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Reconciling support theory and the book-making principle

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Abstract Support theory postulates that an individual's probability judgment for a particular event depends on the description of that event. We analyze decisions based on such a premise and demonstrate the theory's incompatibility with popular models of choice under uncertainty. In particular, we show how support theory's subjective probabilities are at odds with multi-prior beliefs in addition to additive and nonadditive probabilities. We propose a behavioral relaxation of a well-known consistency argument—the book-making principle, in order to accommodate such description-dependent subjective probabilities. As a consequence, we provide a characterization of a set of decisions where the underlying probability judgments follow from support theory. This result offers a unique way for using description-dependent subjective probabilities as consistent inputs for decision analysis and can aid the design of elicitation procedures.

Keywords Book-making principle · Support theory · Nonexpected utility

JEL Classification D81

Support theory (Tversky and Koehler 1994; Rottenstreich and Tversky 1997) is a behaviorally founded theory for probability judgments. It proposes that people's subjective probability for a particular event depends on the accompanying description for that event. As an example, an individual's judged probability that a roll of a six-sided die will result in "a prime number" maybe different than that the outcome will be "either 2 or 3 or 5". Based on the provided description for an event,

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the theory argues that people form a degree of support, which can be considered a summary measure for the evidence in favor of that event. This evaluation process is a fundamental stage that leads to the assignment of subjective probabilities (Fox and Clemen 2005).

Probability judgments derived according to support theory have been used as inputs in decision models (Fox and Tversky 1998; Fox 1999) and have been considered to be at the basis of the decision weights in rank-dependent models of choice: rank-dependent utility, Choquet expected utility, and prospect theory (Kahneman and Tversky 1979; Quiggin 1982; Schmeidler 1989; Tversky and Kahneman 1992). Support theory has received attention in decision theory (Wu and Gonzalez 1999), psychology (Fox 1999; Macchi et al. 1999), and management (Bearden et al. 2006; Clemen and Ulu 2007; Fox and Clemen 2005; Sonnemann et al. 2008). The theory has been tested and its predictions have been confirmed in numerous studies (Brenner and Koehler 1999; Brenner and Rottenstreich 1999; Brenner et al. 2002; Fox et al. 1996; Idson et al. 2001). Fox and Rottenstreich (2003) considered an extension of support theory to include partition dependence (Fox and Clemen 2005). In this paper, we refer to the original version of support theory (Tversky and Koehler 1994; Rottenstreich and Tversky 1997) and leave the extensions (Fox and Rottenstreich 2003) for future studies.

The book-making principle (de Finetti 1931) is a famous argument in decision under uncertainty (Bunn 1984; Burks 1977; Edwards et al. 2007; Fishburn 1989; Nau 1995; Smith and von Winterfeldt 2004; von Winterfeldt 1989). It contends that choices made separately should not result in a combination that will guarantee an undesirable outcome, i.e., a sure loss. For example, consider a gambler betting on the outcome of a coin toss who prefers to bet on “heads” than not betting at all and prefers to bet on “tails” than not betting at all. Suppose she takes the two preferred bets and loses money regardless of the outcome of the coin toss. Then this is an unpalatable case because it contradicts the reasonable idea that a number of good bets should still be good when taken together. The book-making principle simply requires that no such cases exist. This is equivalent to a no arbitrage condition (Hull 2005).

The book-making principle states a natural condition that turns out to imply the existence of coherent subjective probabilities and justifies a model of choice based on them. De Finetti’s idea served as a point of departure for Savage’s (1954) theory of subjective expected utility, and it is, hence, at the basis of Bayesianism. The principle has been modified to accommodate nonadditive probabilities (Diecidue and Wakker 2002) and multiple-prior beliefs (Diecidue and Maccheroni 2003) by restricting the set of gambles considered. These modifications provide characterizations for rank-dependent and multiple-priors models of choice respectively.

This paper has two primary objectives. The first is to systematically show, from a decision theoretic viewpoint, how subjective probabilities according to support theory are incompatible with decision models based not only on additive and nonadditive probabilities, but also multiple-prior beliefs. We do so by demonstrating violations of the book-making principle in modified forms. Our second objective is to propose a behavioral version of the book-making principle that allows probability judgments to be description dependent. This enables us to characterize a set of consistent decisions where the underlying subjective probabilities follow from support theory. The result forms a basis for improvement of the probability elicitation process and specifies conditions which prevent sure losses in decision-making despite description-dependent beliefs.

The paper can be summarized as follows. We recap the essential mathematical foundation of support theory in Section 1. In Section 2, we introduce a class of gambles that accounts for the descriptions of events and we generalize existing forms of the book-making principle. We then illustrate in Section 3, the inconsistencies of subjective probabilities according to support theory with respect to standard models of choice. Violations of strict monotonicity are at the basis of such incoherence and vulnerabilities to losses in decision-making. We propose a behavioral version of the book-making principle in Section 4 to accommodate probability judgments according to support theory. In effect, we characterize a set of consistent decisions where the underlying beliefs are description dependent. Such decisions are closely related to the multiple-priors model of choice. The result can also help minimize biases in the probability elicitation process by controlling for the descriptions of events in gambles. In Section 5, we conclude and suggest possible further research directions.

1 Support theory

Support theory is a descriptive representation of subjective probability originally proposed by Tversky and Koehler (1994) and later revised by Rottenstreich and Tversky (1997). The theory is a popular descriptive alternative to Bayesian probability (Idson et al. 2001). It is mathematically founded and psychologically appealing. The main assumption of support theory is the property of *non-extensionality,* which simply states that an individual's probability judgment for a particular event is dependent on the provided description for that event (Fischhoff et al. 1978; Humphrey 1995; Starmer and Sugden 1993). In general, support theory postulates that the more explicitly an event is described with respect to its composition, the more support it receives and, thus, the higher the judged probability for that event. We now present the essential mathematical formulation of support theory (Tversky and Koehler 1994; Rottenstreich and Tversky 1997) that is relevant to our paper.

Support theory is based on the distinction between *events*, which are subsets of the *state space* T, and *hypotheses* or *descriptions* of the events, which are elements of the *set of hypotheses* H. Support theory assumes that each description $A \in H$ corresponds to a unique event $A' \subseteq T$. A description A is *elementary* if $A' \in T$. Descriptions A, $B \in H$ are *exclusive* if $A' \cap B' = \emptyset$. The assumption of non-extensionality implies that different descriptions A and B can map to the same event, i.e., $A' = B'$. In this case, the two descriptions, A and B, are considered to be *coextensional*.

Support theory proposes a ratio scale s, assigning to each description, a non-negative real number such that for any pair of exclusive descriptions A, $B \in H$,

$$P(A, B) = \frac{s(A)}{s(A) + s(B)}. \tag{1}$$

$P(A, B)$ is the judged probability that description A rather than description B is true and its equivalent counterpart in classical probability is $P\{A'|A' \cup B'\}$. The ratio scale s can be interpreted as the degree of support or the strength of evidence for a particular description that could be based on objective data, subjective impression, or personal reasons (Tversky and Koehler 1994).

A description of an event according to support theory can be categorized as either an *implicit hypothesis* or an *explicit disjunction*. An implicit hypothesis such as $A \in H$ is essentially a holistic description of an event. An explicit disjunction denoted by $(A_1 \vee A_2) \in H$ for example, is in contrast a description of an event as decomposition of two or more of its exclusive and exhaustive subsets. The following condition holds for A and $(A_1 \vee A_2)$ that are coextensional, i.e., $A'=(A_1 \vee A_2)'$.

$$\underline{\text{Implicit and Explicit Subadditivity}} \quad s(A) \leq s(A_1 \vee A_2) \leq s(A_1) + s(A_2). \tag{2}$$

The first inequality in Expression 2, known as *implicit subadditivity*, asserts that the support for an implicit hypothesis A is less than that for its coextensional explicit disjunction $(A_1 \vee A_2)$. The second relation, which was originally an equality in Tversky and Koehler (1994), is the *explicit subadditivity* proposed by Rottenstreich and Tversky (1997). This inequality states that the support for an explicit disjunction $(A_1 \vee A_2)$ is still less than the sum of supports for the exclusive and exhaustive subsets $s(A_1)+s(A_2)$.[1] Implicit subadditivity is due to the fact that people do not "unpack" an implicit hypothesis (Tversky and Koehler 1994). Unpacking is the process of breaking down an event into its exclusive components and adding up their supports. Explicit subadditivity is the result of people tending to "repack" an explicit disjunction (Rottenstreich and Tversky 1997). Repacking is the reverse process of combining exclusive components of an event into its entirety and considering the associated support. The consequence of the two subadditivities of supports in Expression 2 leads to subadditivities of judged probabilities as follows:

$$\underline{\text{Odd inequality}} \quad R(A,B) \leq R(A_1 \vee A_2, B) \leq R(A_1, A_2 \vee B) + R(A_2, A_1 \vee B), \tag{3}$$

where $R(A,B) = \frac{P(A,B)}{P(B,A)} = \frac{s(A)}{s(B)}$ represents the probability ratio.

While the focus of support theory is on numerical judgments of probability, there exist empirical evidence that probability judgments as assumed by support theory can affect people's decisions and evaluations of uncertain prospects in insurance (Johnson et al. 1993), medical decisions (Redelmeier et al. 1995), and sports gambling (Ayton 1997). In a series of experiments, Fox and Tversky (1998) also find that different descriptions of an event may affect a person's willingness to act and that people are willing to pay more for a prospect when its components are evaluated

[1] The main results in this paper hold for both subadditivity and superadditivity of supports. Evidence for subadditivity is presented in Tversky and Koehler (1994), Fox et al. (1996), Rottenstreich and Tversky (1997), Fox and Tversky (1998), Brenner and Koehler (1999), and Idson et al. (2001), while Sloman et al. (2004) and Hadjichristidis and Summers (2006) show violations of subadditivity. Clemen and Ulu (2007) provide evidence for additivity of the support function for continuous variables.

separately. These observations have decision analysis implications. That is, betting on prospects whose subjective probabilities for the payoffs are derived from support theory may lead to sure losses. It is to this implication that we now turn.

2 Subjective probability and the book-making principle

In this section, we propose generalizations of three existing forms of a well-known consistency argument: the *book-making principle* (BMP, de Finetti 1931), the *comonotonic BMP* (Diecidue and Wakker 2002), and the *affine BMP* (Diecidue and Maccheroni 2003). Because probability judgments according to support theory are description-dependent, we first need to augment the standard framework and introduce *extended gambles*, which incorporate the descriptions of events. These new objects are at the basis of our new generalized principles. The original three forms of the BMP become special cases in our construction, but they each have important implications with respect to subjective probabilities.

First, let event $F_k' \subseteq T$ correspond to a description $F_k \in H$, which may not be unique. Assume there are r mutually exclusive and exhaustive events, then $\Phi = \{(F_1', F_1), ..., (F_r', F_r)\}$ represents the event-description correspondence for the entire state space. According to support theory, there may be more than one possible correspondence for a particular partition of the state space $\{F_1', ..., F_r'\}$ (see Fig. 1).

Let $t_1, ..., t_n$ be the n possible states of nature where one and only one of the states will be true. An *extended gamble* $f_\Phi = (f_\Phi(t_1), ..., f_\Phi(t_n))$ is a mapping from the state space to the *outcomes* where $\Phi = \{(F_1', F_1), ..., (F_r', F_r)\}$ is the event-description correspondence. Extended gamble f_Φ will generate a monetary value $f_\Phi(t_i)$ if t_i is the true state of nature. Assume another extended gamble $g_\Gamma = (g_\Gamma(t_1), ..., g_\Gamma(t_n))$ defined on w mutually exclusive and exhaustive events where $\Gamma = \{(G_1', G_1), ..., (G_w', G_w)\}$ is its event-description correspondence. The two extended gambles f_Φ and g_Γ are *descriptively equivalent* if they are based on the same set of events with identical descriptions, i.e., $\Phi = \Gamma$. In addition, extended gambles are considered to be *descriptively invariant* if there exists only one unique description for each possible

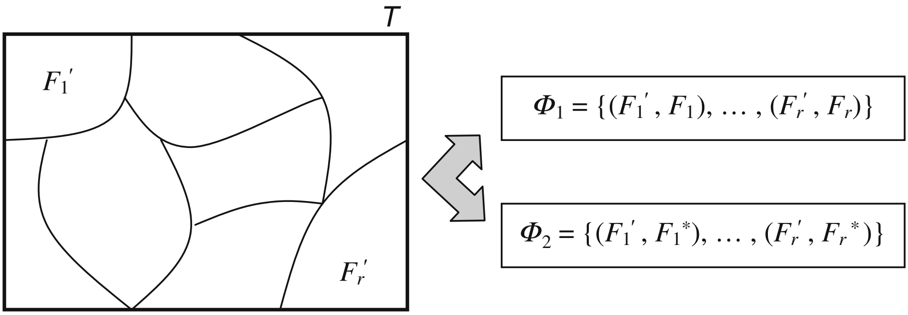

Fig. 1 Partition of the state space T into events $F_1', ..., F_r'$, showing two alternative sets of descriptions given by correspondences Φ_1 and Φ_2

event in the state space. The descriptively invariant extended gambles we consider in this paper will be restricted to only (*standard*) *gambles*. Specifically, we will drop the description notation as in standard gamble f and assume that each event is described by a listing of all the states composing that event. Two extended gambles are *monetary equivalent* if they yield equal outcomes in each and every state of nature. To summarize, extended gambles are n-tuples of contingent outcomes depending only on the state of nature but are also supplemented by an event-description correspondence.

The preference relation $\succeq$ over extended gambles is a *weak order* if it is complete and transitive. Strict preference and indifference are denoted $\succ$ and ~ respectively. A *fair price* for an extended gamble f_Φ is an outcome $\xi = \xi(f_\Phi) \in \Re$ such that $\xi \sim f_\Phi$. Note that this description variant setup allows two monetary equivalent extended gambles f_{Φ_1} and f_{Φ_2} to have different fair prices if their descriptions of events are distinct, i.e., $\Phi_1 \neq \Phi_2$. Finally, a function V, evaluating extended gambles, *represents* the preference relation $\succeq$ if $V(f_\Phi) \geq V(g_\Gamma)$ if and only if $f_\Phi \succeq g_\Gamma$.

Now, consider a set of preferences over extended gambles in m decision situations (where each decision situation involves two extended gambles): $f^1_{\Phi_1} \succeq g^1_{\Gamma_1}, \ldots, f^m_{\Phi_m} \succeq g^m_{\Gamma_m}$. The decision-maker weakly prefers $f^j_{\Phi_j}$ to $g^j_{\Gamma_j}$ in each decision situation j=1, ..., m. Consider 'taking together' these gambles in the sense of state-wise addition and suppose that $\sum_{j=1}^{m} f^j_{\Phi_j}(t) < \sum_{j=1}^{m} g^j_{\Gamma_j}(t)$ for all states t. An *extended* (*Dutch*) *book* is a set of preferences over m decision situations that when taken together yields a loss for each state of nature, i.e., the decision maker is worse off in each state of nature. This is not desirable and therefore the *extended book-making principle* (extended BMP) requires that no such extended book exists.[2] The principle is based on the idea that a number of good decisions, when taken together, should still be good. An extended book is a violation of the extended BMP.

De Finetti's original BMP can be considered a special case of our proposed extended BMP where all the gambles are standard. A *book* is a descriptively invariant counterpart to an extended book. The nonexistence of a book, together with some natural conditions is equivalent to the existence of unique subjective probabilities and the evaluation of a standard gamble f given by $V(f)$, the subjective expected value of f (de Finetti 1931). That is, $V(f)$ is equal to the weighted sum of outcomes by subjective probabilities.

The *comonotonic book-making principle* (Diecidue and Wakker 2002) is the second and more restrictive form of the BMP, which we generalize to apply to extended gambles. The principle restricts the set of gambles considered to be comonotonic. A set of extended gambles is *comonotonic* if for each pair of elements f_Φ, g_Γ there do not exist states t_i, t_j such that $f_\Phi(t_i) > f_\Phi(t_j)$ and

[2] The extended book-making principle is based on strict monotonicity and additivity. Strict monotonicity holds if $f_\Phi(s) > g_\Gamma(s)$, $\forall s$, then $f_\Phi \succ g_\Gamma$. Additivity entails that if $f_\Phi \succeq g_\Gamma$, then $f_\Phi + h_H \succeq g_\Gamma + h_H$. For an example of the additive operation of extended gambles, consider $f_\Phi = (f_\Phi(t_1), \ldots, f_\Phi(t_n))$ and $g_\Gamma = (g_\Gamma(t_1), \ldots, g_\Gamma(t_n))$ where $\Phi = \{(F_1', F_1), \ldots, (F_r', F_r)\}$ and $\Gamma = \{(G_1', G_1), \ldots, (G_w', G_w)\}$. Then $f_\Phi + g_\Gamma = h_H = (f_\Phi(t_1) + g_\Gamma(t_1), \ldots, f_\Phi(t_n) + g_\Gamma(t_n))$. The new event-description correspondence $H = \{(H_1', H_1), \ldots, (H_z', H_z)\}$ is based on a new partition with z mutually exclusive and exhaustive events arising from intersections of $\{F_1', \ldots, F_r'\}$ and $\{G_1', \ldots, G_w'\}$. For a visual illustration, see Appendix A.

$g_{\Gamma}(t_i) < g_{\Gamma}(t_j)$. A *comonotonic extended book* restricts the set of extended gambles $\left\{f^1_{\Phi_1}, \ldots, f^m_{\Phi_m}, g^1_{\Gamma_1}, \ldots, g^m_{\Gamma_m}\right\}$ to be comonotonic. The *comonotonic extended book-making principle* requires that no comonotonic extended book exists.

Under the restriction of descriptive invariance as for the comonotonic BMP, an extended comonotonic book reduces to a *comonotonic book*. The non-existence of this comonotonic book, together with some natural conditions is equivalent to the existence of unique nonadditive probabilities (capacity) and the evaluation of a standard gamble f given by $V(f)$, the Choquet expected value of f. More specifically, $V(f)$ is equal to the weighted sum of outcomes by decision weights derived from the capacity (Theorem 6 in Diecidue and Wakker 2002).

The third and most restricted form of the BMP, which we generalize to accommodate extended gambles is the *affine book-making principle* (Diecidue and Maccheroni 2003). The gambles under consideration for this principle must not only be comonotonic, but also affine. Extended gambles f_{Φ} and g_{Γ} are *affinely related* if there exist $a \geq 0$ and $b \in \Re$ such that $f_{\Phi} = ag_{\Gamma} + b$ or $g_{\Gamma} = af_{\Phi} + b$ (Gilboa and Schmeidler 1989; Ghirardato et al. 1998). The *affine extended book* restricts the set of extended gambles $\left\{f^1_{\Phi_1}, \ldots, f^m_{\Phi_m}, g^1_{\Gamma_1}, \ldots, g^m_{\Gamma_m}\right\}$ to be affinely related. The *affine extended book-making principle* requires that no affine extended book exists.

The implication for standard gambles under the descriptively invariant conditions of the affine BMP is also related to the evaluation form. The evaluation V is *constant linear* when $V(af_{\Phi} + b) = aV(f_{\Phi}) + b$ for $a \geq 0$ and $b \in \Re$. The non-existence of an *affine book* and some additional natural conditions is equivalent to the evaluation of a standard gamble f given by $V(f)$, the maxmin expected value of f based on multiple-prior beliefs (Gilboa and Schmeidler 1989).

To accommodate description-dependent subjective probabilities such as those derived according to support theory, we propose in this section, generalizations of three forms of the BMP. We will now proceed to show that such probability judgments do not fulfill basic consistency requirements. In particular, support theory's probabilities can lead to violations of not just the extended BMP, but also the comonotonic extended BMP and the affine extended BMP.

3 Support theory's violations of the extended book-making principle

In this section, we examine decisions where the underlying probability judgments are based on support theory. We demonstrate through four examples how such decisions are incompatible with additive, nonadditive, and even multiple-prior models of choice. The first two examples represent violations of the extended BMP, exposing support theory's vulnerabilities from implicit and explicit subadditivities (Expression 3). Example 3 and Example 4 illustrate violations of the comonotonic extended BMP and the affine extended BMP respectively. The structure for all the examples is based on Fox and Tversky (1998) where they find empirical evidence consistent with support theory in decisions under uncertainty. We then show how subjective probabilities following from support theory can lead to a fundamental violation of strict monotonicity in decision-making, which is the primary reason for all the consistency violations.

Example 1 Violation of the extended BMP from implicit subaddivity

Consider extended gambles on the team that will win the NBA Playoffs. There are a total of eight competing teams: the four teams from the western conference are w_1, w_2, w_3, w_4, and the four teams from the eastern conference are e_1, e_2, e_3, e_4. Hence, there are eight possible states of nature for the winning team: w_1', w_2', w_3', w_4', e_1', e_2', e_3', e_4'.

Let $f^1_{\Phi_1}$ be an extended gamble that pays \$1 if a western conference team wins, and −\$1 otherwise. Extended gamble $g^1_{\Gamma_1}$ pays \$1 if either team w_1, team w_2, team w_3, or team w_4 wins, and −\$1 otherwise. In addition, $f^2_{\Phi_2}$ is an extended gamble that pays \$1 if an eastern conference team wins, and −\$1 otherwise. Similarly, $g^2_{\Gamma_2}$ is an extended gamble that pays \$1 if either team e_1, team e_2, team e_3, or team e_4 wins, and −\$1 otherwise. Below is a summary of the winning events and their corresponding descriptions for the four extended gambles.

Extended gambles	Description of the winning event[a]	The corresponding winning event
$f^1_{\Phi_1}$	W="a team from the western conference wins"	$W'=\{w_1', w_2', w_3', w_4'\}$
$g^1_{\Gamma_1}$	$(w_1 \vee w_2 \vee w_3 \vee w_4)$="either w_1, w_2, w_3, or w_4 wins"	$(w_1 \vee w_2 \vee w_3 \vee w_4)'=\{w_1', w_2', w_3', w_4'\}$
$f^2_{\Phi_2}$	E="a team from the eastern conference wins"	$E'=\{e_1', e_2', e_3', e_4'\}$
$g^2_{\Gamma_2}$	$(e_1 \vee e_2 \vee e_3 \vee e_4)$="either e_1, e_2, e_3, or e_4 wins"	$(e_1 \vee e_2 \vee e_3 \vee e_4)'=\{e_1', e_2', e_3', e_4'\}$

[a] The alternate description is always "otherwise", which corresponds to the complement of the winning event.

Note that extended gambles $f^1_{\Phi_1}$ and $f^2_{\Phi_2}$ are monetary equivalent to extended gambles $g^1_{\Gamma_1}$ and $g^2_{\Gamma_2}$ respectively. W and E are implicit hypotheses, whereas $(w_1 \vee w_2 \vee w_3 \vee w_4)$ and $(e_1 \vee e_2 \vee e_3 \vee e_4)$ are explicit disjunctions. Nevertheless, their descriptions are coextensional, i.e., $W'=(w_1 \vee w_2 \vee w_3 \vee w_4)'$ and $E'=(e_1 \vee e_2 \vee e_3 \vee e_4)'$.

According to support theory, a decision-maker will judge the probability of the implicit hypothesis $P\left(W, \overline{W}\right)$ to be less likely than that of the explicit disjunction $P\left(w_1 \vee w_2 \vee w_3 \vee w_4, \overline{w_1 \vee w_2 \vee w_3 \vee w_4}\right)$. This behavioral assumption of implicit subadditivity from unpacking in Expressions 2 and 3 is at the basis of support theory. Consequently, she would prefer extended gamble $g^1_{\Gamma_1}$ over extended gamble $f^1_{\Phi_1}$ in decision situation 1. By a similar line of argument, the decision-maker will judge $P\left(E, \overline{E}\right)$ to be less likely than $P\left(e_1 \vee e_2 \vee e_3 \vee e_4, \overline{e_1 \vee e_2 \vee e_3 \vee e_4}\right)$ and prefer $g^2_{\Gamma_2}$ over $f^2_{\Phi_2}$. These judgments are consistent with experimental findings from Fox and Tversky (1998).

Consider adding a constant $\varepsilon \geq 0$ to the payoffs in some states of the four gambles as exhibited below. Then, preferences of $g^j_{\Gamma j}$ over $f^j_{\Phi j}$ in the two decision situations j=1, 2 hold as strict preference for ε=0, and for some ε>0 sufficiently small, the preferences will still hold weakly as indicated below.

$$f^1_{\Phi_1}\ (1+\varepsilon, 1+\varepsilon, 1+\varepsilon, 1+\varepsilon, -1, -1, -1, -1) \preceq (+1, +1, +1, +1, -1, -1, -1, -1)\ g^1_{\Gamma_1}$$
$$f^2_{\Phi_2}\ (-1, -1, -1, -1, 1+\varepsilon, 1+\varepsilon, 1+\varepsilon, 1+\varepsilon) \preceq (-1, -1, -1, -1, +1, +1, +1, +1)\ g^2_{\Gamma_2}$$

but

$$f^1_{\Phi_1} + f^2_{\Phi_2}(\ \varepsilon\ ,\varepsilon\ ,\varepsilon\ ,\varepsilon\ ,\varepsilon\ ,\varepsilon\ ,\varepsilon\ ,\varepsilon\ ) > (\ 0\ ,0\ ,0\ ,0\ ,0\ ,0\ ,0\ ,0\ )\ g^1_{\Gamma_1} + g^2_{\Gamma_2}.$$

When the two decision situations are taken together, they will yield an extended book. The result is a sure loss for the decision-maker in each state. Hence, this is a violation of the extended BMP that is due to support theory's implicit subadditivity assumption.

Example 2 Violation of the extended BMP from explicit subadditivity

As in Example 1, we consider extended gambles on the winning team of the NBA Playoffs based on the experimental study in Fox and Tversky (1998). Extended gambles $g^1_{\Gamma_1}$ and $g^2_{\Gamma_2}$ are as in Example 1. They yield \$1 when the true state of nature is either w_1', w_2', w_3', or w_4' and e_1', e_2', e_3', or e_4' respectively. The alternative payoffs in both extended gambles are −\$1. We now introduce extended gambles $h^1_{H_1}$ and $h^2_{H_2}$. Let $h^1_{H_1}$ be an extended gamble consisting of four perfectly correlated subgambles. The first subgamble corresponds to the elementary description that the winning team will be "w_1". The payoff is \$1 if state w_1' occurs, 0 if state w_2', w_3', or w_4' occurs, and else −\$$^1/_4$ if any of the remaining states occurs. Likewise, the second, third, and fourth subgambles are based on elementary descriptions "w_2", "w_3", and "w_4" respectively. Note that at most one of these four subgambles will yield a positive payoff. For example, if the true state of nature is w_1', only the first subgamble $h^1_{H_1}$ will yield a positive payoff of \$1 whereas all the other three will yield zero. On the contrary, a true state of e_1' will yield −\$$^1/_4$ in each of the four subgambles, resulting in a total loss of −\$1. We construct $h^2_{H_2}$ similarly as four perfectly correlated gambles corresponding to the elementary descriptions that the state will be "e_1", "e_2", "e_3", and "e_4". The following table lists the winning events and their corresponding descriptions for the two new extended gambles, $h^1_{H_1}$ and $h^2_{H_2}$.

Extended gambles	Description of the winning event[a]	The corresponding winning event
$h^1_{H_1}$	(w_1, w_2, w_3, w_4)="w_1", "w_2", "w_3", "w_4"	$(w_1, w_2, w_3, w_4)' = \{w_1', w_2', w_3', w_4'\}$
$h^2_{H_2}$	(e_1, e_2, e_3, e_4)="e_1", "e_2", "e_3", "e_4"	$(e_1, e_2, e_3, e_4)' = \{e_1', e_2', e_3', e_4'\}$

[a] The alternate description is always "otherwise", which corresponds to the complement of the winning event.

We observe that extended gambles $h^1_{H_1}$ and $h^2_{H_2}$ are described by sums of exclusive and exhaustive components, (w_1, w_2, w_3, w_4) and (e_1, e_2, e_3, e_4). These two gambles are monetary equivalent and coextensional with $g^1_{\Gamma_1}$ and $g^2_{\Gamma_2}$.

Due to the assumption of repacking explicit disjunctions in Expressions 2 and 3, a decision-maker whose probability judgment follows from support theory will judge \$1 returns to be more likely in $h^1_{H_1}$ and $h^2_{H_2}$ than in $g^1_{\Gamma_1}$ and $g^2_{\Gamma_2}$ respectively. This is because explicit disjunctions receive less support than their coextensional sums of exclusive components, and thus $P(w_1 \vee w_2 \vee w_3 \vee w_4, \overline{w_1 \vee w_2 \vee w_3 \vee w_4}) < P(w_1, \overline{w}_1) + P(w_2, \overline{w}_2) + P(w_3, \overline{w}_3) + P(w_4, \overline{w}_4)$. Therefore, in decision situation 1, the decision-maker will prefer extended gamble $h^1_{H_1}$ over extended gamble $g^1_{\Gamma_1}$ strictly for ε=0 and for some ε>0, as shown below. Similar arguments can be made for preference of extended gamble $h^2_{H_2}$ over extended gamble $g^2_{\Gamma_2}$. These preferences illustrated below are also supported by the experimental results in Fox and Tversky (1998).

$$g^1_{\Gamma_1}(+1,\ +1,\ +1,\ +1,\ -1,\ -1,\ -1,\ -1)\preceq \left\{\begin{array}{l}(1-\varepsilon, 0, 0, 0,\ -1/4,\ -1/4,\ -1/4,\ -1/4)\\(0, 1-\varepsilon, 0, 0,\ -1/4,\ -1/4,\ -1/4,\ -1/4)\\(0, 0, 1-\varepsilon, 0,\ -1/4,\ -1/4,\ -1/4,\ -1/4)\\(0, 0, 0, 1-\varepsilon,\ -1/4,\ -1/4,\ -1/4,\ -1/4)\end{array}\right\} h^1_{H_1}$$

$$g^2_{\Gamma_2}(-1,\ -1,\ -1,\ -1,\ +1,\ +1,\ +1,\ +1)\preceq \left\{\begin{array}{l}(-1/4,\ -1/4,\ -1/4,\ -1/4, 1-\varepsilon, 0, 0, 0)\\(-1/4,\ -1/4,\ -1/4,\ -1/4, 0, 1-\varepsilon, 0, 0)\\(-1/4,\ -1/4,\ -1/4,\ -1/4, 0, 0, 1-\varepsilon, 0)\\(-1/4,\ -1/4,\ -1/4,\ -1/4, 0, 0, 0, 1-\varepsilon)\end{array}\right\} h^2_{H_2}$$

but

$$g^1_{\Gamma_1} + g^2_{\Gamma_2}(0, 0, 0, 0, 0, 0, 0, 0) > (-\varepsilon,\ -\varepsilon,\ -\varepsilon,\ -\varepsilon,\ -\varepsilon,\ -\varepsilon,\ -\varepsilon,\ -\varepsilon)h^1_{H_1} + h^2_{H_2}.$$

If we aggregate the decision-maker's preferences over the two decision situations above, the result is an extended book and a violation of the extended BMP. In this case, the violation is due to support theory's explicit subadditivity property.

Example 3 Violation of the comonotonic extended BMP

Assume extended gambles on the winning team of the NBA Playoffs as in Fox and Tversky (1998). Extended gambles $f^1_{\Phi_1}$ and $g^1_{\Gamma_1}$ are as introduced in Example 1. Recall that the two extended gambles yield \$1 if the state is "a western conference wins" and "either team w_1, team w_2, team w_3, or team w_4 wins" respectively. The alternative payoffs are −\$1 in both gambles. Let extended gamble $f^2_{\Phi_1}$ be descriptively equivalent to $f^1_{\Phi_1}$ but differing on the states in which $\varepsilon \geq 0$ is added. Specifically, let the addition of some sufficiently small $\varepsilon > 0$ be made to improve upon the negative payoffs in $f^2_{\Phi_1}$ to counterbalance the higher judged probability in $g^1_{\Gamma_1}$.

As in Example 1, because $g^1_{\Gamma_1}$ is expressed as an explicit disjunction, it will be preferred to both $f^1_{\Phi_1}$ and $f^2_{\Phi_1}$, which are based on implicit hypotheses. According to the Expressions 2 and 3 from Section 2, this is a behavioral phenomenon arising from unpacking of implicit hypotheses that leads to greater support and consequently, higher judged probability for $g^1_{\Gamma_1}$ than for $f^1_{\Phi_1}$ or $f^2_{\Phi_1}$. The result is the set of preferences below.

$$f^1_{\Phi_1}\ (1+\varepsilon,\ 1+\varepsilon, 1+\varepsilon, 1+\varepsilon,\ -1,\ -1,\ -1,\ -1)\preceq (+1,\ +1,\ +1,\ +1,\ -1,\ -1, -1, -1)\, g^1_{\Gamma_1}$$

$$f^2_{\Phi_1}\ (+1, +1, +1, +1, -1+\varepsilon, -1+\varepsilon, -1+\varepsilon, -1+\varepsilon)\preceq (+1,\ +1,\ +1,\ +1,\ -1,\ -1,\ -1, -1)\, g^1_{\Gamma_1}$$

but

$$f^1_{\Phi_1} + f^2_{\Phi_1}\ (2+\varepsilon, 2+\varepsilon, 2+\varepsilon, 2+\varepsilon, 2+\varepsilon, 2+\varepsilon, 2+\varepsilon, 2+\varepsilon) > (2, 2, 2, 2, 2, 2, 2, 2) g^1_{\Gamma_1} + g^1_{\Gamma_1},$$

In this example, all three extended gambles, $f^1_{\Phi_1}$, $f^2_{\Phi_1}$, and $g^1_{\Gamma_1}$ are comonotonic, and summing over the two decision situations, generates a sure loss state-wise for the decision-maker, i.e., a comonotonic extended book. Thus, the result is a violation of the comonotonic extended BMP.

Example 4 Violation of the affine extended BMP

As in the previous three examples and based on the study in Fox and Tversky (1998), the extended gambles are on the winning team of the NBA Playoffs. The four affinely related extended gambles f_Φ, g_Γ, f_Γ, g_Φ are as presented in the table below.

Extended gambles	Description of the winning event[a]	The corresponding winning event
f_Φ	W="a team from the western conference wins"	$W'=\{w_1', w_2', w_3', w_4'\}$
g_Γ	$(\mathrm{w}_1 \vee \mathrm{w}_2 \vee \mathrm{w}_3 \vee \mathrm{w}_4)$="either w_1, w_2, w_3, or w_4 wins"	$(\mathrm{w}_1 \vee \mathrm{w}_2 \vee \mathrm{w}_3 \vee \mathrm{w}_4)'=\{\mathrm{w}_1', \mathrm{w}_2', \mathrm{w}_3', \mathrm{w}_4'\}$
f_Γ	$(w_1 \vee w_2 \vee w_3 \vee w_4)$="either w_1, w_2, w_3, or w_4 wins"	$(w_1 \vee w_2 \vee w_3 \vee w_4)'=\{w_1', w_2', w_3', w_4'\}$
g_Φ	W="a team from the western conference wins"	$W'=\{w_1', w_2', w_3', w_4'\}$

[a] The alternate description is always "otherwise", which corresponds to the complement of the winning event.

For the extended gambles f_k, $k=\Phi$, Γ, the decision-maker gains \$5 if the winning event $\{w_1', w_2', w_3', w_4'\}$ occurs. Otherwise, she loses −\$1. In comparison, the payoffs for extended gambles g_k, $k=\Phi$, Γ, are \$4 and 0 corresponding to winning and losing events respectively. (The payoffs in g_k are obtained by multiplying the payoffs in f_k by $a=2/3$ and adding a constant $b=2/3$ for $k=\Phi$, Γ.)

The same winning event is described by an implicit hypothesis W in Φ, but by an explicit disjunction $(w_1 \vee w_2 \vee w_3 \vee w_4)$ in Γ. Thus, according to support theory and Expression 3, the winning event would be judged more likely in Γ than in Φ. Consequently, it is plausible that a decision-maker would strictly prefer g_Γ over f_Φ, but f_Γ weakly over g_Φ. Adding a sufficiently small $\varepsilon>0$ to f_Φ results in the following preferences over the extended gambles.

$$f_\Phi\ (5+\varepsilon, 5+\varepsilon, 5+\varepsilon, 5+\varepsilon, -1+\varepsilon, -1+\varepsilon, -1+\varepsilon, -1+\varepsilon) \preceq (4,\ 4,\ 4,\ 4,\ 0,\ 0,\ 0,\ 0)\, g_\Gamma$$

$$g_\Phi\ (4, 4, 4, 4, 0, 0, 0, 0) \preceq (5,5,5,5,-1,-1,-1,-1)\, f_\Gamma$$

but

$$f_\Phi + g_\Phi\ (9+\varepsilon, 9+\varepsilon, 9+\varepsilon, 9+\varepsilon, -1+\varepsilon, -1+\varepsilon, -1+\varepsilon, -1+\varepsilon) > (9,9,9,9,-1,-1,-1,-1)\, g_\Gamma + f_\Gamma.$$

What is essentially a preference reversal between f_k and g_k results in an affine extended book and hence a violation of the affine extended BMP.

The four examples above demonstrate how preferences over extended gambles in which probability judgments follow from support theory can violate the extended BMP, the comonotonic extended BMP, and the affine extended BMP. As stated in Section 3, these principles form the necessary and sufficient conditions for models of choice based on additive, nonadditive, and multiple-prior beliefs respectively. Hence, this implies that support theory's subjective probabilities are compatible with neither one of these beliefs.

We are aware that Tversky and Koehler (1994) clearly asserted that probability judgments according to support theory do not follow the norms of standard additive or nonadditive probabilities. The theory's incoherence is also cited in subsequent literature (eg., Ayton 1997). However, what we have shown through the four examples is an examination of support theory's subjective probabilities from a decision theoretic perspective (rather than judgment). Furthermore, to our knowledge, there has been no prior study which shows that such description-dependent subjective probabilities are also incompatible with multiple-prior beliefs. This is somewhat surprising as the multiple-priors model by definition can accommodate more than one set of probability distributions over the state space. We will investigate this issue further in the next section.

We will now formally show that the root cause for support theory's incoherence stems from its susceptibility to violation of strict monotonicity in decision-making.

Observation 1
Probability judgments with the assumption of non-extensionality as in support theory can lead to violations of strict monotonicity in outcomes in extended gambles. This violation can occur even when probability judgments do not violate monotonicity with respect to set inclusion. Consequently, such description-dependent subjective probabilities are prone to violations of the extended (comonotonic/affine) book-making principle.

Proof: Appendix B. We should first remark that Tversky and Koehler (1994) explicitly assumed that the support function is nonmonotonic with respect to set inclusion. Specifically, it is possible for $s(A) \leq s(B)$ even though $A' \supset B'$ and this can lead to nonmonotonicity in probability judgments in Expression 3. However, this property is omitted from Rottenstreich and Tversky (1997). More importantly, nonmonotonicity with respect to set inclusion is not part of the necessary and sufficient conditions for support theory in either of the two papers (Appendices of Tversky and Koehler 1994 and Rottenstreich and Tversky 1997). Fox and Tversky (1998) pointed out that violation of monotonicity in outcomes can and do occur, but their statement also involves violation of monotonicity in probabilities with respect to set inclusion. In particular, they state that $C(x, A) < C(x, A_1 \vee ... \vee A_n)$ where $C(x, A)$ is the certainty equivalent of the prospect that pays \$$x$ if event A' occurs and event $(A_1' \vee ... \vee A_n')$ is a proper subset of A. Observation 1 shows that under the assumption of non-extensionality, violation of strict monotonicity in outcomes can occur even if monotonicity in probability judgments with respect to set inclusion is not violated. Furthermore, our result covers non-extensionality assumptions of both subaddivity (Tversky and Koehler 1994; Rottenstreich and Tversky 1997) and superadditivity (Sloman et al. 2004) of supports.

In this section, we show that probability judgments with the assumption of non-extensionality can lead to violations of various forms of the extended BMP. We will propose in the next section a descriptive version of the consistency principle, which can accommodate description-dependent subjective probabilities as postulated by support theory.

4 The behavioral book-making principle

In this section, we propose a weakened version of the affine extended BMP, which can accommodate support theory's probability judgments. However, we will first explain the intuition as to why the affine framework is the most appropriate one for such description-dependent subjective probabilities.

Recall from Section 3 that under the assumption of descriptive invariance when all the gambles are standard, the BMP and the comonotonic BMP entail necessary and sufficient conditions for unique subjective probabilities and unique decision weights respectively. For each state of nature and each event composed of some or all of those states, the subjective probabilities or the decision weights are fixed for a particular decision-maker under these conditions. Because probability judgments are description dependent under support theory, there is no such consistency with respect to either the states or the events even for a single decision-maker. Hence, neither the BMP nor the comonotonic BMP can accommodate the non-extensionality assumption of support theory. In comparison, the affine BMP for standard gambles is compatible with non-unique subjective probabilities in the form of multiple-priors as stated in Section 2. We argue that this is the structure which is most applicable and natural for varying subjective probabilities derived from support theory. The affine BMP, however, requires one consistent set of probability distributions for all gambles under consideration. This is the primary reason why support theory probability judgments violate the affine extended BMP as illustrated by Example 4 in the previous section. Therefore, we need to amend the affine extended BMP by imposing additional restrictions on the descriptions of extended gambles, which we present formally below.

Consider a set of affinely related extended gambles $\left\{f^1_{\Phi_1},\ldots,f^m_{\Phi_m},g^1_{\Gamma_1},\ldots,g^m_{\Gamma_m}\right\}$. However, we restrict the extended gambles $f^j_{\Phi_j}$, $g^j_{\Gamma_j}$, j=1, ..., m, in each decision situation to be descriptively equivalent, i.e., $\Phi_j=\Gamma_j$. Then two monetary equivalent extended gambles must have the same fair price when they are in the same decision situation, but they may have different fair prices if they are in different decision situations. Specifically, let $u_j=\xi\left(f^j_{\Phi_j}\right)$ and $v_k=\xi\left(g^k_{\Gamma_k}\right)$ be the respective fair prices of extended gambles $f^j_{\Phi_j}=(x_1,\ \ldots,x_n)_{\Phi_j}$ and $g^k_{\Gamma_k}=(x_1,\ \ldots,x_n)_{\Gamma_k}$ where $x_i=f_{\Phi_j}(t_i)=g_{\Gamma_k}(t_i)$ are equal payoffs for both f_{Φ_j} and g_{Γ_k} when t_i is the true state of nature. If j=k, then $\Phi_j=\Gamma_k$ and $u_j=v_k$ and vice versa. But if $j\neq k$, then $\Phi_j\neq\Gamma_k$ implies $u_j\neq v_k$ and vice versa. Hence, we allow the descriptions of extended gambles to vary across different decision situations, but not within the same one. This setting captures the description dependent nature of support theory. A *behavioral book* is essentially an affine extended book when the extended gambles in each decision situation are restricted to be descriptively equivalent as in our current setting. The *behavioral book-making principle* requires that no behavioral book exists. This leads to the result below.

Theorem 2 *A binary relation over m pairs of affinely related extended gambles satisfying weak order and the fair price property allows no behavioral book if and only if the evaluation V is constant linear.*

Proof: Appendix B. The basic intuition behind this result is that there is an underlying structure in the evaluation V of extended gambles that are described identically, but may not be monetary equivalent. For deeper insights into the V function, note that a decision-maker's probability assessments for descriptively equivalent extended gambles in each decision situation are unique only at the level of events and not for each state of nature. Hence, this is equivalent to the decision-maker considering more than one probability distribution over the states as in the multiple-priors model of choice. We can further interpret this as the decision-maker having a set of beliefs in each decision situation based on the common descriptions of events. And because the descriptions of events can vary across different decision situations, so can the set of beliefs for the decision-maker. This is the manner in which the premise of support theory is represented in the current setting: alternative descriptions of events translate to distinct sets of beliefs. The implication is that the decision-maker evaluates extended gambles through the function V in the same decision situation based on the same set of beliefs and across decision situations with possibly different sets of beliefs. In comparison to the affine BMP for standard gambles in Diecidue and Maccheroni (2003), the evaluation form V for extended gambles in the current setting is the multiple-priors model where the convex set of probabilities may be different in each decision situation depending on the description of events. To our knowledge, this is the first study that examines the implied model of choice in which probability judgments follow from support theory. We will now proceed to show the empirical implications of this result.

While there exists consensus on the descriptive content of support theory, Tversky and Koehler (1994) discuss the prescriptive implications of support theory. They argue that the theory can aid the design of elicitation procedures and reconcile inconsistent assessments of subjective beliefs. To this important point, is the empirical contribution of the above result in our opinion. Inconsistent measurement of beliefs may arise when the elicitation is based on a decision situation in which the descriptions of the underlying events vary. In fact, one of the most studied implications of support theory is partition dependence (Clemen and Ulu 2007; Fox and Clemen 2005; Sonnemann et al. 2008). This stream of literature shows that the subjective probability for an event depends on how the state space is partitioned. Consequently, the elicitation and measurement of subjective beliefs can also be biased by the same process. In the behavioral BMP, we specify a setting that is immune from the influence of partitions and avoids distortions arising from the descriptions of events. By restricting description equivalence within each decision situation, the proposed principle provides conditions under which decision analysts can consistently elicit and incorporate beliefs that follow from support theory. Note that decision situations based on the same descriptions of the events are typical in real life, eg., in sports gambling (Ayton 1997) and prediction markets (Sonnemann et al. 2008). Therefore, empirical validity of the behavioral BMP can be tested in experiments and field studies as descriptively equivalent gambles are simply those whose outcomes are based on the same partition of the state space and where each event is described identically. We leave this for further studies.

We introduce in this section a weakened version of the affine extended BMP: the behavioral BMP. The modified principle allows for individuals' probability judgments to be dependent on the descriptions of events and yet provide a form of consistency in the overall decision process. The principle characterizes a consistent set of decisions where the underlying probability judgments are based on support theory. The result provides insights into the implied evaluation form of extended gambles and form a basis for improvement of elicitation procedures for subjective probabilities.

5 Conclusion

Decision theorists have long been familiar with consistency requirements for subjective probabilities. One of the most popular and elegant is the BMP by de

Finetti (1931). Behavioral decision theory is becoming increasingly popular and there are a number of modifications of the BMP to accommodate descriptive evidence. By learning from normative violations, decision analysts can, for example, improve the prescription of decision analysis (Bleichrodt et al. 2001).

Support theory is a psychologically and mathematically founded theory for the assessment of subjective probabilities. We show in this paper how support theory's necessary and sufficient conditions can lead to violations of various forms of the (extended) BMP. In effect, we demonstrate that subjective probabilities following from support theory are incompatible with popular decision models that can accommodate additive, nonadditive, and even multiple-prior beliefs.

We propose a behavioral version of the BMP that captures the description dependent characteristic of support theory. The result specifies conditions under which support theory can respect weakened consistency requirements. In terms of empirical implications, the result also provides a basis for improving measurements of probabilities. The behavioral BMP, to our knowledge, is the first attempt to characterize and analyze decisions based on support theory probabilities.

We hope that this paper will help clarify the decision theoretic implications of support theory and uncover the potential of support theory in constructive use by analysts and practitioners. We further believe that the behavioral BMP can have more general implications outside the domain of support theory. This is due to the general framework of the principle, which can become a foundation for characterizations of context dependence and framing effects. However, we leave these directions for future research.

Appendix A

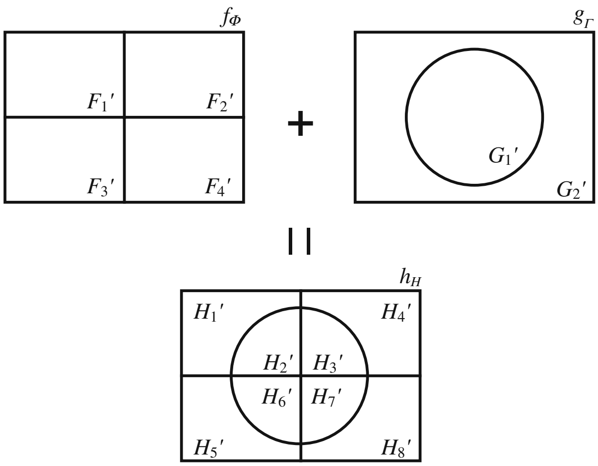

Fig. 2 An example of the addition of two extended gambles $f_\Phi = (f_\Phi(t_1), \ldots, f_\Phi(t_n))$ and $g_\Gamma = (g_\Gamma(t_1), \ldots, g_\Gamma(t_n))$ where $\Phi=\{(F_1', F_1), \ldots, (F_4', F_4)\}$ and $\Gamma=\{(G_1', G_1), (G_2', G_2)\}$. Here, $f_\Phi + g_\Gamma = h_H = (f_\Phi(t_1) + g_\Gamma(t_1), \ldots, f_\Phi(t_n) + g_\Gamma(t_n))$ where $H=\{(H_1', H_1), \ldots, (H_8', H_8)\}$

Appendix B: proofs

Observation 1 Let $T=\{t_1,..., t_n\}$ be the state space. Consider two monetary equivalent extended gambles $f_\Phi = (x_1, \ldots, x_n)_\Phi$ and $g_\Gamma = (x_1, \ldots, x_n)_\Gamma$ where $x_i = f_\Phi(t_i) = g_\Gamma(t_i)$ are equal payoffs for both f_Φ and g_Γ when t_i is the true state. Assume that $\Phi = \{(Y_1', Y_1^F), \ldots, (Y_r', Y_r^F)\} \neq \Gamma = \{(Y_1', Y_1^G), \ldots, (Y_r', Y_r^G)\}$, denoting that an event Y_k', $k=1, ..., r$ is described by different descriptions Y_k^F and Y_k^G in extended gambles f_Φ and g_Γ, respectively.

Note that the two extended gambles f_Φ and g_Γ are affinely related (monetary equivalent extended gambles are a special case of affine extended gambles where $a=1$ and $b=0$) and hence their payoffs are based on the same partition of the state space $\{Y_1', \ldots, Y_r'\}$ as indicated. Without loss of generality, assume $Y_1', \ldots, Y_{w-1}', Y_w', Y_{w+1}', \ldots, Y_r'$ are categorized into positive, negative, and zero payoff events as follows: for $k=1, \ldots, w-1$, $x_i>0$, $\forall t_i \in Y_k'$, for $k=w+1, \ldots, r$, $x_i<0$, $\forall t_i \in Y_k'$, and $x_i=0$, $\forall t_i \in Y_w'$. Furthermore, let each positive event Y_k', $k=1, \ldots, w-1$ be described by implicit hypotheses Y_k^F in f_Φ and explicit disjunctions Y_k^G in g_Γ. Conversely, let the descriptions Y_k^F in f_Φ be explicit disjunctions and Y_k^G in g_Γ be implicit hypotheses for negative payoff events Y_k', $k=w+1, \ldots, r$.

We now proceed by assuming subadditivity of supports as in Expression 2. According to support theory, the decision-maker will prefer g_Γ over f_Φ. This is because the events corresponding to positive payoffs will be judged to be more likely under g_Γ when they are described by explicit disjunctions than under f_Φ. Conversely, events with negative payoffs will be judged less likely under g_Γ when they are described by implicit hypotheses than under f_Φ. Therefore, we can compensate the less attractive gamble f_Φ by adding a sufficiently small constant $\varepsilon>0$ to all the states and still maintain a weak preference. Specifically, we will have $f_\Phi(t_i) = x_i + \varepsilon > x_i = g_\Gamma(t_i)$, $\forall t_i$, and yet $f_\Phi \preceq g_\Gamma$, which violates strict monotonicity. Violation of strict monotonicity when the supports are assumed to be superadditive can be shown similarly, but with the reverse preference order of $f_\Phi \succeq g_\Gamma$.

The above illustration is in fact a violation of the affine extended BMP, which also implies violations of both the comonotonic extended BMP and the extended BMP. ■

Theorem 2 *Assume a finite state space* $T=\{t_1, \ldots, t_n\}$. *Let* $f_{\Phi_1} = (f_1, \ldots, f_n)_{\Phi_1}, \ldots, f_{\Phi_m} = (f_1, \ldots, f_n)_{\Phi_m}$ *where* $f_i = f_{\Phi_j}(t_i)$, $i=1, \ldots, n$ and $j=1, \ldots, m$ *be monetary equivalent extended gambles with possibly different descriptions. Consider two sets of affinely related extended gambles based on each of these* f_{Φ_j} *variants,* $g_{\Phi_j}^j = a_j f_{\Phi_j} + b_j$ and $h_{\Phi_j}^j = c_j f_{\Phi_j} + d_j$ *where* $a_j, c_j \geq 0$ *and* $b_j, d_j \in \Re$ *for* $j=1, \ldots, m$. *Furthermore, let the fair price of each extended gamble* f_{Φ_j} *be* $\xi_j = \xi(f_{\Phi_j})$, $j=1, \ldots, m$ *and also* $V(f_{\Phi_j}) = \xi_j$. *From constant-linear evaluation,* $V\left(g_{\Phi_j}^j\right) = a_j\, \xi j + b_j$ *and* $V\left(h_{\Phi_j}^j\right) = c_j\, \xi_j + d_j$, $j=1, \ldots, m$.

We will show by contradiction by first supposing that a behavioral book exists, i.e., without loss of generality $g_{\Phi_j}^j \preceq h_{\Phi_j}^j$, $j=1, \ldots, m$, *but that* $\sum_{j=1}^m g_{\Phi_j}^j(t) > \sum_{j=1}^m h_{\Phi_j}^j(t)$. *The first part of the statement implies that* $V\left(g_{\Phi_j}^j\right) = a_j\xi_j + b_j \leq c_j\xi_j + d_j = V\left(h_{\Phi_j}^j\right)$, $j=1, \ldots, m$. *Then we have,* $(a_j - c_j)\xi_j \leq (d_j - b_j)$, $j=1, \ldots, m$, *and summing over all m rows yields* $\sum_{j=1}^m (a_j - c_j)\xi_j \leq \sum_{j=1}^m (d_j - b_j)$. *However, the second part of the statement* *implies that* $f_i \sum_{j=1}^{m} a_j + \sum_{j=1}^{m} b_j > f_i \sum_{j=1}^{m} c_j + \sum_{j=1}^{m} d_j$, $i=1, \ldots, n$. *Hence,* $f_i \sum_{j=1}^{m} (a_j - c_j) > \sum_{j=1}^{m} (d_j - b_j)$, $i=1, \ldots, n$.

For the two parts of the statement to be congruent, it must be that $\sum_{j=1}^{m} (a_j - c_j)\xi_j < f_i \sum_{j=1}^{m} (a_j - c_j)$, $i=1, \ldots, n$. *Rearranging and aggregating over all the n states, we get* $n \sum_{j=1}^{m} (a_j - c_j)\xi_j - \sum_{i=1}^{n} f_i \sum_{j=1}^{m} (a_j - c_j) < 0$.

However, because $V(\xi_j, \ldots, \xi_j) = V(f_{\Phi}^{j}) = \xi_j$, $j=1, \ldots, m$, *we know that* $n(a_j - c_j)\xi_j - (a_j - c_j)\sum_{i=1}^{n} f_i = 0$, $j=1, \ldots, m$. *Hence,* $n \sum_{j=1}^{m} (a_j - c_j)\xi_j - \sum_{i=1}^{n} f_i \sum_{j=1}^{m} (a_j - c_j) = 0$ *and this is a contradiction with the previous statement* $n \sum_{j=1}^{m} (a_j - c_j)\xi_j - \sum_{i=1}^{n} f_i \sum_{j=1}^{m} (a_j - c_j) < 0$. *The reverse direction is obtained by adapting the proof of Theorem (1) in Diecidue and Maccheroni (2003) to a set of extended gambles that are descriptively equivalent in each decision situation.* ■

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