Catwalk:

Simulation-Based Re-insurance Risk Modelling

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This case was written by Theodoros Evgeniou, Associate Professor of Decision Sciences and Technology Management at INSEAD, with Aon Benfield Analytics, Asia Pacific. It is intended to be used as a basis for class discussion rather than to illustrate either effective or ineffective handling of an administrative situation. All numbers in this case are not real.

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At eleven in the morning, on a sunny day in May, you are sitting outside a classic Parisian café, working with mathematical models. Partly due to the sun, and partly due to the models, you are getting more and more confused about the report you are preparing.

A few weeks earlier, the Property and Casualty division of Catwalk, a local insurance company, asked you to help them select a reinsurance product. By buying reinsurance, Catwalk can pass some of its own risk to the reinsurer, hence freeing up some of its own capital to use for selling more insurance products and (why not?) pay your fees. Catwalk’s goals are straightforward: to decrease its overall risk exposure and free up as much capital as possible at the lowest cost.

Your meeting with the management team of Catwalk is in a few hours. They expect your final analysis and recommendation on which reinsurance product to select. How should you analyse the reinsurance options Catwalk is considering? Which one should you recommend to Catwalk? What issues do you expect the management team of Catwalk to raise about your analysis?

Key Decision Parameters – and Some Vocabulary

A reinsurance product, much like any insurance product, is characterised by many parameters (summarized in Exhibit 1) – clear to an insurance industry insider but cryptic to outsiders. The type of product that Catwalk is considering is a so-called *Excess of Loss (XOL or XL) Reinsurance Layer (RL)*. As the name suggests, Catwalk would pay a pre-agreed *Reinsurance Premium* to the reinsurance company and, in exchange, for each Catwalk’s *claim* the reinsurer would pay Catwalk an amount exceeding the *Attachment Point* up to a maximum of the specified *Limit*. This amount is referred to as the *Ceded Loss* of the reinsurer. Twelve such XOL Reinsurance Layers – the ones Catwalk managers want you to help them select from – are described in Table 1. For example, in the first case, “€9.75 million in excess of €250,000” means that a loss of up to €9.75 million above €250,000 would be covered by the reinsurance company. Anything below €250,000 or above €10 million would be covered by Catwalk. These are shown in Exhibit 2.

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Table 1: Twelve Excess of Loss Reinsurance Layers (all numbers are in €1,000s)

Of course, each of these XOL RL has a price tag that Catwalk will need to pay to the reinsurer, the so-called *Reinsurance Premium* – this is normally paid up-front in the form of a minimum deposit. Different reinsurance companies may charge different Reinsurance Premiums for the same XOL RL, depending on a number of factors such as the cost of the
capital, the existing relationship the reinsurer has with Catwalk, the eagerness of the reinsurer to enter a market, the reinsurer’s risk tolerance, profitability, capital capacity, etc. Despite all these factors, the Reinsurance Premium for an XOL RL would naturally also depend on the risk-return that XOL RL would offer Catwalk. Having a good estimate of the risk-return of each XOL RL is crucial for Catwalk before they start any negotiations with the reinsurers.

Estimating a “fair” Reinsurance Premium for each XOL RL requires, among others, a careful assessment of the claims Catwalk expects to send to the reinsurer – which are effectively claims of Catwalk’s clients to Catwalk. Nobody can forecast the future, so insurance companies go through major pains to best assess the future losses against which to insure. Data from past claims of Catwalk’s clients as well as years of experience and intuition support every such “exercise” to decide which is the best (re)insurance product.

To begin with, Catwalk needs to have a good estimate of the severity of the claims it will receive from its Property and Casualty clients, as well as the number of claims per year. Severity is the amount of loss of a given claim. Obviously, nobody can forecast the actual claims received, but statistical studies of past claims of Catwalk’s clients allow some educated guesses about the severity of each claim as well as the frequency with which claims will occur.

The severity of each claim is typically modelled with a lognormal distribution (unfortunately, severity always has one sign, negative) with mean $\mu$ and standard deviation $\sigma$ – these are the mean and standard deviation of the logarithm of the severity. In addition, each claim is capped at some maximum value – €20 million in this case – thus Catwalk would not cover claims of its clients in excess of €20 million. The annual number of claims is often modelled as a Poisson random variable with parameter $\lambda$ equal to the historic average annual number of claims. All parameters, $\mu$, $\sigma$, and $\lambda$, are estimated using standard statistical distribution fitting techniques to Catwalk’s historical claim payment data. (Isn’t history the best predictor of the future anyway?) In this case, Catwalk uses $\mu=11$, $\sigma=2.10$, and $\lambda=10$.

The Reinsurance Premium to be paid to the reinsurer depends on the expected Ceded Losses of the reinsurer as well as the risk taken by the reinsurer (and removed from Catwalk). The latter is measured in various ways, the standard deviation of the Ceded Losses being one of them. In this case, the Reinsurance Premium is estimated as:

$$\text{Reinsurance Premium} = (\text{Expected Ceded Losses}) + (\text{Reinsurer’s Risk Loading Factor}) \times (\text{Standard Deviation of Ceded Losses})$$

where

Reinsurer’s Risk Loading Factor is a parameter that the reinsurer would internally define (based on past experience). It would differ across reinsurance companies

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1 In the case of XOL treaties, which are non-proportional treaties, the claims are made as they arise. This is unlike the case of proportional treaties for which a quarterly or half-yearly aggregation of claims is done.

2 To get a random sample from the lognormal distribution in a simulation, one can generate a normally distributed variable with mean $\mu$ and standard deviation $\sigma$ first, and then take the exponential (e.g. “exp” in Excel) of that number. More information about the lognormal distribution can be found, for example, at http://en.wikipedia.org/wiki/Log-normal_distribution.
according to their cost of capital, internal margin requirements, or risk appetite among other factors. In this particular case, based on past experience, Catwalk expects that the reinsurers would use a Risk Loading Factor of about 30%, although this would vary from one reinsurer to another.

Meanwhile, Catwalk would receive some Premium Income from its own clients to insure their claims – similar to the Reinsurance Premium Catwalk will, in turn, need to pay to the reinsurer. The Premium Income Catwalk would collect is again based on the frequency and severity of claims paid – with the same parameters as above – and on Catwalk’s own expenses involved in directly underwriting these risks. It is the ratio of the expected claims of Catwalk’s clients divided by Catwalk’s so-called Direct Pricing Loss Ratio, that is, 

\[ \text{exp}(\mu + \sigma^2/2) \times \lambda / [\text{Catwalk’s Direct Pricing Loss Ratio}] \]

The latter is similar to the reinsurer’s Risk Loading Factor. It is defined (roughly) as the historical ratio of Catwalk’s “losses” (claims paid + other expenses + profit margin + margin for risk) to the premium collected in the past. In this case it is 60%.

The choice of the XOL Layer affects a key parameter for Catwalk: its Capital Requirements. These are typically set by considering a number of factors: (i) the company’s appetite for risk, (ii) regulatory bodies of the country that impose capital/solvency requirements, and (iii) the need to obtain a desired level of rating from rating agencies such as S&P, Moody’s, A. M. Best and Fitch, which have their own definitions of capital adequacy. One way to define Capital Requirements is by using the 99th percentile Value at Risk (VaR); this is the 99% worst case scenario for which the company would set aside its capital to sufficiently cover its losses (net of the Premium Income received from clients). It is therefore important that Catwalk calculates the impact each possible XOL RL would have on its Capital Requirements – this is the Capital Relief that an XOL RL would provide to Catwalk.

The Capital Relief that a particular XOL RL would provide Catwalk is simply the difference between the Capital Requirements Catwalk would have with and without the XOL RL. Without reinsurance, Catwalk’s Capital Requirements would be equal to the capital required to sustain the gross underwriting loss (= premium income – claims paid) corresponding to a probability of exceedance of 1% (the 99% VaR) Similarly, the Capital Requirements of Catwalk after buying the reinsurance is equal to the capital required to sustain the net underwriting loss with reinsurance (= premium income – claims paid – reinsurance premium + ceded loss).

By buying reinsurance effectively, Catwalk pays a Reinsurance Premium (as calculated above) to free up capital equal to the Capital Relief. The ratio (Reinsurance Premium – Expected Ceded Losses)/(Capital Relief) is the Ceded ROE that Catwalk effectively pays for the reinsurance: it is similar to an interest rate Catwalk would pay the reinsurer for “borrowing” capital equal to the Capital Relief from the reinsurer. The lower the Ceded ROE, the better the terms Catwalk gets for the reinsurance.

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3 The use of Risk Loading Factors is a simplistic, approximate approach to guesstimating the way reinsurers actually do pricing. The Risk Loading Factor varies by reinsurer, line of business and peril, among other things.
Analysis and Decision

The decision to select the best XOL RL may be based on a number of considerations. In particular:

1. Layers that give higher reward for lower risk are preferred.
2. Layers with Ceded ROE greater than a threshold value may not be considered as they may be considered prohibitively expensive relative to internal ROE requirements.
3. Other conditions that are company specific and are internal to the company, such as the rating of a reinsurance company, Catwalk’s risk appetite, and other financial considerations.

To help Catwalk select an XOL RL, you will need to calculate the following:

1. Catwalk’s 99% VaR Capital Requirements without buying any reinsurance.
2. Catwalk’s 99% VaR Capital Requirements after buying an XOL RL – for each of the 12 options.
3. The Capital Relief provided by each XOL RL (difference between (1) and (2)).
4. The Reinsurance Premium Catwalk would need to pay the reinsurer for each XOL RL.
5. The risk-reward profile of each XOL RL. A risk-reward curve (for the 12 XOL RLs) can be plotted. Two such plots are:
   a. The Reinsurance Premium plotted against the Capital Relief provided by each XOL RL;
   b. The Ceded ROE plotted against (1 – the ratio of the standard deviation of Catwalk’s losses before and after reinsurance), measuring the volatility reduction provided by the XOL RL.

To estimate these, you can use (among other tools) a simulation tool to simulate the possible future outcomes (e.g., the severity and annual number of claims). In particular, a simulation is run for each possible XOL RL of Table 1. For each simulation, 10,000 trials (“possible future outcomes”) are run. For each trial, you should:

1. Randomly generate the number of claims in a year using the Poisson distribution with parameter $\lambda = 10$.
2. For each claim, generate a random severity loss (this is the loss for that particular claim) using the lognormal distribution with mean $\mu = 11$ and standard deviation $\sigma = 2.10$.
3. Calculate the total losses across all claims;
4. Calculate the Ceded Losses that will be passed to the reinsurer.

After running the 10,000 trials (you need to run all trials in order to estimate the mean, variance, and 99% VaR of the Ceded Losses, for example), you should calculate:

1. The mean, standard deviation, and 99th percentile value of the reinsurance’s Ceded Losses.
2. The Reinsurance Premium Catwalk would pay for each XOL RL.
Based on these steps (and a few others you need to figure out), you now need to prepare your final presentation to Catwalk’s management team. You must be ready to answer the following questions.

**Questions**

1. Following the steps outlined above, generate the suggested risk-return plots required to recommend an XOL RL, as well as any other plots you may find useful to make a decision.

2. Which XOL RL would you recommend? Why?

3. What issues would you expect Catwalk’s management team to raise about your analysis?
**Exhibit 1**

**Glossary**

**Excess of Loss Reinsurance:** A contract between an insurer and a reinsurer, whereby the insurer agrees to pay a specified proportion of a claim and the reinsurer to pay all or a part of the claim above that amount and up to some maximum.

**Attachment Point:** The amount of loss at which reinsurance begins to apply.

**Limit Point:** The maximum amount of loss that is covered by the reinsurance beyond the Attachment Point.

**Ceded Losses:** The losses covered by the reinsurance, hence transferred (ceded) from the insurance to the reinsurance. The Ceded ROE of the insurance company is like “the interest” insurance companies pay for the freed up capital, hence it should be minimised.

**Reinsurance Premium:** The premium paid by the insurer to the reinsurer for the liability assumed by the reinsurer.

**Loading Factors:** Multipliers used to scale a “fair” reinsurance premium. They reflect, for example, administrative costs, risk appetite, margin requirements, and other factors.

**Lognormal Distribution:** When the logarithm of a variable is normal, the variable itself is lognormal.

**Direct Pricing Loss Ratio:** The historical ratio of the “losses” (claims paid + other expenses + profit margin + margin for risk) to the premium collected in the past.

**Capital Requirement:** The minimum amount of capital required of an insurer to support its operations and write coverage.

**Non-exceedance Probability:** The likelihood that a particular level of loss will not be exceeded in some time period

**Claim Severity:** The average amount of loss per loss claim.

**Probable Maximum Loss:** This is defined by taking a particular exceedance probability (typically 1% or 0.5% corresponding to 1 in 100 years or 1 in 200 years) and then putting sufficient capital into place to assure the ability to withstand that level of loss.
**Exhibit 2**


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**LA**: Attachment Point

**LL**: Limit Point

**PML**: Probable Maximum Loss

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