "threshold phenomenon," i.e., "a single discrete step to a higher utility level," which we show is a direct consequence of the aspiration level.

4.4. *The Mean-Variance Trade-Off.* In standard portfolio theory, it is argued that managers trade off higher returns against higher variances (Markowitz, 1952). Risk is usually identified with variance. Hence, among portfolios with the same mean, the one with the lowest variance is to be preferred, and vice versa. More specifically, it is often assumed that a prospect X is evaluated as

$$E(X) - k\sigma^2, \tag{5}$$

where σ^2 is the variance, and the constant k a measure of risk aversion. A closely related version that is sometimes proposed replaces the variance by a semivariance. The semivariance is calculated with respect to a target outcome rather than the expected value and is restricted to outcomes below the target (Eeckhoudt and Gollier, 1995).

This mean-variance trade-off is not implied by Equation (2). A portfolio with the same mean but a higher variance may give a higher probability of reaching the aspiration level. Conversely, lowering the variance of a given portfolio (keeping the mean constant) may result in a lower valuation if some probability mass is shifted away from above the aspiration level to below (Hakansson and Ziemba, 1995, p. 80). The same applies to the version based on semivariance.

Such a phenomenon was found in the experimental results of Payne et al. (1980, 1981). Recall from Section 2.2 that many subjects prefer prospect $X'' = (0.5, 14; 0.1, -30; 0.4, -85)$ to $Y'' = (0.3, -20; 0.5, -30; .2, -45)$. The prospects have the same mean but the variance of X'' is higher. At the same time, X'' gives a higher overall probability of success. This choice cannot be explained by Equation (5) irrespective of the degree of risk-aversion k. By contrast, it can be explained by Equation (2) if the weight given to the overall probability of success is sufficiently high.

More generally, it can be expected that with an aspiration level, the portfolio choice will not be mean-variance efficient. Subjects with a high aspiration level will tend to construct portfolios carrying more risks but with a higher overall probability of success. Although a thorough portfolio analysis is out of the scope of this article, it is interesting to note that Shefrin and Statman (2000) also use an approach in which the subject cares about the probability of failure. They find indeed that efficient portfolios have few risky elements for subjects with a low aspiration level and more risky elements for subjects with a high aspiration level.

5. CONCLUSIONS AND FUTURE

There is evidence that subjects are particularly sensitive to success and failure. There is a reluctance to take prospects that may entail the possibility of failure measured with respect to an aspiration. This reluctance cannot be fully explained by risk aversion. A great deal of evidence describes that subjects perceive prospects in terms of the probability of being above or below an aspiration level. It seems that very small changes in outcomes that increase the overall probabilities of success and failure can lead to significant changes in preference over prospects (Payne, 2005). We have discussed and investigated the psychological intuition underlying the aspiration levels, presented a model, characterized the importance of the overall probabilities of success and failure, and explored some of the decision theoretical implications.

Expected utility has since its emergence been a classical benchmark in decision under risk and decision analysis. Parsimony, elegance, and broad applicability are its strong points. In this article we present a model of decision under risk that includes the overall probabilities of success and failure into expected utility. Building on the psychologically appealing probabilities of success and failure, we then come up with a model that is equivalent to a discontinuous EU. The model allows for a number of predictions that can better describe the behavior of individuals. However, we do not aim at explaining all the possible behavioral irregularities in decision making under risk: Other complicating, but behaviorally relevant, factors (i.e., probability transformation) have been abstracted from for the sake of conceptual clarity. We have shown that insights into the significance of an aspiration level deriving form cognitive psychology can be integrated in decision theory.

One first obvious extension is to study decision under uncertainty. In this article we have assumed that probabilities are known to the decision maker. In reality, this may often not be the case and it is interesting to see how agents deal with such uncertainty. We conjecture that the importance of the overall probabilities of success and failure will be enhanced by such uncertainties (Fellner, 1961; Kahneman and Tversky, 1979; Kahn and Sarin, 1988; Weber, 1994). Another natural extension is to characterize a model where the aspiration level is combined with elements from prospect theory such as probability transformations (Tversky and

Kahneman, 1992). The interest there is to investigate how the model changes once the probability transformation is considered. Then, both the model with expected utility and prospect theory can be generalized to any finite number of aspiration levels. Finally, we conjecture that the more complex the problem is to a subject, the more attention the overall probabilities of success and failure receive (a similar conjecture is made by Payne, 2005). If the focus on the aspiration level and the overall probabilities of success and failure is a heuristic used to simplify problems, one expects that the weights attached to these overall probabilities increase in importance in more complex settings. One way to test this is to vary the complexity of the prospects by modifying the number of outcomes. The elicited weights μ and λ should be higher for more complex prospects.

APPENDIX (PROOFS)

PROOF OF LEMMA 1. Necessity of the preference condition easily follows. Sufficiency: By observation 1, one can take a β where utility is continuous. Then lower/upper continuity of utility at each α follows by simple substitution of EU. Part *a* and *b* similar to above. ■

PROOF OF THEOREM 1. The implication that Equation (2) satisfies A1–A6 is straightforward. For the implication in the opposite direction any of the aforementioned EU axiomatization provides a constructive proof for the existence of a utility function $\tilde{u}$ (unique up to location and a positive scaling factor) such that the EU form holds true, i.e., $X \mapsto \sum_{j=1}^{n} p_j \tilde{u}(x_j)$ represents preferences. By A5 and A6 and by Lemma 1, $\tilde{u}$ is lower/upper continuous everywhere outside zero and, thus, $u(0) - \lim_{x\uparrow 0} \tilde{u}(x) = \lambda \geq 0$, and $u(0) - \lim_{x\downarrow 0} \tilde{u}(x) = -\mu \leq 0$ with at least one of the inequalities being strict. By stochastic dominance μ and λ are nonnegative. Define $u = \tilde{u} - \mu$ for positive outcomes and $u = \tilde{u} + \lambda$ for negative outcomes.

Part *a* is a special case of the above and its proof is similar. To prove part *b* of the theorem,

$\Rightarrow$ Suppose condition 1 holds: $\exists x$ s.t. $\forall y(.5, x; .5, -y) \preccurlyeq 0$. This implies

$$.5u(x) + .5u(-y) \leq \lambda - \mu. \tag{A.1}$$

Let $u(x) = c$. By continuity and nondrecreasingness of $u(\cdot)$ and with $u(0) = 0$, we know that for $\varepsilon, \delta > 0, \forall \varepsilon$ there exists a δ s.t. $-u(-y) < \varepsilon$ for $y < \delta$. In particular, if $c = \varepsilon$, then $-u(-y) < u(x)$ for $y < \delta$. Then (A.1) above can only be satisfied for $\lambda > \mu$.

$\Leftarrow$ Let $\lambda > \mu$. Then the RHS of (A.1) is strictly positive. Note that the least upper bound of the LHS is given by $.5u(x)$. Hence, by continuity of $u(x)$, for small enough values of x the LHS never exceeds the RHS for any y. This implies condition 1 is satisfied.

Similar arguments can be used to show that condition 2 is satisfied if and only if $\lambda < \mu$. It then follows in a straightforward manner that if neither condition 1 nor condition 2 is satisfied, it must be the case that $\lambda = \mu$. ■

PROOF OF PROPOSITION 1. Part (*a*). The utility of the sure outcome z is given by $u(px) + \mu$. The utility of the prospect is given by $pu(x) + p\mu$. The risky prospect is preferred if $pu(x) - u(px) \geq (1 - p)\mu$. As $x \to 0, pu(x) - u(px) \to 0$. Since the RHS is negative, the inequality is always satisfied for sufficiently small x. Part (*b*) can be proved along similar lines. ■

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