

Learning from Stock Prices and Economic Growth

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A competitive stock market is embedded into a neoclassical growth economy to analyze the interplay between the acquisition of information about firms, its partial revelation through stock prices, capital allocation, and income. The stock market allows investors to share their costly private signals in a cost-effective incentive-compatible way. It contributes to economic growth by raising total factor productivity (TFP). A calibration indicates the effect on TFP to be large but that on income to be modest. Several predictions on the evolution of real and financial variables are derived. Finally, the growth impact of two common forms of investor irrationality, overconfidence and inattention, are analyzed. (*JEL* O16, G11, G14)

Economic institutions are widely believed to play a crucial role for economic growth. In particular, there is now considerable evidence that financial institutions, once considered a sideshow, promote economic growth by relaxing constraints undermining the efficiency of investments. In this paper, we analyze the role of one such institution, the stock market, in alleviating one such constraint, investors' inability to perfectly communicate their private information. Economists have long argued that stock prices improve the allocation of capital by aggregating dispersed information and pointing to the most promising investment opportunities. Although several aspects of the relation between the stock market and the real economy have been examined, "existing theories have not yet assembled the links in the chain from the functioning of stock markets, to information acquisition, and finally to aggregate long-run economic growth" (Levine 1997, 695; see also Levine 2005, p.9). This paper assembles these links.

We merge two standard frameworks—the neoclassical overlapping generations growth model (e.g., Diamond's 1965) and the noisy rational expectations model of the stock market (Grossman and Stiglitz 1980)—in order to present a fully integrated model of information acquisition and dissemination through

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prices, capital allocation, and economic growth. Merging is not straightforward because capital, and output are nonlinear functions of productivity shocks in the neoclassical economy, which make the extraction of information from stock prices an intractable problem. We circumvent this issue by making a small risk approximation.¹

The main features of the model are the following. The economy is composed of two sectors—a final and an intermediate goods sector, and overlapping generations of agents who work in the final goods sector and invest their wage in the intermediate goods sector. Decreasing returns to intermediate goods employed as capital in the final goods sector produce the neoclassical tendency for income to grow at a decreasing rate until it reaches a steady state in which it no longer grows (there is neither technological progress nor population growth in the model). The intermediate goods sector is composed of many firms that raise capital from the young through the stock market. Firms' productivity is unknown, but agents can collect private signals about it at a cost. Specifically, they are endowed with one unit of free time, which they can devote either to analyzing stocks or to enjoying leisure. This specification implies that the cost of learning about the stock market—foregone leisure—varies endogenously over time as a function of the relative valuation of goods and leisure. Agents' information is reflected in stock prices, but only partially because of the presence of noise. Prices in turn guide investors in their portfolio allocations.

We are interested in situations in which sharing information is highly beneficial; for example because the intermediate firms operate complex new technologies, about which opinions diverge widely. Accordingly, we make two assumptions on the information structure. First, private signals have errors that are independent across agents. Second, their cost is an increasing and convex function of their precision.² As a result, agents are better off collecting imprecise signals and pooling them. In fact, the first-best capital allocation is reached when agents collect signals of infinitesimal precisions (arbitrarily close, but not equal, to zero) and combine them to discover firms' productivity, thanks to the Law of Large Numbers. But it may not be possible to share dispersed information that is produced at a cost and privately because of issues with incentives and communication. In the current setup, the first best is not a Nash equilibrium, unless signal precisions are contractible: agents' best response is to set their precisions to zero and report noise, which results in no learning.

¹ Rational expectations models of competitive stock trading under asymmetric information typically conjecture that equilibrium stock prices or trades are linear functions of random variables. This conjecture is not valid in the neoclassical framework with productivity shocks, because productivity and capital interact multiplicatively in the production of goods; that is, the production function is of the type $Y \equiv Af(K)$, where Y denotes output, A productivity, and K capital. We solve for a rational expectations equilibrium in closed form when productivity shocks are small.

² The model assumes further that each generation consists of a continuum of agents and that the marginal cost of information equals zero at the origin.

The stock market provides the means to share private information in a cost-effective and incentive-compatible way. For example, when agents receive optimistic signals about a firm, they buy its shares and bid up its stock price. The high stock price in turn indicates that investors collectively believe the firm to have good prospects. Thanks to stock prices, agents are better informed even though no new information is actually produced. Of course, the effectiveness of the stock market is limited by the very existence of informative prices, which undermine the incentive to collect costly information in the first place. Indeed, investors' cannot fully appropriate the benefit of their signals as they are leaked to competitors through prices (the Grossman-Stiglitz paradox). Thus, informative stock prices have an impact that is beneficial *ex post* but detrimental *ex ante* to capital efficiency. Noise trading—some trades are motivated by random shocks unrelated to fundamentals—provides the smoke screen behind which investors can conceal their informed trades and reap some benefit. The first contribution of the paper is to dissect this trade-off, highlighting its real effects on the allocation of capital. We find that the information-sharing benefit outweighs the disincentive cost in our setup. That is, agents, though they reduce the precision of their private signals in response to a decline in the intensity of noise trading, are nevertheless better informed on the whole, thanks to the increased accuracy of stock prices. The allocation of capital improves and, moreover, converges to the first best as the intensity of noise trading approaches zero.

The second contribution we make is to characterize the dynamics of information and income in the stock market economy. Income is governed by a standard neoclassical law of motion: it grows at a decreasing rate until it reaches a steady state. The learning process thus has no bearing on long run growth—it does not counter the diminishing returns to capital, but it does influence the long-run level of income and therefore its transitory growth rate. The dynamics of learning in turn are shaped by two competing forces. On one hand, the cost of learning—forgone leisure enjoyment—grows with income. Indeed, agents with a higher wage consume more of the final good because they saved more, which reduces its marginal utility. Hence, they would rather spend less time on information collection and more on leisure (the substitution effect). On the other hand, information generates increasing returns to scale—its benefit, unlike its cost, rises with the amount to be invested. The substitution effect leads wealthier agents to learn less, while the scale effect of information induces them to learn more.

If the scale effect of information dominates the substitution effect, then investors produce more private information as their income grows, which then allows them to invest more efficiently in the intermediate firms. The resulting increased supply of intermediate goods in turn enhances the marginal product of labor and makes the next generation of workers richer. In this case, income grows at an accelerated rate compared with a standard neoclassical economy; that is, the growth rate of income falls less quickly. If instead the substitution

effect dominates, wealthier investors collect less private information and invest less efficiently, so the growth rate of income declines faster. In both cases, the revelation of information through prices increases the steady state level of income and its transitory growth rate.³ In this way, the stock market contributes to economic growth.

The model, in the case in which the scale effect of information dominates the substitution effect, is consistent with broad features of the data. First, it predicts that the stock market develops (e.g., as measured by the time spent analyzing stocks) in tandem with income and contributes to economic growth. Empirically, [Levine and Zervos \(1998\)](#), [Rousseau and Wachtel \(2000\)](#), and [Carlin and Mayer \(2003\)](#) document that income grows faster in countries with better functioning stock markets.⁴ Second, in the model, capital is more efficiently allocated across firms as the economy grows; that is, more (less) capital is channelled to more (less) productive firms. This superior efficiency translates into higher total factor productivity (TFP), even though there is no technological progress.⁵ Empirically, [Wurgler's \(2000\)](#) documents that investments are more responsive to value-added in more financially developed countries and in particular in countries with a more informative stock market. Furthermore, [Levine and Zervos \(1998\)](#) show that stock markets promote TFP growth, rather than capital growth. Third, in the model, the stock market is particularly useful for investing in innovative technologies, consistent with [Carlin and Mayer's \(2003\)](#) finding that the impact of the stock market is stronger for industries with high R&D investments and skilled labor. Finally, the model implies that the stock market processes information only when income exceeds a threshold, again a consequence of the increasing returns to information. This matches the casual observation that financial institutions only emerge once a critical level of income is reached.

A calibration of the model gives a sense of how important a role the stock market plays. It uses data for the United States, assuming that the United States has reached a steady state. Firms in the model are interpreted as (four-digit SIC) industries, and shocks to their productivity are calibrated to the U.S. manufacturing sector, assuming a factor share of capital of one-third. To parametrize the information environment in a way that, as much as possible, is independent of nonobservables such as the cost of information and the variance

³ Strictly speaking, this statement requires the intensity of noise trading to be low enough. The reason is that noise trading, in addition to making investments less efficient, has a direct influence on the average income, which is unrelated to the learning process and is beneficial, thanks to a Jensen inequality effect, in which positive noise shocks increase output more than negative shocks decrease it. This effect diminishes as the intensity of noise trading weakens.

⁴ In the model, the effect of the stock market on growth is only transitory because of the absence of technological progress. Section 6.4 discusses in detail the evidence in light of the model.

⁵ TFP, also known in the growth literature as the "Solow residual," is defined as the residual from a regression of income growth on factor growth. It encompasses any factor, beyond labor growth and capital growth, that contributes to output growth. Empirically, most of the differences in income across countries and periods stem from differences in TFP (e.g., [Hall and Jones 1999](#)).

of noise trading, we rely on Wurgler's (2000) study of capital efficiency. Wurgler measures the efficiency of investments in a country by running a regression of investment growth in an industry on value-added growth in the industry. The slope coefficient represents the elasticity of investment to value-added in the country. The baseline estimate of 0.723 for the United States implies that investment will increase (respectively, decrease) by 7.23%, on average, in an industry subject to a +10% (respectively, -10%) shock to value-added growth. This elasticity, together with the R^2 of the regression, can be computed explicitly in the model and used to infer the precision of private and price signals. Thus, the data reported in Wurgler's (2000), together with the productivity parameters, allow us to completely characterize the information environment. The baseline calibration indicates that private and price signals account for, respectively, 41% and 7% of agents' total information (the residual comes from their priors). It also yields reasonable estimates of stock return volatility and forecast errors. It reveals that the investment elasticity and TFP equal about half of their first-best levels but that income per capita is only 8% below. The muted response for income is the result of the strongly decreasing returns to capital, which limit the benefit from a more efficient allocation.

The third contribution we make is to derive additional observable properties of the economy during its transition to the steady state, particularly properties characterizing the evolution of the stock market.⁶ As the economy grows starting from an initial wage below its steady state level, (1) the economy specializes. Indeed, agents invest more selectively, leading capital and profits to become more concentrated across firms. (2) Income inequality follows a "Kuznets curve," widening at first and then narrowing. (3) Stock market liquidity (the inverse of the sensitivity of stock prices to uninformative noise shocks) and share turnover (the ratio of the value of shares traded to the total capitalization of the market) increase at first and then decrease. Inequality, liquidity, and turnover display similar nonmonotonic behaviors because all three are driven by the extent to which investors disagree about stocks. At the early stage of development, agents follow mostly price signals because their private signals are imprecise, so disagreement is low. As their private signals become more accurate, agents rely more on them, so disagreement, inequality, trading volume, and liquidity rise with income. But they decrease beyond a level of income because private signals that are more precise are also more similar. (4) The volatility of stock prices rises with income as they track technology shocks more closely. As a result, stock returns, which absorb residual shocks, fluctuate less and display lower idiosyncratic and total volatility. In contrast, the volatility of the market (price and return) is constant. It follows that the cross-correlation of stock prices falls, whereas that of stock returns rises. The evidence

⁶ The patterns (1) through (4) are reversed if the substitution effect dominates the scale effect.

about these implications is missing or mixed and calls for more empirical work (see Section 6.4 for a discussion).

The final contribution we make is to analyze the growth impact of investor irrationality. We consider two forms of irrationality among the most pervasive, inattention and overconfidence. They yield opposite implications for capital efficiency. Inattention, on one hand, hurts capital efficiency because having fewer stocks to choose from implies less value from being able to discriminate between them and less information in equilibrium. Overconfidence, on the other hand, improves capital efficiency: overconfident investors overestimate the precision of their private signals so they trade on them more aggressively, leading to more informative prices. Moreover, the calibrated model suggests that the growth impact of both types of investor irrationality is modest.

1. Related Literature

Our work relates to three main strands of theory. First and foremost, it contributes to the theoretical literature on finance and growth. Most closely related is the seminal paper by [Greenwood and Jovanovic \(1990\)](#). In their setup, investors choose whether to invest directly in their own project or through a financial intermediary in exchange for a fee. The intermediary, by pooling numerous individual projects, discovers the state of the economy and offers a higher return on capital for less risk, thereby promoting growth. In both papers, economic and financial development feed on each other, but there are three important differences. First and foremost, Greenwood and Jovanovic neither specify where investors' private signals (projects) come from, nor how they are pooled. In particular, they do not study agents' incentives to produce and communicate private information. In contrast, we explicitly address these issues: we model how investors make their decisions to collect costly signals, and how the stock market aggregates and transmits these signals. Putting it differently, [Greenwood and Jovanovic \(1990\)](#) examine an economy free from contracting and communication frictions, whereas we consider an economy in which these frictions are so severe that eliciting effort and exchanging information between investors is impossible. Moreover, we can characterize the evolution of several observable features of the stock market as the economy grows, such as the volatility of stock returns and the trading intensity.

The second difference concerns the cost of financial intermediation, assumed to be constant in [Greenwood and Jovanovic \(1990\)](#), whereas its counterpart in our model, the cost of information, grows endogenously with income. Indeed, information is produced at the expense of leisure, whose value rises with income. As a result, the financial sector in [Greenwood and Jovanovic's \(1990\)](#) setting always develops with income, when in our setting, it does so only if the value of information increases faster than its cost. Finally, [Greenwood and Jovanovic \(1990\)](#) obtain a permanent growth effect, whereas we do not. But

this difference arises only because they assume that capital displays constant returns to scale, whereas we assume that it is subject to diminishing returns.

Second, our work is connected to the endogenous growth literature (see [Aghion and Howitt \(1998\)](#) for an overview). This literature models the discovery of technologies by profit-maximizing agents. In contrast to this literature, we endow the economy with technologies and focus instead on their selection by investors trading on the stock market. Similar issues arise nonetheless. In particular, technical innovations and information about stocks both give rise to increasing returns to scale, limited by the incomplete appropriability of the rents generated.⁷ Whether or not long-run growth is possible depends on the law of motion postulated for technological progress rather than on the structure of the models. When technological progress is assumed away, we find that the information technology cannot generate any permanent growth effect.

Finally, our work belongs to the literature on trading under endogenous and asymmetric information, and in particular to the subset emphasizing the real benefits of informational efficiency. Several authors argue that stock markets are best suited for aggregating information that is dispersed and serendipitous, whereas banks, on the other hand, can more efficiently produce standardized information and avoid duplication costs. Unlike much of this research, the real effect studied here stems from stock prices helping investors allocate capital across firms rather than assisting managers with their firm's investment plans.⁸ Our model contributes to this literature by developing a rational expectations framework in which learning from stock prices and income interact dynamically. Moreover, it offers a quantitative assessment of this effect, and considers departures from investor rationality.

2. Economic Environment

We embed a competitive stock market à la [Grossman and Stiglitz \(1980\)](#) into [Diamond's \(1965\)](#) neoclassical growth economy. The economy is composed of two sectors—a final and an intermediate goods sector—and overlapping generations of agents. Firms in the intermediate goods sector raise capital on the stock market by issuing claims to their future profits. Young agents save by purchasing these claims.

⁷ Increasing returns arise from the nonrivalry of information: information is costly to generate but costless to replicate. Endogenous growth models preserve incentives to do research by granting market power to innovators, whereas models of the stock market introduce noise into the price system. For applications to finance, see, for example, [Acemoglu and Zilibotti \(1999\)](#), [Veldkamp \(2005, 2006\)](#), and [Zeira \(1994\)](#).

⁸ On the financial structure of the economy, see, for example, [Allen \(1993\)](#), [Allen and Gale \(2000\)](#), [Dow and Gorton \(1997\)](#), [Boot and Thakor \(1997\)](#), and [Subrahmanyam and Titman \(1999\)](#). On the feedback effect from financial markets to the real economy, see, for example, [Fishman and Hagerty \(1992\)](#), [Holmström and Tirole \(1993\)](#), [Goldstein and Guembel \(2008\)](#), [Dow, Goldstein, and Guembel \(2010\)](#), and [Bond and Goldstein \(2012\)](#).

2.1 Agents

The economy is populated by overlapping generations of agents who live for two periods. There is no population growth. Each generation consists of a continuum of agents with mass L indexed by $l \in [0, L]$. Young agents are each endowed with one unit of labor time and one unit of free time. Utility, derived from the consumption of the final good g and leisure j , is represented by a function $U(g, j)$, increasing and concave in each argument and with a positive cross-derivative, $\partial^2 U / \partial g \partial j$. Two aspects of preferences are of particular relevance to our analysis: risk aversion and the degree of substitutability between final goods and leisure. We define the following functions:

$$\tau(g) \equiv -\frac{\frac{\partial U}{\partial g}(g, 1)}{\frac{\partial^2 U}{\partial g^2}(g, 1)} \quad \text{and} \quad \rho(g) \equiv \frac{\frac{\partial U}{\partial j}(g, 1)}{\frac{\partial U}{\partial g}(g, 1)}.$$

$\tau(g)$ measures the absolute risk tolerance of an agent consuming g units of the final good and one unit of leisure. τ captures attitudes toward risk because leisure consumption is deterministic. We assume that τ is increasing in g , as supported by most empirical studies. The function ρ measures the marginal rate of substitution between final goods and leisure, again for an agent consuming g units of the final good and one unit of leisure. Naturally, ρ is increasing in g because the marginal utility of the final good declines, whereas that of leisure rises when more final goods are consumed.

For example, $U(g, j) \equiv (\varpi g^\sigma + (1 - \varpi)j^\sigma)^{1/\sigma}$, where ϖ is in $(0, 1)$ and $\sigma < 1$, displays a constant elasticity of substitution (CES). As σ goes to 0, the function converges to a Cobb-Douglas utility ($U(g, j) \equiv g^\varpi j^{1-\varpi}$). Under these preferences, $\tau(g) = g(\varpi g^\sigma + 1 - \varpi)/(1 - \sigma)/(1 - \varpi)$ and $\rho(g) = g^{1-\sigma}(1 - \varpi)/\varpi$, and the elasticity of substitution between goods and leisure equals $1/(1 - \sigma)$.

Young agents are employed in the final good sector, to which they supply their unit of labor time inelastically for a competitive wage w_t , so aggregate labor supply equals L . They save their entire labor income by investing in the stock market to consume in the next period when they are old.⁹ They divide their unit of free time between enjoying leisure and analyzing stocks. There are no short-sales constraints, and no riskless asset.¹⁰

⁹ These assumptions simplify the model and are broadly consistent with evidence. Concerning labor supply, Francis and Ramey (2009) estimate that leisure per capita has remained constant in the United States throughout the 20th century. With regard to the savings rate exogenously set to one, Bonser-Neal and Dewenter (1999) report that there is no relation between savings rates and stock market development, and Levine and Zervos (1998) and Beck, Levine and Loayza (2000) report that financial development enhances growth through higher productivity rather than through higher saving rates.

¹⁰ We assume that there is no storage technology and that final wealth is not verifiable. The latter assumption rules out lending because final wealth equals zero with a nonzero probability. Borrowers would simply claim that they are unable to repay their loans.

2.2 Technologies

2.2.1 Final good sector. The final good is produced according to a riskless technology that employs labor and intermediate goods: $G_t \equiv L^{1-\beta} \sum_{m=1}^M (Y_t^m)^\beta$, where G_t is final output; L is labor; M is the number of types of intermediate goods; Y_t^m is the employment of the m 'th type; and $0 < \beta < 1$ is the factor share of intermediate goods in the production of the final good. The production function follows Dixit and Stiglitz (1977) and Romer (1990), among others. Many identical firms compete in the final good sector and aggregate to one representative firm. The final good is used as the numeraire. It can be consumed by agents or invested to produce intermediate goods in the following period.

2.2.2 Intermediate good sector. M firms operate in the intermediate goods sector. Firm m is the exclusive producer of good m . Its production is determined by a risky technology that displays constant returns to capital: $\tilde{Y}_{t+1}^m \equiv \tilde{A}_t^m K_t^m$ for $m = 1, \dots, M$, where \tilde{Y}_{t+1}^m is the quantity of intermediate goods produced in period $t + 1$ by firm m net of capital depreciation; \tilde{A}_t^m is its random productivity; and K_t^m is the amount of capital (which consists of final goods) it raises in period t . Tildes denote random variables not yet realized. Firms are liquidated immediately after production.¹¹

The productivity shocks \tilde{A}_t^m are log-normally distributed and independent from one another and over time. Because there is no closed-form solution to investors' portfolio choice under general preferences, we resort to a small-risk expansion to solve the model. We consider small productivity shocks and log-linearize the return on investors' portfolio. Specifically, we assume that $\ln \tilde{A}_t^m \equiv \tilde{\alpha}_t^m z$, where $\tilde{\alpha}_t^m z$ is normally distributed with mean $\tilde{\alpha}_t^m z$ and variance $\sigma_a^2 z$; $\tilde{\alpha}_t^m$ is normally distributed with mean 0 and variance σ_a^2 ; and z is a scaling factor. The model is solved in closed form by driving z toward zero. Throughout the paper, we assume that z is small enough for the approximation to be valid.¹²

Firms raise capital in the stock market. Firm m issues one perfectly divisible share—a claim to its entire future profit, for a price P_t^m . The productivity shock \tilde{a}_t^m is not observed at the time agents invest, but they can learn about its average $\tilde{\alpha}_t^m$ as we describe next.

2.2.3 Information technology. At the time they invest, agents do not observe intermediate firms' productivity. Instead, they receive private signals about

¹¹ Assuming firms are liquidated just after production simplifies the dynamics of the economy and allows us to focus on the early stage of a firm's development. It is well known that young firms, because they have little retained earnings, are more dependent on external financing than mature firms. Several empirical studies confirm that financial development fosters growth mainly through young firms (Rajan and Zingales 1998; Beck, Demirgüç-Kunt, and Maksimovic 2005; Brown, Fazzari, and Petersen 2008).

¹² Rational expectations models of competitive stock trading under asymmetric information typically conjecture that equilibrium stock prices are linear functions of random variables. This conjecture is not valid in a neoclassical framework because productivity and capital interact multiplicatively in the production of goods, and capital itself is a function of stock prices. For examples of small risk expansions applied to portfolio choice, see Campbell and Viceira (2002) and Peress (2010a).

its mean. The private signal $s_{l,t}^m$ received by agent l in period t about firm m 's average productivity shock is given by $s_{l,t}^m = \beta \tilde{\alpha}_t^m + \tilde{\varepsilon}_{l,t}^m$, where $\tilde{\varepsilon}_{l,t}^m$ is an agent-specific disturbance independent of $\tilde{\alpha}_t^m, \tilde{\alpha}_t^m$, across firms and time. $\tilde{\varepsilon}_{l,t}^m$ is normally distributed with mean 0 and variance $1/x_{l,t}^m$ (precision $x_{l,t}^m$). Investors choose the precision of their signals before the stock market opens. Observing a signal of precision $x_{l,t}^m$ costs $C(x_{l,t}^m)z$ units of free time, where C is continuous, increasing, convex and $C(0)=C'(0)=0$. We emphasize that the information technology neither leads to the discovery of new physical technologies nor improves existing ones. Instead, it allows society to invest more efficiently in the physical technologies.

2.2.4 Noise trading. Agents know that stock prices reflect other investors' private information in equilibrium, and they learn from them. Some noise is needed to blur price signals and avoid the Grossman-Stiglitz paradox, that is, preserve incentives to collect costly information. We assume that a fraction q of agents form their portfolio guided by exogenous shocks. Specifically, they believe that the expected return on stock m equals $\tilde{\theta}_t^m$, where $\tilde{\theta}_t^m$ is normally distributed with mean 0 and variance σ_θ^2 , and is independent of $\tilde{\alpha}_t^m, \tilde{\varepsilon}_{l,t}^m$, across firms and time.¹³

2.3 Timing

The time line is summarized in Figure 1. An agent lives one period as a young agent (as a worker, then as an investor) and one period as an old agent (as a consumer). After earning a wage and before the stock market opens, workers choose how to divide their free time between stock analysis and leisure, by setting the precision of their signals. Then, they invest their wage across the different stocks, guided by stock prices and their private signals. In the following period, the young become old, productivity shocks are revealed, final goods are produced, and old agents consume their share of profits.

2.4 Notation

For any firm-specific variable, $\psi_t^m, \bar{\psi}_t \equiv \frac{1}{M} \sum_{m=1}^M \psi_t^m$ denotes its average across firms, $\Delta \psi_t^m \equiv \psi_t^m - \bar{\psi}_t$ its deviation from the average, and $\{\psi_t^m\}$ the vector of stacked variables for $m=1$ to M . Finally, we adopt the following notation to keep track of the quality of the approximation: $o(1), o(z)$, and $o(z^2)$ capture, respectively, terms of an order of magnitude smaller than 1, z , and z^2 .

¹³ Alternatively, noise shocks could stem from liquidity needs, preference shifts, random stock endowments, or private risky investment opportunities. The formulation adopted here has the following properties. First, the accuracy of noise traders' beliefs is arbitrary and does not affect our findings. Second, including an agent-specific component to these beliefs has no incidence on the equilibrium. Third, the intensity of noise trading remains commensurate with that of rational trading as the economy grows. As Equation (9) shows, portfolio holdings are scaled by a function of income, $\tau(\varphi(w))/\varphi(w)$. If, for example, this function increases with income (e.g., $\sigma > 0$ under CES utility), then trades, both rational and noise-motivated, grow with the economy. If we assume instead that noise trades equal an exogenous constant, then they will shrink relative to rational trades. This will mechanically make stock prices more informative and the allocation of capital more efficient, and reinforce the usefulness of the stock market.

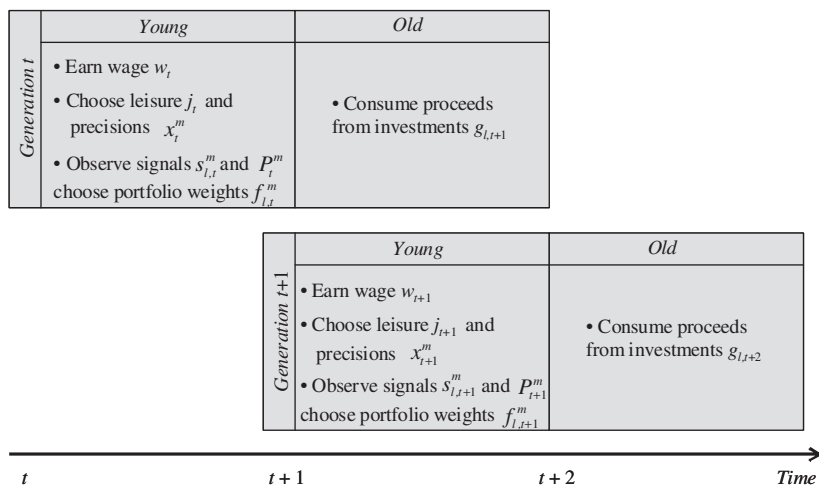


Figure 1
Timing

2.5 Equilibrium concept

We describe the equilibrium concept working backwards from production in period $t + 1$, to capital allocation and information acquisition in period t . The gains from trade depend on how much information is collected in aggregate and revealed through prices. We denote $X_t^m \equiv \int_1 x_{l,t}^m / L$ as the average precision of private information about firm m . A rational expectations equilibrium satisfies the following conditions.

- (1) Market clearing in the intermediate goods sector

Final goods producers maximize their profit. Because labor and intermediate goods trade in competitive markets and aggregate labor supply equals L , the following equilibrium factor prices (denominated in units of the final good) obtain in period $t + 1$:

$$\tilde{w}_{t+1} = (1 - \beta) \sum_{m=1}^M (\tilde{Y}_{t+1}^m / L)^\beta \quad \text{and} \quad \tilde{\rho}_{t+1}^m = \beta (L / \tilde{Y}_{t+1}^m)^{1-\beta}, \quad (1)$$

where $\tilde{\rho}_{t+1}^m$ denotes the $t + 1$ price of intermediate good m and $\tilde{\Pi}_{t+1}^m = \tilde{\rho}_{t+1}^m \tilde{Y}_{t+1}^m$ is firm m 's profit.

- (2) Capital allocation

Let $f_{l,t}^m$ denote the fraction of her wage that agent l invests in stock m in period t or her “portfolio weights.” She sets $\{f_{l,t}^m\}$ to maximize her expected utility, guided by stock prices and private signals, and taking as given her income w_t , her leisure time j_t , the precision of her signals $\{x_{l,t}^m\}$, the average precisions

$\{X_t^m\}$, share prices, and capital stocks:

$$\max_{\{f_{l,t}^m\}} E[U(\tilde{g}_{l,t+1}, j_t) | \mathcal{F}_{l,t}] \quad \text{subject to} \quad \begin{cases} \tilde{g}_{l,t+1} = w_t \tilde{R}_{l,t+1} \\ \tilde{R}_{l,t+1} = \sum_{m=1}^M f_{l,t}^m \tilde{R}_{t+1}^m \\ \sum_{m=1}^M f_{l,t+1}^m = 1 \end{cases}, \quad (2)$$

where $\mathcal{F}_{l,t} \equiv \{s_{l,t}^m, P_t^m \text{ for } m = 1 \text{ to } M\}$, $\tilde{g}_{l,t+1}$, $\tilde{R}_{l,t+1}$ and $\tilde{R}_{t+1}^m = \tilde{\Pi}_{t+1}^m / P_t^m$ denote, respectively, agent l 's information set, her consumption of the final good, the return on her portfolio, and the return on stock m . The time subscripts on j_t and $\tilde{g}_{l,t+1}$ make clear that leisure time is set at t before private signals are observed, whereas the consumption of final goods is determined at $t + 1$, once the return on the portfolio is realized. We call $U_0(\{x_t^m, X_t^m\}, j_t, w_t)$ the value function for this problem.

In equilibrium, prices clear the stock market. Because each firm issues one share, its capital stock coincides with its stock price: Formally,

$$\int w_t f_{l,t}^m = K_t^m = P_t^m \quad \text{for } m = 1, \dots, M,$$

where the integral sums the demand emanating from rational and noise traders.

(3) Precision choice

An agent's optimal precisions $x_{l,t}^m = x(w_t, \{X_t^m\})$ maximize her ex ante expected utility subject to her free time budget constraint, taking her income w_t and the average precisions $\{X_t^m\}$ as given:

$$\max_{\{j_t \geq 0, x_{l,t}^m \geq 0\}} E[U_0(\{x_{l,t}^m, X_t^m\}, j_t, w_t)] \quad \text{subject to} \quad \sum_{m=1}^M C(x_{l,t}^m)z + j_t = 1,$$

where $C(x_{l,t}^m)z$ is the time spent investigating stock m and $1 - \sum_{m=1}^M C(x_{l,t}^m)z$ is the time left for leisure. In equilibrium, the average and optimal precisions must be consistent:

$$X_t^m = x(w_t, \{X_t^m\}) \quad \text{for } m = 1, \dots, M.$$

3. First Best

Before we proceed to the general case, we describe a benchmark, the first-best outcome, in which agents perfectly share their information. The first best is achieved when signal precisions are contractible and there is no noise trading. In that case, agents all commit to infinitesimal precisions that are very close, but not equal, to zero, and reveal their private signals to a central planner who invests on their behalf. The central planner can perfectly infer average

productivity shocks $\{\tilde{\alpha}_t^m\}$ thanks to the Law of Large Numbers because there is a continuum of signals with finite variances and uncorrelated errors ($\int_t \varepsilon_{l,t}^m = 0$). The central planner chooses a capital allocation $\{K_t^{mFB}\}$ to maximize agents' expected utility subject to an economy-wide resource constraint, taking as given their income w_t :

$$\max_{\{K_t^{mFB}\}} E[U(\tilde{g}_{l,t+1}, 1) | \{\tilde{\alpha}_t^m\}] \quad \text{subject to} \quad \begin{cases} \tilde{g}_{l,t+1} = \sum_{m=1}^M \tilde{\Pi}_{t+1}^{mFB} / L \\ \sum_{m=1}^M K_t^{mFB} = Lw_t \end{cases}, \quad (3)$$

where $\tilde{\Pi}_{t+1}^{mFB} = \beta L^{1-\beta} (\tilde{A}_t^m K_t^{mFB})^\beta$ denotes the profit generated by firm m , to be divided equally between agents. The following lemma describes the capital allocation in this economy.

Lemma 1. In the first-best outcome, firm m 's capital stock equals $K_t^{mFB} = \frac{Lw_t}{M} \exp(\Delta k_t^{mFB} z)$, where

$$\Delta k_t^{mFB} = \frac{1}{1-\beta} \Delta \beta \tilde{\alpha}_t^m + o(1). \quad (4)$$

When z , the factor that scales shocks, equals zero, the firms are perfectly identical so capital is equally distributed, each firm receiving Lw_t/M units of goods.¹⁴ When $z > 0$, the allocation depends on firms' productivity relative to one another. The more productive firms (higher $\Delta \tilde{\alpha}_t^m \equiv \tilde{\alpha}_t^m - \bar{\alpha}_t$) receive more capital. The elasticity of investments to productivity shocks, $\partial(\ln K_t^{mFB}) / \partial \tilde{\alpha}_t^m = (1 - 1/M)\beta / (1 - \beta)$, captures the efficiency of the capital allocation. It increases with β , the factor share of capital, because a higher β indicates that more capital can be channeled to the better firms without immediately damaging their marginal product. Efficiency also increases with the number of stocks M because there is a wider choice of uses for capital.

Given its capital stock, firm m produces $\tilde{Y}_{t+1}^m = \tilde{A}_t^m K_t^{mFB}$ intermediate goods. As a result, the number of final goods produced and the wage equal, respectively,

$$\tilde{G}_{t+1} = Lw_t^\beta M^{1-\beta} \exp(\beta(\tilde{\alpha}_t^m z + k_t^{mFB} z)),$$

$$\tilde{w}_{t+1} = (1 - \beta) \tilde{G}_{t+1} / L = (1 - \beta) w_t^\beta M^{1-\beta} \exp(\beta(\tilde{\alpha}_t^m z + k_t^{mFB} z)).$$

The wage is random as it depends on the realizations of the productivity shocks. The following lemma characterizes the dynamics of the economy along its average path; that is, assuming that the wage realized in any period equals its mean. This is a good description of the economy if the number of firms is large.

¹⁴ Firm m 's marginal profit, $\partial \Pi_{t+1}^{mFB} / \partial K_t^{mFB} = \beta^2 L^{1-\beta} A_t^{m\beta} K_t^{mFB m\beta - 1}$, is a decreasing function of K_t^{mFB} . Hence, if firms are identical, the central planner distributes capital equally across the M firms.

Lemma 2. In the first-best outcome, average income evolves according to the following equation:

$$E(\tilde{w}_{t+1}) = \Lambda \exp(\lambda^{FB} z^2) w_t^\beta, \tag{5}$$

where Λ and λ^{FB} are positive constants given by

$$\Lambda \equiv (1 - \beta) M^{1-\beta} \exp\left(\frac{1}{2} \beta^2 (\sigma_a^2 z + \sigma_\alpha^2 z^2)\right), \tag{6}$$

$$\text{and } \lambda^{FB} \equiv \frac{M-1}{M} \frac{\beta^3}{(1-\beta)^2} \left(1 - \frac{\beta}{2}\right) \sigma_\alpha^2 + o(1). \tag{7}$$

Average income converges to a steady state w^{FB} , given by:

$$w^{FB} = \Lambda^{1/(1-\beta)} \exp\left(\frac{\lambda^{FB}}{1-\beta} z^2\right). \tag{8}$$

The average wage evolves according to a standard neoclassical law of motion, illustrated by the dashed curves in Figures 6 and 7. The marginal product of labor increases with current income (assuming income is initially below its steady state value) but at a decreasing rate, until it reaches a steady state at which it no longer grows. The growth rate of income is given by $\Gamma^{FB}(w_t) \equiv E(\tilde{w}_{t+1})/w_t = \Lambda w_t^{-(1-\beta)} \exp(\lambda^{FB} z^2)$. It declines at the rate $d \ln \Gamma^{FB}(w_t) / d \ln w_t = -(1-\beta)$. The steady state level of income w^{FB} solves $w^{FB} = \Lambda w^{FB\beta} \exp(\lambda^{FB} z^2)$, which leads to Equation (8).

In Greenwood and Jovanovic’s (1990) setting, agents are endowed with projects (signals) that they supply to a financial intermediary, allowing it to discover the state of the economy and achieve the first-best allocation. Here in contrast, signals are costly to produce. Moreover, their precisions are not contractible. Suppose all investors agree to acquire information about a stock, however imprecise, and to disclose it. Given that the cost of information is not zero, the optimal strategy for an agent is to deviate from the agreement, that is, to not collect any information and make a random announcement. But if all agents make random announcements, then the productivity shock cannot be learned. Thus, the first-best outcome cannot be reached if signal precisions are not contractible.

4. Equilibrium Characterization

The remainder of the paper assumes that signal precisions are not contractible but that some trades are motivated by noise. In that case, the stock market offers a way to share information, albeit imperfectly. We characterize first investors’ portfolios and the allocation of capital, then various aspects of the economy, and finally information acquisition decisions. Throughout this section, we take as given investors’ income w_t , which we endogenize in Section 6.

4.1 Capital Allocation

As usual with a noisy rational expectations equilibrium, we guess that capital is a log-linear function of shocks, solve for portfolios, derive the equilibrium capital allocation, and check that the guess is valid. The following lemma displays investors' portfolio composition for the conjectured capital allocation.

Lemma 3. Assume that firm m 's capital stock takes the form $K_t^m = \frac{Lw_t}{M} \exp(\Delta k_t^m z)$, where $k_t^m \equiv k_{\alpha t}^m (\beta \Delta \tilde{\alpha}_t^m + \mu_t^m \Delta \tilde{\theta}_t^m) + o(1)$ and μ_t^m is a deterministic scalar. The portfolio weights for agent l are given by

$$f_{l,t}^m = \frac{1}{M} + \frac{\tau(\varphi(w_t))}{\varphi(w_t)\beta^2\sigma_a^2 z} E(\Delta \ln R_{t+1}^m | \mathcal{F}_{l,t}) + o(1), \quad (9)$$

$$\text{where } \varphi(w) \equiv \beta M^{1-\beta} w^\beta. \quad (10)$$

- For a rational agent who receives private signals of precision $\{x_{l,t}^m\}$, weights equal

$$f_{l,t}^m = \frac{1}{M} + \frac{\tau(\varphi(w_t))}{\varphi(w_t)\beta^2\sigma_a^2} \left\{ \frac{x_{l,t}^m}{H(\mu_t^m) + x_{l,t}^m} \Delta s_{l,t}^m + \left(\frac{1}{(H(\mu_t^m) + x_{l,t}^m)\mu_t^m \sigma_\theta^2 k_{\alpha t}^m} - (1-\beta) \right) \Delta k_t^m \right\} + o(1). \quad (11)$$

$$\text{where } H(\mu) \equiv \frac{1}{\beta^2\sigma_a^2} + \frac{1}{\mu^2\sigma_\theta^2}. \quad (12)$$

- For a noise trader, weights equal:

$$f_t^m = \frac{1}{M} + \frac{\tau(\varphi(w_t))}{\varphi(w_t)\beta^2\sigma_a^2} \Delta \tilde{\theta}_t^m + o(1). \quad (13)$$

Stock m 's portfolio weight equals the weight it would receive if firms were identical, $1/M$, tilted by a measure of the stock's expected excess performance relative to the market, $E(\Delta \ln \tilde{R}_{t+1}^m | \mathcal{F}_{l,t})$. The deviation from equal portfolio shares is more pronounced when stocks are less risky (lower β or σ_a^2) and when agents are relatively more risk tolerant. $\tau(\varphi(w_t))$ measures investors' absolute risk tolerance in a neighborhood of their consumption; to a first approximation (i.e., at the order 0 in z), they consume $\varphi(w_t)$ units of the final good. Relative risk tolerance, $\tau(\varphi(w_t))/\varphi(w_t)$, determines how aggressively investors trade on their information. Though absolute risk tolerance $\tau(\varphi(w))$ rises with income by assumption, this need not be the case for relative risk tolerance, $\tau(\varphi(w))/\varphi(w)$. For example, under CES preferences $\tau(\varphi(w))/\varphi(w) = (\varpi\beta^\sigma M^{\sigma(1-\beta)} w^{\sigma\beta} + 1 - \varpi)/[(1-\sigma)(1-\varpi)]$. If $\sigma > 0$ (< 0), then $\tau(\varphi(w))/\varphi(w)$ increases (decreases) with income and wealthier investors' portfolio weights deviate more (less) from

equal shares. If $\sigma = 0$ (Cobb–Douglas utility), then $\tau(\varphi(w))/\varphi(w)$ is a constant, $1 - \varpi$, so portfolio weights are independent of wealth.

Equation (11) expresses portfolio weights as a combination of the stock price (the Δk_t^m term) and the relative private signal (the $\Delta s_{l,t}^m$ term). In this expression, the stock price plays a dual role: it clears the stock market and provides information about the firm’s productivity. Given our conjecture, observing stock prices is equivalent to observing $\beta \Delta \alpha_t^m + \mu_t^m \Delta \theta_t^m$ for each firm, a signal about $\beta \Delta \alpha_t^m$ with error $\mu_t^m \Delta \theta_t^m$. Thus, μ_t^m represents the noisiness of stock m ’s price. The function $H(\mu_t^m) + x_{l,t}^m = 1 / \text{Var}(\beta \alpha_t^m | F_{l,t})$ measures the total precision of an investor’s information about a stock. She receives information from three sources—her priors (the $1/(\beta^2 \sigma_\alpha^2)$ term), the price (the $1/(\mu_t^m \sigma_\theta^2)$ term), and her private signal (the $x_{l,t}^m$ term)—and their precisions simply add up. The next proposition describes the equilibrium allocation of capital for an arbitrary level of noisiness μ_t^m . Equivalently, the equilibrium can be characterized in terms of the average precisions about stocks, X_t^m , because X_t^m and μ_t^m are connected one for one (Equation (16)).

Proposition 4. Let $\mu_t^m (> \frac{q}{1-q})$ be the noisiness of stock m ’s price. There exists a log-linear rational expectations equilibrium in which firm m ’s capital stock and its share price equal $K_t^m = P_t^m = \frac{Lw_t}{M} \exp(\Delta k_t^m z)$, where

$$\Delta k_t^m \equiv k_\alpha(\mu_t^m)(\beta \Delta \tilde{\alpha}_t^m + \mu_t^m \Delta \tilde{\theta}_t^m) + o(1), \tag{14}$$

$$k_\alpha(\mu) \equiv \frac{1}{1-\beta} \left(1 - \frac{1}{\beta^2 \sigma_\alpha^2 (H(\mu) + X(\mu))} \right) > 0, \tag{15}$$

$$\text{and } X(\mu) \equiv \frac{H(\mu)}{\left(\frac{1-q}{q}\right)\mu - 1}. \tag{16}$$

Capital and stock prices are approximately log-linear functions of productivity and noise shocks. As in the first best, they equal those that would obtain if firms were identical (Lw_t/M), disturbed by an order- z function of relative shocks. Productivity shocks appear directly in the price function, though they are not known by any agent, because individual signals, $\tilde{s}_{l,t}^m$, once aggregated, collapse to their mean, $\beta \tilde{\alpha}_t^m$. Noise traders’ introduce noise $\tilde{\theta}_t^m$ into the price system through their trades. For simplicity, the conditions that characterize k_α and X (Equations (15) and (16)) are stated under the assumption that signal precisions are identical across agents for any stock m ($x_{l,t}^m = X_t^m$ for all l), a property that holds when signal precisions are chosen optimally (see Lemma 5 below). Equation (A10) in the Appendix displays these conditions for arbitrary precisions. The average precision X_t^m and stock price noisiness μ_t^m are inversely related one for one through Equation (16), as Figure 2 illustrates.

Proposition 4 outlines the allocative role of the stock market. Equation (14) implies that capital and technology shocks are positively correlated. The key

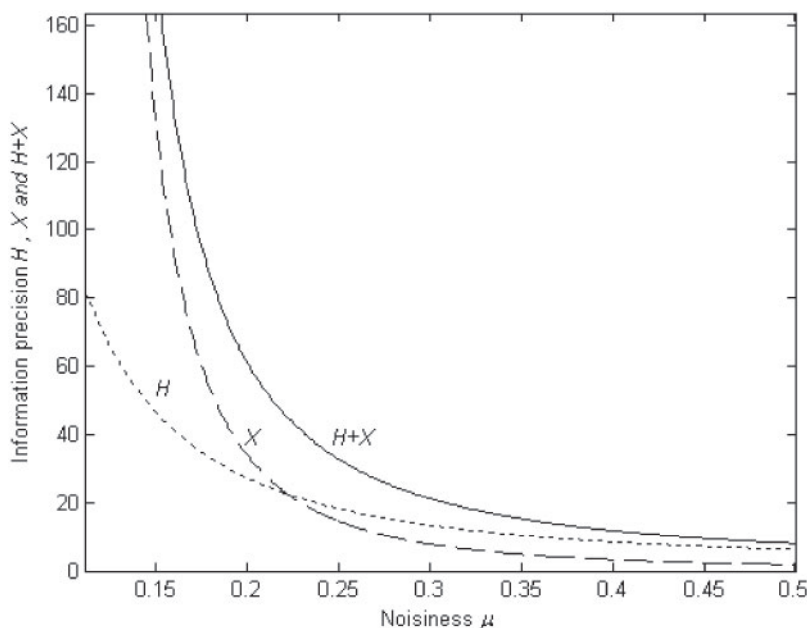


Figure 2
Signal precisions and the noisiness of the price system

This figure depicts the precision of the stock price H (dotted curve), the precision of an investor's private signal X (dashed curve) and an investor's total precision $H+X$ (solid curve) as a function of the stock price noisiness μ . Utility is CES ($U(g, j) \equiv (\varpi g^\sigma + (1-\varpi)j^\sigma)^{1/\sigma}$) with $\sigma=0.5$. The other parameters are $\beta=2/3$, $C(x)=x^2$, $q=0.1$, $\sigma_a^2=0.01$, $\sigma_\theta^2=\sigma_\alpha^2=1$, $\omega=0.5$, $M=50$, and $z=0.5$.

parameter is k_α , which controls the elasticity of investments to productivity shocks, $\partial(\ln K_t^m)/\partial \tilde{\alpha}_t^m = (1 - 1/M)\beta k_\alpha$. k_α is positive, meaning that funds flow to the most productive firms, and monotonically increasing with the quality of information. It starts from zero when there is no information (μ_t^m is infinite and $X_t^m=0$), so capital is allocated independently from productivity shocks, and reaches $1/(1-\beta)$ under perfect information ($\mu_t^m=q/(1-q)$ and X_t^m is infinite), where the elasticity coincides with that of the first best.

4.2 Impact of noisiness on properties of the economy

In this section, we describe how information about firms influences real and financial aspects of the economy, holding income fixed.

Lemma 5. When information is more accurate (noisiness is lower), investments are more responsive to productivity shocks, TFP is higher, and capital and profits are more concentrated across firms.

Better-informed agents distribute capital more efficiently across firms, leading to a higher elasticity of investments to productivity shocks,

$\partial \ln K_t^m / \partial \tilde{\alpha}_t^m$. This superior efficiency translates into higher TFP, defined from the following economy-wide production function:

$$E(\tilde{G}_{t+1}) = ML^{1-\beta} E[(\tilde{A}_t^m K_t^m)^\beta] = ML^{1-\beta} E(\tilde{A}_t^{m\beta}) E(K_t^{m\beta}) \times \exp(\text{Cov}(\beta \tilde{a}_t^m z, \beta \Delta k_t^m z)). \tag{17}$$

We interpret the factor $\exp(\text{Cov}(\beta \tilde{a}_t^m z, \beta \Delta k_t^m z))$ as TFP. It captures the additional output obtained from distributing capital in relation to productivity shocks, in comparison to an economy in which capital is randomly allocated. The concentration of economic activity is measured using Herfindahl indices, $Her(K_t^m) \equiv E(K_t^{m2}) / [E(K_t^m)]^2$ and $Her(\tilde{\Pi}_{t+1}^m) \equiv E(\tilde{\Pi}_{t+1}^{m2}) / [E(\tilde{\Pi}_{t+1}^m)]^2$. When agents invest more selectively, they channel more (less) capital to the more (less) productive firms. As a result, fewer firms account for a larger fraction of the economy’s stock of capital. Profits are more concentrated than capital because they compound the effect of a high productivity shock with that of a large capital stock.

Lemma 6. Income is larger on average in the next period when information is more accurate (noisiness is lower), for a given level of current income.

More accurate information leads to more efficient investments and hence to a larger supply of intermediate goods in the subsequent period. This in turn increases the marginal product of labor and the next generation’s average income.

Lemma 7. Wealth inequality widens at first and then narrows as information improves (noisiness declines).

Final wealth, that is, consumption $\tilde{g}_{l,t+1}$, is unequal because agents, guided by their private signals, choose different portfolios. Two forces work in opposite directions when information improves. On one hand, agents increase the weight on their private signals relative to public information, which tends to increase portfolio heterogeneity. On the other hand, idiosyncratic signal errors shrink, so private signals, and therefore portfolios, are less diverse. The first effect tends to dominate for low precisions and the second for high precision, making inequality nonmonotonic in precision.

Lemma 8. Trading and liquidity on the equity market intensify at first and then weaken as information improves (noisiness declines).

The value of shares traded equals $\sum_{m=1}^M \int_l |f_{l,t}^m w_t| / 2$, where the factor 2 avoids double counting matching buys and sells. The share turnover is defined as the ratio of the value of shares traded to the total capitalization of the market, $\sum_{m=1}^M K_t^m$. Lemma 8 resembles Lemma 7: agents trade because they disagree,

and their disagreement is a source of inequality. More accurate information leads, on the one hand, to more disagreement because agents use their private signals more aggressively, but, on the other hand, it leads to more consensual private signals. The resulting relation is nonmonotonic.

The inverse of the sensitivity of stock prices to (uninformative) noise shocks, $1/(\partial(\ln K_t^m)/\partial(\tilde{\theta}_t^m z)) = 1/[(1 - 1/M)k_\alpha(\mu_t^m)\mu_t^m]$, measures liquidity. It has two components. The first is the sensitivity to technology shocks (the k_α term in the formula), which, from Lemma 5, rises with information accuracy, thereby reducing liquidity. The second component is the sensitivity to noise shocks relative to that of technology shocks (the μ_t^m term). Thanks to this factor, liquidity tends to improve when information is more accurate. As a result, liquidity is nonmonotonic in accuracy. The first factor (sensitivity to technology shocks) tends to dominate for high precision levels, and the second for low levels.

Lemma 9. When information is more accurate (noisiness is lower), stocks' prices are more volatile, whereas the idiosyncratic and total volatility of their returns are lower. In contrast, the volatility of the market is unchanged.

Stock prices fluctuate more as they incorporate technology shocks more fully. Returns, which absorb residual shocks, fluctuate less, whether fluctuations are measured as total or idiosyncratic volatility. Because the market return (price), in contrast, does not see its volatility change, a rise (decline) in the cross-correlation of stock returns (prices) offsets the reduction (rise) in individual stock volatility.

4.3 Information acquisition

We turn to the information acquisition decision. The following lemma characterizes how much free time an investor devotes to learning about productivity shocks for an arbitrary level of stock price noisiness μ_t^m and given her income w_t .

Lemma 10. Let $\mu_t^m (> \frac{q}{1-q})$ be the noisiness of stock m 's price. Investors set the precision of their private signal about stock m , x_t^m such that

$$\rho(\varphi(w_t))C'(x_t^m) = \tau(\varphi(w_t)) \frac{1 - 1/M}{2\beta^2\sigma_a^2} \frac{1}{(H(\mu_t^m) + x_t^m)^2} + o(1). \quad (18)$$

Investors choose a signal precision that equates the marginal benefit of information to its marginal cost, taking into account how much is revealed through stock prices. The left-hand side of Equation (18) measures the utility cost, denominated in units of the final good, of a marginal increase in the signal precision. Indeed, increasing the precision of a signal from x to $x + \delta$ requires cutting leisure time by $C'(x)\delta$ units and suffering a utility loss of $\frac{\partial U}{\partial j} C'(x)\delta$. The same loss would occur if the consumption of the final good were

to fall by $\frac{\partial U}{\partial j} C'(x)\delta / \frac{\partial U}{\partial g} = \rho(\varphi(w_t))C'(x)\delta$ units, where $\rho(\varphi(w_t))$ measures the marginal rate of substitution between goods and leisure in a neighborhood of consumption. This coefficient, and therefore the cost of information, increase with income because of a substitution effect: wealthier agents invest more and hence consume more of the final good, which decreases its marginal utility and makes leisure more enjoyable.

The right-hand side of Equation (18) represents the utility benefit from a marginal increase in precision, again denominated in units of the final good. It rises with investors' income through their absolute risk tolerance, τ , because discriminating across firms is more valuable when one has more to invest. Thanks to its nonrival nature, information can be applied to every dollar of investment without requiring its cost to be incurred repeatedly. Putting it differently, information generates increasing returns with respect to the scale of investments, captured by $\tau(\varphi(w))$.

The benefit of private information also rises when public information is less accurate (i.e., when priors are less precise: σ_α^2 is larger; or when stock prices are more noisy: μ_t^m or σ_θ^2 are larger) because stock prices, by revealing private signals, limit investors' ability to appropriate the full benefit from their information expenditures (the ex ante disincentive effect). Moreover, it is lower when the conditional variance of productivity shocks σ_a^2 is higher because portfolio weights deviate less from equal shares.

Equation (18) admits a unique solution and implies that signal precisions are identical across agents for any stock m ($x_{l,t}^m = X_t^m$ for all l). They are higher when σ_α^2 , μ_t^m , and σ_θ^2 are larger, and when σ_a^2 and C' are lower. Most of these properties obtain in the usual framework with exponential utility, normally distributed random variables and a riskless asset (e.g., Verrecchia 1982).¹⁵ The influence of income on the signal precision is more novel. It depends on which, of the marginal rate of substitution and risk tolerance, is the more sensitive to income, as outlined in Lemma 12 below. The following proposition characterizes the degree of noisiness in equilibrium, μ_t^m , for a given level of income w_t .

Proposition 11. In equilibrium, the noisiness of stock prices, μ_t , is the unique solution to

$$\rho(\varphi(w_t))C' \left(\frac{H(\mu_t)}{\frac{1-q}{q}\mu_t - 1} \right) = \tau(\varphi(w_t)) \frac{1-1/M}{2\beta^2\sigma_a^2} \left(\frac{1 - \frac{q}{(1-q)\mu_t}}{H(\mu_t)} \right)^2 + o(1). \tag{19}$$

¹⁵ In an economy similar to ours except that (1) preferences display constant absolute risk aversion with a coefficient of absolute risk tolerance τ , (2) stocks have normally distributed payoffs with variance σ_Π^2 , and (3) a riskless asset with gross return R^f is available, the equilibrium precision of private signals solves $2R^f C'(x_t) = \tau/(H_t + x_t)$, where $H_t \equiv 1/\sigma_\Pi^2 + 1/(\mu_t^2\sigma_\Theta^2)$ and σ_Θ^2 is the variance of noise trading. From this equation, x_t rises when σ_Π^2 , τ or $\mu_t^2\sigma_\Theta^2$ increase or when C decreases.

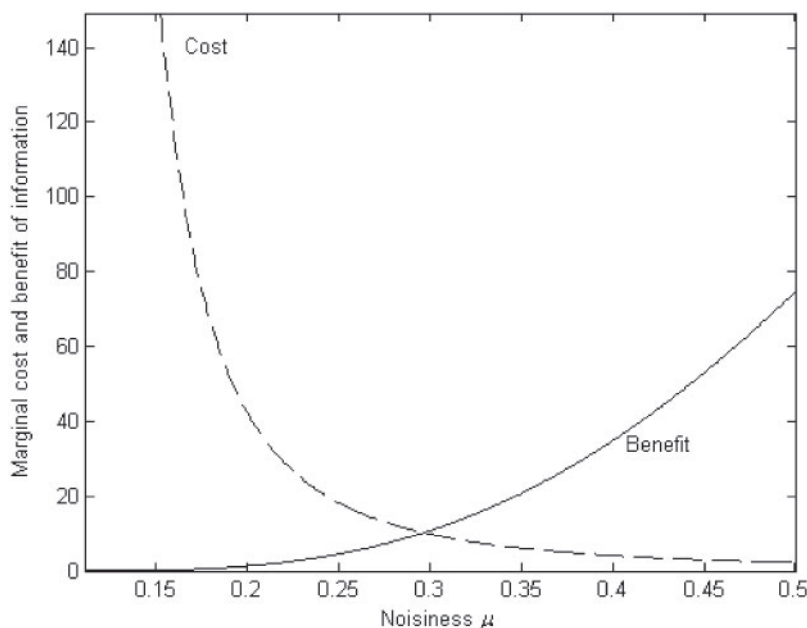


Figure 3

The benefit and cost of information in equilibrium

This figure depicts the marginal benefit of private information (solid curve) and its marginal cost (dashed curve) in Equation (19) as a function of the equilibrium noisiness μ . See Figure 2 for the parameters of this figure.

Equation (19) is obtained by equating individual and average precisions, x_t^m and X_t^m , because agents all choose the same precisions, and by substituting Equation (16) into the first-order condition (18). It admits a unique solution for any level of income, as Figure 3 illustrates. Noisiness, average and individual precisions are identical across stocks, allowing us to drop the superscript m from now on ($X_t^m \equiv X_t$, $x_t^m = x_t$ and $\mu_t^m = \mu_t$ for all m).

The properties of the average precision X_t are identical to those of individual precisions x_t , discussed above. Those of the equilibrium noisiness μ_t follow. It decreases (i.e., stock prices are more informative) when priors are more accurate (σ_a^2 smaller), when the variance of noise trades σ_θ^2 is larger, when the conditional variance of productivity shocks σ_a^2 , or the marginal cost of information C' are lower. In contrast, μ_t increases with the fraction of noise traders q (q has a direct effect on μ_t that dominates its indirect effect through X_t). We conclude this section with an analysis of the influence of income on X_t .

Lemma 12. If τ/ρ is an increasing (decreasing) function of consumption, then the noisiness of stock prices falls (rises) with income.

We observed in the discussion following Lemma 10 that current income increases both the marginal cost of information (through a substitution effect)

and its marginal benefit (through a scale effect). The impact of income on the equilibrium precision of information depends on which of these two effects dominates. If the scale effect dominates, that is, the marginal benefit rises with income faster than the marginal cost does (τ/ρ increasing in consumption), then agents collect more information as they grow wealthier, so $d\mu_t/dw_t < 0$. If instead the substitution effect dominates (τ/ρ decreasing in consumption), then agents collect less information, so $d\mu_t/dw_t > 0$. Under CES utility, for example, information improves with income if $\sigma > 0$, deteriorates if $\sigma < 0$, and is unchanged if $\sigma = 0$ (Cobb-Douglas utility or constant relative risk aversion). There is no scale effect under constant absolute risk aversion—the preferences usually assumed in rational expectations models of trading under asymmetric information (e.g., $U(g, j) = (-\exp(-\tau g))v(j)$), so the precision of information is a decreasing function of income. Figure 4 (top left panel) illustrates Lemma 12.

Empirically, it is not clear which of the scale or substitution effect dominate. In the model, the elasticity of substitution between goods and leisure should be interpreted as a mix of intertemporal and intratemporal elasticities. Indeed, it represents not only the (within-period) substitutability between goods and leisure but also the intertemporal substitutability of consumption, given that agents consume leisure when young and goods when old. The literature estimating these elasticities yields mixed results. For example, Hall (1988), relying on aggregate data, reports an intertemporal elasticity of substitution close to zero (so $\sigma < 0$ under CES utility), whereas Attanasio and Vissing-Jorgensen (2003), exploiting disaggregated data, report that it is above one (so $\sigma > 0$).¹⁶

5. The Role of the Stock Market

This section delves into the information processing role of the stock market. Stock prices, by aggregating dispersed private signals about technology shocks into public signals, affect capital efficiency in two conflicting ways. On one hand, they help investors evaluate firms and deploy their capital. As such, the stock market can be viewed as a mechanism for sharing costly private information. Importantly, this mechanism is incentive-compatible and inexpensive because investors “communicate” through their trades.¹⁷ On the

¹⁶ A more general model would allow for both goods and time to be used as inputs in the production of information (rather than time only in the current model) and for investment gains to buy both goods and time (e.g., by retiring early) (rather than goods only in the current model). In such a setting, we conjecture the intertemporal elasticity to matter more than the intratemporal elasticity.

¹⁷ This effect can be best understood by comparison to a fictitious economy in which agents collect the same private signals but stock prices do not reveal any of their content. In such an economy, the average precision $X(\mu_t^m)$ is the same as in the “normal” economy but an investor’s total precision is lower because the precision of the price signal, $1/(\mu_t^{m2}\sigma_\beta^2)$, is lost: the total precision equals $1/(\beta^2\sigma_\alpha^2) + X(\mu_t^m) < H(\mu_t^m) + X(\mu_t^m)$. Accordingly, the elasticity of investments to productivity shocks falls to $(1 - 1/(1 + \beta^2\sigma_\alpha^2 X(\mu_t^m)))/(1 - \beta)$ which is below $k_\alpha(\mu_t^m)$.

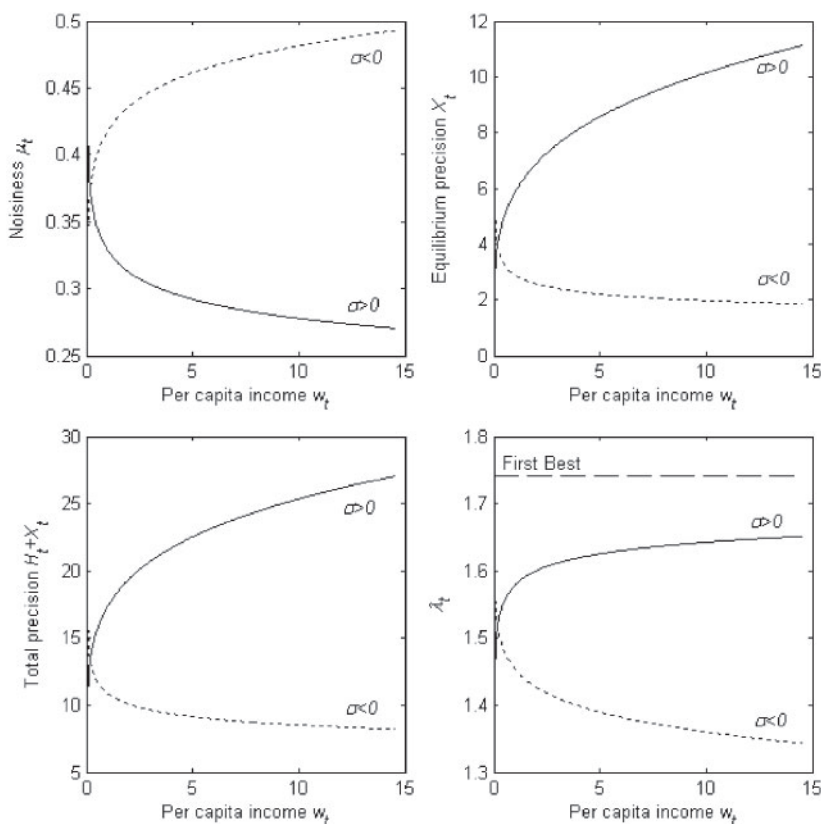


Figure 4
The impact of income on the equilibrium

This figure depicts the equilibrium noisiness μ_t (top left panel), the precision of private information X_t (top right panel), the total precision $H_t + X_t$ (bottom left panel), and λ_t , which captures the effect of learning on income (bottom right panel) as a function of current income w_t . See Figure 2 for the parameters of the picture. The solid curves correspond to $\sigma = 0.5$, and the dotted curves correspond to $\sigma = -0.5$.

other hand, the very existence of informative prices undermines the incentive to collect costly information in the first place. Indeed, investors' cannot appropriate the full benefit of their signals as they are leaked to competitors through prices. Thus, informative stock prices have an impact that is beneficial ex post but detrimental ex ante to capital efficiency. Noise trading plays a crucial part in this trade-off as its intensity determines how much information is revealed. By varying the fraction of noise traders q , one can get a sense of the net informational effect of the stock market. The next lemma shows that

The allocation of capital is not as efficient, though the same private signals are produced because investors do not share them.

the ex post information sharing effect more than compensates for the ex ante disincentive effect.

Lemma 13. When the fraction of noise traders q decreases, less information is produced but more is shared through stock prices. The net effect is an improvement in total information, $H_t + X_t$, and in the efficiency of investments, captured by a higher elasticity, k_{at} .

Next, we relate the allocation of capital achieved through the stock market to the first best. Because noise trading was introduced into the stock market economy to avoid the Grossman-Stiglitz paradox, we make the comparison in the limiting situation in which noise vanishes, that is, as the fraction of noise traders goes to zero.

Lemma 14. The allocation of capital achieved through the stock market converges to the first best allocation as the fraction of noise traders goes to zero:

$$\lim_{\substack{q \rightarrow 0 \\ q > 0}} k_t^m = k_t^{mFB} \quad \text{for } m = 1, \dots, M.$$

The capital allocation achieved through the stock market can be made arbitrarily close to the first-best allocation by reducing the fraction of noise traders q .¹⁸ It follows that the dynamics of income, as described in the next section, also can be made arbitrarily close to those obtained in the first-best economy. Lemmas 13 and 14 are illustrated in Figure 5, which displays μ_t , X_t , $H_t + X_t$, and k_{at} as a function q under CES utility.

6. Dynamics

In this section, we tie together learning, investments, and income, analyze the evolution of the economy along its average path, and relate the model's predictions to the evidence.

6.1 Observable properties of the growth path

The following proposition determines the dynamics of income by combining Lemmas 6 and 12.

Proposition 15. Average income evolves according to the following equation:

$$E(\tilde{w}_{t+1}) = \Lambda \exp(\lambda(w_t)z^2) w_t^\beta, \tag{20}$$

¹⁸ However, q cannot exactly equal zero; otherwise, there is no equilibrium (the Grossman-Stiglitz paradox). Two assumptions underlie this lemma. First, each generation consists of a continuum of agents. Second, the marginal cost of information equals zero at the origin. We conjecture that it would not hold if either assumption were relaxed.

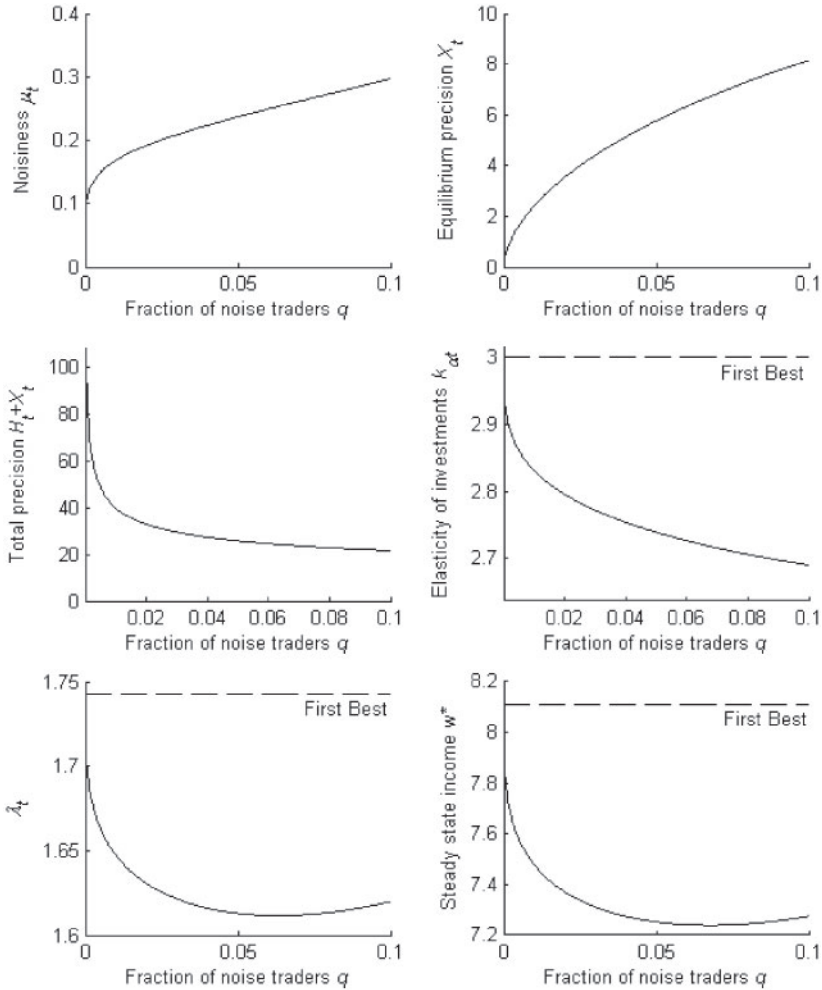


Figure 5
The impact of the fraction of noise traders on the equilibrium

This figure depicts the equilibrium noisiness μ_t (top left panel), the precision of private information X_t (top right panel), the total precision $H_t + X_t$ (middle left panel), the elasticity of investments to productivity shocks $k_{\alpha t}$ (middle right panel), λ_t , which captures the effect of learning on income (bottom left panel), and the steady state level of income w^* (bottom right panel) as a function of the fraction of noise traders q . See Figure 2 for the parameters of this figure.

where

$$\lambda(w_t) \equiv \left(1 - \frac{1}{M}\right) \beta^2 \left(k_{\alpha}(\mu_t) \beta \sigma_{\alpha}^2 + \frac{k_{\alpha}(\mu_t)^2}{2} (\beta^2 \sigma_{\alpha}^2 + \mu_t^2 \sigma_{\theta}^2) \right) + o(1) > 0, \quad (21)$$

and Λ , k_{α} , and $\mu_t = \mu(w_t)$ are defined respectively in Equations (6), (15), and (19).

- The economy converges to a steady state. The steady-state level of income w^* is given by:

$$w^* = w^{FB} \exp\left(-\frac{\lambda^{FB} - \lambda((1-\beta)^{1/(1-\beta)}M)}{1-\beta} z^2\right). \quad (22)$$

- If τ/ρ is an increasing (decreasing) function of consumption, then λ increases (decreases) with income. Moreover, if there exists a scalar u such that $\lim_{g \rightarrow u} \tau(g)/\rho(g) = \infty$, then $\lim_{w_t \rightarrow u} \lambda(w_t) = \lambda^{FB} + \lambda^{Noise}$, where $\lambda^{Noise} \equiv (1-1/M)(\beta q/(1-q))/(1-\beta)^2 \sigma_\theta^2/2$. For example, under CES preferences, λ is an increasing function of income and $\lim_{w_t \rightarrow \infty} \lambda(w_t) = \lambda^{FB} + \lambda^{Noise}$ if $\sigma > 0$, whereas λ is a decreasing function and $\lim_{w_t \rightarrow 0} \lambda(w_t) = \lambda^{FB} + \lambda^{Noise}$ if $\sigma < 0$.

To a first approximation (at the order 0 in z), the dynamics of income are similar to those obtained under the first best: income grows at a declining rate until it reaches a steady state w^* (assuming the wage is initially below w^*). Thus, the dynamics of income continue to be dominated by the neoclassical force of diminishing returns to capital, whereby learning only generates a small deviation (of order z^2) from the neoclassical path. Though this is the case by construction in our model—learning about productivity shocks generates benefits that are small because we assume these shocks to be small—we conjecture that this property extends to large shocks because income admits the first best as an upper bound (starting from the same arbitrary level of income, income in the next period is lower than in the first best in which capital is most efficiently allocated) and income in the first best eventually reaches a steady state. Proposition 15 is illustrated in Figure 6.

The effect of learning on income is captured by the function illustrated in the bottom right panel of Figure 4. We note that income may be higher in steady state in the stock market economy compared with the first-best economy because of the presence of noise (which was assumed away in the first best). Indeed, noise trading may be beneficial to income in spite of making investments less efficient because it increases the variability of the capital allocation and therefore the average income, a convex function thereof (positive noise shocks increase output more than negative shocks decrease it). This effect, reflected in the term $\mu_t^2 \sigma_\theta^2$ in Equation (21), vanishes as the intensity of noise trading q approaches zero. If it were not for this direct influence of noise trading, steady state income would always be lower than in the first best. The growth rate of income during the transition to the steady state, $\Gamma(w_t) \equiv E(\tilde{w}_{t+1})/w_t$, differs from the first best, by a factor $\exp[-(\lambda^{FB} - \lambda(w_t))z^2]$, and is lower than in the first best when the intensity of noise trading is weak.

Figure 7 depicts $\Gamma(w_t)$ for various utility functions as well as the first-best economy. When the scale effect of information dominates the substitution effect (e.g., when $\sigma > 0$ under CES utility), investors collect more information as the

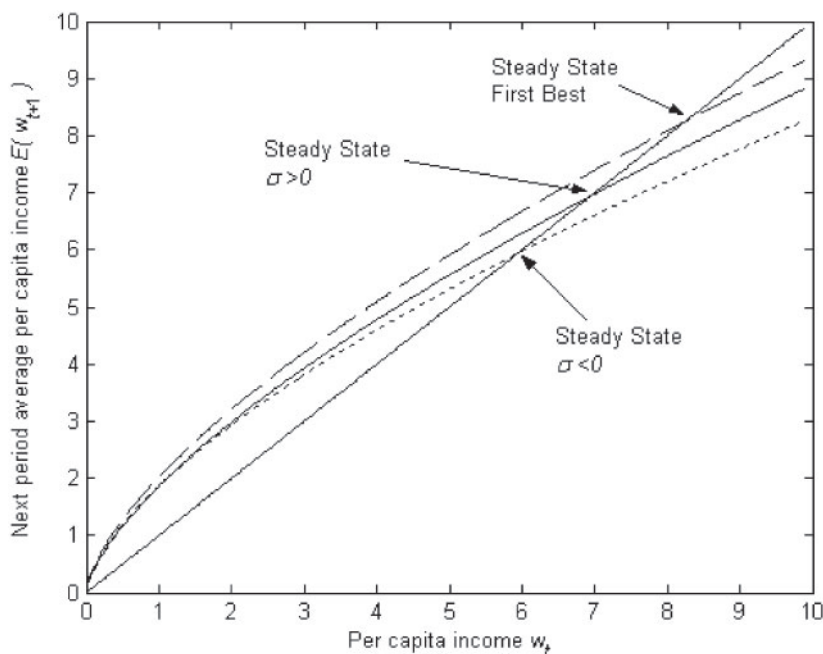


Figure 6
The dynamics of income in an economy along its average path

The curves represent the average income in period $t+1$, $E(w_{t+1})$, as a function of income in period t , w_t . See Figure 2 for the parameters of this figure. The solid curve corresponds to $\sigma = 0.5$, and the dotted curve corresponds to $\sigma = -0.5$. The dashed curve corresponds to the first-best economy. The economies' steady states are located at the intersections of these curves with the 45° line, represented as a solid line.

economy grows, which contributes to growth. As a result, the growth rate of income declines less quickly than in the first best:

$$\frac{d \ln \Gamma(w_t)}{d \ln w_t} = -(1 - \beta) + \frac{d \lambda(w_t)}{d \ln w_t} z^2 > -(1 - \beta) = \frac{d \ln \Gamma^{FB}(w_t)}{d \ln w_t}.$$

That is, the growth rate of income is typically (i.e., when the intensity of noise trading is weak) lower than in the first best (capital is not as efficiently deployed), but it declines less quickly (the allocation of capital improves over time). Thus, in this case, learning has a transitory beneficial effect on growth, that mitigates the negative neoclassical force. When instead the scale effect of information dominates the substitution effect (e.g., when $\sigma < 0$ under CES utility), investors collect less information as the economy grows, which slows growth. So, the growth rate of income falls at a faster rate than in the first best:

$$\frac{d \ln \Gamma(w_t)}{d \ln w_t} = -(1 - \beta) + \frac{d \lambda(w_t)}{d \ln w_t} z^2 < -(1 - \beta).$$

We derive next various observable properties of the economy during its transition to the steady state (for an initial wage below its steady state level), by combining Lemmas 5 to 9 with Lemma 12.

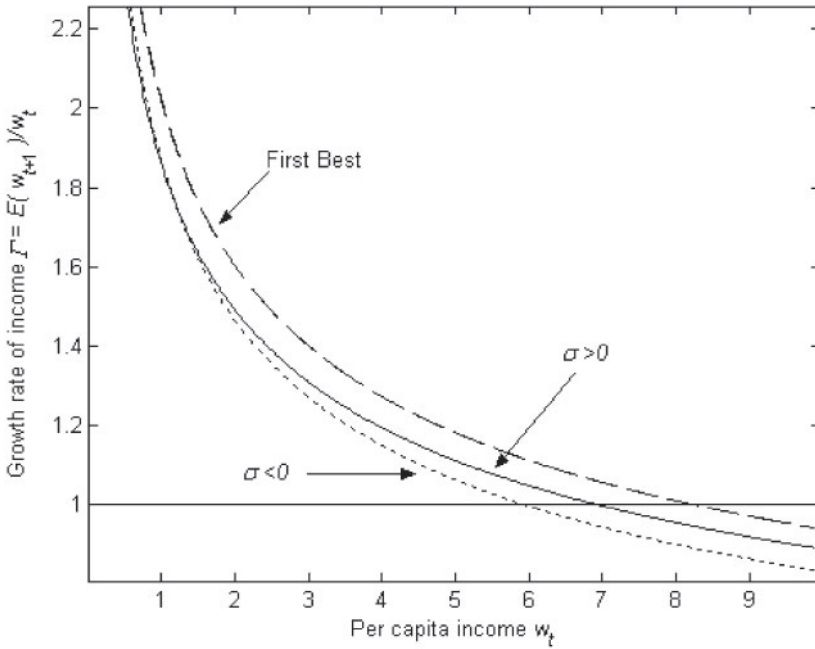


Figure 7
The growth rate of income
 This figure depicts the growth rate of income, $\Gamma(w_t) \equiv E(\tilde{w}_{t+1})/w_t$, during the transition to the steady state. See Figure 2 for the parameters of this figure. The solid curve corresponds to $\sigma = 0.5$, and the dotted curve corresponds to $\sigma = -0.5$. The dashed curve corresponds to the first-best economy.

Proposition 16. Suppose that the scale effect of information dominates the substitution effect (e.g., $\sigma > 0$ under CES utility). As the economy grows,

1. the elasticity of investments to productivity shocks and TFP increase,
2. capital and profits are more concentrated across firms,
3. income inequality widens at first and then narrows,
4. trading on the equity market and liquidity intensify at first and then weaken, and
5. the volatility of stock prices rises, the idiosyncratic and total volatility of stock returns fall, and the volatility of the market is constant.

These patterns are reversed if instead the substitution effect dominates (e.g., $\sigma < 0$ under CES utility).

The predictions of Proposition 16 for a growing economy when the scale effect dominates can be interpreted as follows. (1) Capital is more efficiently allocated across firms, that is, more (less) capital is channelled to more (less) productive firms. This superior efficiency leads to higher TFP, even though

there is no technological progress. (2) The economy specializes, as agents invest more selectively, leading capital and profits to become more concentrated across firms. (3) Income inequality follows a “Kuznets curve,” widening at first and then narrowing. (4) Stock market liquidity and the share turnover increase at first and then decrease. Inequality, liquidity, and turnover display similar nonmonotonic behaviors because all three are driven by the extent to which investors disagree about stocks. At the early stage of development, agents follow mostly price signals because private signals are imprecise, so disagreement is low. Agents rely more on private signals as their precision improves, so disagreement rises with income. A consensus re-emerges beyond a level of income because private signals that are closer to the truth are also more similar. (5) The volatility of stock prices rises with income as they track technology shocks more closely. As a result, stock returns, which absorb residual shocks, fluctuate less, as reflected in their idiosyncratic and total volatility. In contrast, the volatility of the market is constant. It follows that the cross-correlation of stock prices falls, whereas that of stock returns rises to offset, respectively, the rise in the volatility of individual stock prices and the reduction in the volatility of individual stock returns. Effects (1) to (5) are reversed when the substitution effect dominates the scale effect of information. In this case, agents prefer to enjoy more leisure and produce less information as they grow wealthier.

6.2 No-information trap

Agents always collect private signals in the model because an infinitesimal amount of private information is costless, that is, $C'(0)=0$. Empirically, however, financial institutions only emerge once a critical level of income has been reached. In this section, we show that information production only takes place for sufficiently developed economies if $C'(0) > 0$. The following proposition describes how investors’ learning decisions are altered.

Proposition 17. Suppose that $C'(0) > 0$. Investors collect information if and only if $\frac{\tau(\varphi(w_t))}{\rho(\varphi(w_t))} > \frac{2\sigma_a^2(\sigma_\alpha^2)^2}{(1-1/M)\beta^2}C'(0)$. In that case, the equilibrium noisiness is the unique solution to Equation (19).

If $C'(0) > 0$, then Equation (19), which determines the equilibrium noisiness may admit no solution. For example, when $\rho(\varphi(w_t))$ is large relative to $\tau(\varphi(w_t))$, the marginal cost of information may exceed its marginal benefit at all levels of noisiness. In that case, no information is collected in equilibrium. The condition on τ/ρ for learning to take place leads to a condition on income, as can easily be seen in the case of CES utility.

Lemma 18. Suppose that $C'(0) > 0$ and that utility is CES. Let

$$\underline{w} \equiv \left(\frac{1-\varpi}{2\varpi} \left(\sqrt{1 + \frac{8\varpi(1-\sigma)\sigma_a^2(\sigma_\alpha^2)^2}{(1-\varpi)(1-1/M)\beta^2}C'(0)} - 1 \right) \right)^{1/\sigma}.$$

When $\sigma > 0$, investors collect information if and only if their income exceeds the threshold \underline{w} .

When instead $\sigma < 0$, they collect information if and only if their income is below the threshold \underline{w} .

The threshold \underline{w} is the unique income level such that $\tau(\varphi(\underline{w}))/\rho(\varphi(\underline{w}))=C'(0)2\sigma_a^2(\sigma_\alpha^2)^2/(1-1/M)/\beta^2$. When $\sigma > 0$, the scale effect of information dominates so wealthier investors collect information only if their income w_t is large enough. When $\sigma < 0$, the substitution effect dominates so investors stop collecting information when their income exceeds \underline{w} . The properties of \underline{w} mirror those of the equilibrium precision X_t : the factors that increase (decrease) X_t tend to decrease (increase) \underline{w} . Assuming that $\sigma > 0$ and that $w^* > \underline{w} > w_0$, where w_0 is the initial level of income, the economy goes through two stages of development. At first, it behaves as a standard neoclassical economy with no information. Once income reaches a threshold, agents start collecting private signals and growth accelerates by a factor $\exp(\lambda(w_t)z^2)$. Thus, in this case, the stock market only operates as an information processor if the economy is sufficiently developed. If instead $w_0 < w^* < \underline{w}$, then no information is ever collected.

6.3 Calibration

In this section, we calibrate the model to data for the United States, assuming that the United States has reached a steady state. Obviously, choosing parameter values in such a stylized model is a challenge. Starting with firms, we set to 1/3 the factor share of intermediate goods in the production of the final good, β (it also equals the factor share of capital for the economy as a whole as shown in Equation (17)). For productivity parameters, we rely on Syverson's (2004) study of the distribution of TFP across U.S. manufacturing plants. Sorting plants into 443 industries according to their four-digit SIC code, he reports that the dispersion in log TFP is 0.342 across industries and 0.254 within industries (inferred from the reported 90-10 percentile range of 0.651). One interpretation of the model in light of these data is that a firm in the model represents one of the 443 industries, that is, $M=443$, and that a productivity shock has two components: the first is a shock to the average which can be learned, that is, $\tilde{\alpha}_t^m z$ is the industry average shock; the second is a disturbance around this average, which cannot, $\tilde{a}_t^m z - \tilde{\alpha}_t^m z$. Assuming that productivity follows a random walk, the variance of these shocks grows linearly with time (Syverson measures TFP over one year). Hence, setting $z=1$ for illustrative purposes and the duration of a period to 30 years, variances are given by $\sigma_a^2 z^2=0.254^2 \times 30=1.935$ and $\sigma_\alpha^2 z^2=0.342^2 \times 30=3.589$. These estimates are sufficient to determine the first best, which we shall use as a benchmark. Table 1 displays various aspects of this economy.

Several features of the model, in particular the cost of information and the intensity of noise trading, are difficult to parametrize. To characterize

Table 1
Calibration

| Variable | First best | Baseline elasticity | Elasticity projected on <i>Synchronicity</i> | Elasticity purged from projection on <i>SOE, Rights, FD</i> and 1960-GDP |
|---|------------|---------------------|--|--|
| | (1) | (2) | (3) | (4) |
| Elasticity | 1.489 | 0.723 | 0.870 | 0.307 |
| (pct. deviation from FB) | | (-51%) | (-42%) | (-79%) |
| R² | 1.000 | 0.126 | 0.182 | 0.023 |
| <i>Fraction of noise traders q</i> | | 0.1 | 0.1 | 0.1 |
| <i>Relative Risk Aversion</i> | | 1 | 1 | 1 |
| <i>Elasticity of substitution</i> | | 0.8 | 0.8 | 0.8 |
| <i>Information cost - scale</i> | | 0.01 | 0.01 | 0.10 |
| Square forecast error (pct. annual) | 0.70% | 1.36% | 1.23% | 1.71% |
| (pct. deviation from FB) | | (93%) | (75%) | (144%) |
| Total precision | Infinite | 5.087 | 6.294 | 3.297 |
| Prior precision (pct. of total precision) | | 51% | 42% | 79% |
| Price precision (pct. of total precision) | | 7% | 9% | 2% |
| Private precision (pct. of total precision) | | 41% | 49% | 19% |
| Log TFP | 19% | 9% | 11% | 4% |
| (pct. deviation from FB) | | (-51%) | (-42%) | (-79%) |
| Income per capita | 538 | 494 | 508 | 456 |
| (pct. deviation from FB) | | (-8%) | (-5%) | (-15%) |
| Herfindhal index | 2.34 | 4.881 | 4.900 | 4.787 |
| (pct. deviation from FB) | | (109%) | (110%) | (105%) |
| Stock return volatility (pct. annual) | 8.37% | 17.64% | 16.93% | 19.47% |
| (pct. deviation from FB) | | (111%) | (102%) | (133%) |
| Variance of noise trading Var(θ) | | 36.198 | 33.507 | 46.079 |
| Utility - σ | | -0.250 | -0.250 | -0.250 |
| Utility - ω | | 0.453 | 0.453 | 0.453 |
| Information cost - exponent | | 2.121 | 1.355 | 2.263 |

In this table, the model is fitted to the U.S. economy assuming that the United States has reached a steady state. Column 1 describes the first best. Columns 2 to 4 refer to the economy under asymmetric information. The elasticity and R^2 are based on data reported in Wurgler (2000). Column 2 uses Wurgler's baseline estimates for the United States. The last two specifications isolate the contribution of firm-specific information, as measured by Morck, Yeung, and Yu's (2000) stock price synchronicity (*Synchronicity*), to the elasticity, by either projecting the baseline elasticity estimate on *Synchronicity* (Column 3), or by subtracting from the baseline elasticity its projection on four variables that capture other mechanisms that improve capital allocation (the fraction of an economy's output due to state-owned enterprises *SOE*, an index of effective investor rights *Rights*, a measure of financial development *FD*, and the 1960-value of log per capita GDP) (Column 4). Each period in the model lasts 30 years. All four columns assume the following parameter values $z = 1$, $\beta = 1/3$, $M = 443$, $\sigma_a^2 = 1.94/\text{period}$, $\sigma_\theta^2 = 3.51/\text{period}$, and an information cost function of the form $C(x) = \text{scale} \times x^{\text{exponent}}$. Bold letters indicate an estimate drawn from Wurgler (2000), and italic letters indicate parameters chosen arbitrarily.

the information environment as fully as possible without taking a stand on these functions and parameters, we exploit the estimates reported by Wurgler (2000). Using data covering 28 manufacturing industries in 65 countries from 1963 to 1995, Wurgler measures the efficiency of capital allocation in a country by running a regression of investment growth in an industry on value-added growth in the industry (that is, growth in sales minus cost of intermediate goods). The slope coefficient represents the elasticity of investment to value-added in the country. For example, the estimated elasticity of 0.723 for the United States (we refer to it as the "baseline" estimate) implies that investment will increase (respectively, decrease) by 7.23%, on average, in an industry subject to a +10%

(respectively, -10%) shock to value-added growth. The average elasticity across countries equals 0.429, indicating that the same shock in the “average” country leads to a 40% smaller increase (respectively, decrease) in investment.

Wurgler identifies several mechanisms that improve capital allocation other than through better information about firms, in particular better incentives and investor rights. To assess their importance, he runs cross-country regressions of the elasticity on proxies for these mechanisms, including a measure of stock price synchronicity developed by [Morck, Yeung, and Yu \(2000\)](#), which is inversely related to the amount of firm-specific information impounded into stock prices (see Lemma 9 above). These regressions allow us to isolate the contribution of firm-specific information to the elasticity. Specification 1 of his Table 5 only includes *Synchronicity* and a constant as regressors, and yields coefficient estimates of -3.185 and 2.714 . Given that *Synchronicity* in the United States is 57.9% (the lowest number among Wurgler’s 40 countries), the elasticity attributable to information equals $0.870 = 2.714 - 3.185 \times 57.9\%$. Wurgler’s most elaborate specification (specification 6 of his Table 5) includes a constant and five regressors, namely, *Synchronicity*, the fraction of an economy’s output due to state-owned enterprises (*SOE* with a coefficient estimate of -1.094), an index of effective investor rights (*RIGHTS* with a coefficient estimate of -0.033), a measure of financial development (*FD* with a coefficient estimate of 0.434), and the 1960-value of log per capita GDP (with a coefficient estimate of 0.121). Deducting from the elasticity for the United States these coefficient estimates, multiplied by the value of the corresponding variable for the United States, yields a more conservative measure of the elasticity attributable to information: $0.307 = 0.723 - 1.094 \times 0.046 - 0.033 \times 6 + 0.434 \times \ln(1 + 1.440) + 0.121 \times \ln(9.910)$. These elasticities are 42% to 79% lower than in the first best (Table 1).

Wurgler’s investment elasticities have a direct counterpart in the model. Equation (14) implies that $\Delta k_t^m z \equiv k_\alpha (1 - \frac{1}{M}) \beta \tilde{\alpha}_t^m z + \eta_t^m$, where $\Delta k_t^m z$ is log investment; $\beta \tilde{\alpha}_t^m z$ is log value-added (up to an additive constant and valuing capital at book value); and $\eta_t^m \equiv k_\alpha (\mu \Delta \tilde{\theta}_t^m z - \frac{1}{M} \sum_{m' \neq m}^M \beta \tilde{\alpha}_t^{m'} z)$ is an error term orthogonal to the regressor $\beta \tilde{\alpha}_t^m z$. Therefore, the elasticity equals $k_\alpha (1 - \frac{1}{M})$. Knowing this elasticity allows to estimate the total precision of information about stocks $H + X$ through Equation (15). For example, the baseline estimate of 0.723 leads to $H + X = 5.09$. Data on financial analysts’ earnings forecasts allow us to cross-check this estimate. Indeed, the average square forecast error is given in the model by $E \left[\left(E(\ln(\tilde{\Pi}_{t+1}^m z) | \mathcal{F}_{t,t}) - \ln(\tilde{\Pi}_{t+1}^m z) \right)^2 \right] = \frac{z^2}{H+X} + \beta^2 \sigma_a^2 z$ over one (30-year) period, and equals 1.36% per year for the baseline estimate. [Harris \(1999\)](#) reports that the mean square error of analysts’ long-run earnings growth forecasts is 7.15%. Because his forecast horizon is 5 years, the mean

square error equals about a fifth, 1.43%, over a one-year horizon, consistent with the model.

The R^2 of the investment value-added regression can be used to further characterize the information environment. Wurgler (2000) reports an R^2 of 12.6% for the United States.¹⁹ In the model, it equals $(1 - \frac{1}{M}) \frac{\beta^2 \sigma_\alpha^2}{\beta^2 \sigma_\alpha^2 + \mu^2 \sigma_\theta^2}$, which implies a variance of noise $\mu^2 \sigma_\theta^2 = 2.64$, and a precision of public information $H = 3.00$ for the baseline estimate. The average precision, $X = 2.09$, then follows from the knowledge of $H + X$. Thus, the data reported in Wurgler (2000) (elasticity and R^2), together with the technological parameters (σ_α^2 , σ_θ^2 , M , and β), are sufficient to completely characterize the information environment, namely X and H . In particular, the cost of information, the variance of noise trading, and preferences need not be specified. Table 1 presents inferences for the three elasticity estimates.

Armed with these estimates, we can quantify the effect of information on the economy. Deviations of the equilibrium from the first best in terms of investment elasticity, TFP, Herfindahl index, income per capita and stock return volatility are shown in Table 1. A remarkable feature is that modest variations in income per capita are compatible with large variations in other variables. In particular, though elasticity and TFP are considerably smaller than in the first best (e.g., -51% for the baseline estimate), income per capita is only moderately lower (-8%).²⁰ The reason is that technology displays strongly decreasing returns to capital, limiting the benefit from a more efficient allocation. If β is increased from one-third to, for example, one-half, then for the baseline estimate, the elasticity, TFP, and income are, respectively, 63%, 63%, and 66% lower than in the first best.

When examining Table 1, it is important to bear in mind some important hypotheses the model makes. First, shocks are assumed to be small. Hence, normalized variables (e.g., elasticities or relative deviations from a benchmark such as the first best or the baseline) are more informative than are raw levels. Second, shocks are assumed to be i.i.d. across firms. Therefore, predictions should be interpreted as pertaining to variables purged from economy and industry factors. Thus, the volatility inferred from the model, for example, 17.64% in the baseline estimation, should be compared to estimates of idiosyncratic volatility. It is reasonably close to 25%, the estimate reported by Campbell et al. (2001) for the United States. Finally, the model cannot match, by construction, several variables such as the equity premium and the

¹⁹ For the elasticity estimates projected on *Synchronicity* and purged from their projections on *SOE*, *Rights*, *FD*, and 1960-GDP, we divide the R^2 reported in Wurgler (2000) by the squared ratio of the baseline elasticity estimate (0.723) to the adjusted elasticity estimate (0.870 or 0.307).

²⁰ The percentage deviation in log TFP from the first-best equals the percentage deviation in investment elasticity, $(k_\alpha - 1/(1 - \beta))/(1/(1 - \beta))$, and equals minus the prior precision as a percentage of total precision, $-(1/\beta^2 \sigma_\alpha^2)/(H + X)$.

expected return on the market (both equal zero since all shocks are assumed to have zero means), trading volume (investors are assumed to only trade once in their lifetime), and the growth rate of income in steady state (technological progress is assumed away).

So far, we have neither determined noise trading nor preference parameters. The data at our disposal do not allow us to identify separately q (the fraction of noise traders) and σ_θ^2 (the variance of their beliefs shocks). To proceed, we need to choose an arbitrary value for q or for σ_θ^2 . Supposing that $q=0.1$ implies that $\sigma_\theta^2=36.2$ in the baseline estimation. Preferences are, to a large extent, independent of information variables in this calibration exercise. Assuming a CES utility, $U(g, j) \equiv (\varpi g^\sigma + (1 - \varpi)j^\sigma)^{1/\sigma}$, the elasticity of substitution between goods and leisure equals $1/(1 - \sigma)$ and the coefficient of relative risk aversion $g/\tau(g) = (1 - \sigma)(1 - \varpi)/(\varpi g^\sigma + 1 - \varpi)$. We calibrate σ and ϖ so that they lead to an elasticity of substitution of 0.8 and to a coefficient of relative risk aversion of one. Finally, we assume that the cost of information is a power function of precision, $C(x) = cx^{\hat{c}}$, where $\hat{c} > 1$. We choose an arbitrary value for the scale parameter c (0.01) and solve for the exponent \hat{c} using the equilibrium condition for the average precision (Equation (19)).

We analyze in Table 2 the sensitivity of the economy to the four parameters describing firms' technology (β , M , σ_a^2 and σ_α^2), maintaining the elasticity and R^2 at their baseline values (0.723 and 12.6%). Generally, the economy is the most sensitive to the choice of β . For example, the total precision has to increase by 72% to maintain the elasticity at 0.723 when β declines by 20% to 0.27. Qualitatively, investments tend to be less responsive to productivity shocks when β is lower (e.g., the elasticity equals $(1 - \frac{1}{M})\frac{1}{1-\beta}$ in the first best), but more responsive when investors are better informed. A lower β also reduces TFP strongly as $\text{Log TFP} = \beta^3 \sigma_\alpha^2 z^2 \times \text{Elasticity}$. But its impact on income is dampened because workers earn a larger fraction of GDP. A 20% smaller number of firms M reduces income by 20% because of the decreasing marginal product of capital but does not influence other aspects of the economy. Finally, the economy is more sensitive to σ_α^2 than to σ_a^2 because of the impact of learning.

6.4 Evidence

In this section, we relate the model's predictions to the evidence. Cross-country studies of the finance and growth nexus, such as [Levine and Zervos's \(1998\)](#) or [Rousseau and Wachtel's \(2000\)](#), should be interpreted with caution because of the small sample size and a potential omitted variable bias. Cross-country/cross-industry studies in the spirit of [Rajan and Zingales \(1998\)](#), such as [Carlin and Mayer's \(2003\)](#), provide more reliable evidence. Several aspects of the model appear to be broadly consistent with the data, assuming that the scale effect of information dominates the substitution effect.

First, [Levine and Zervos \(1998\)](#), [Rousseau and Wachtel \(2000\)](#), and [Carlin and Mayer \(2003\)](#) document that income grows faster in countries with better functioning stock markets as measured by capitalization, trading volume,

Table 2
Sensitivity analysis of the calibrated model

| Variable | β | | M | | σ_d^2 | | σ_a^2 | |
|--|---------|--------|--------|--------|--------------|--------|--------------|--------|
| | -20% | 20% | -20% | 20% | -20% | 20% | -20% | 20% |
| Square forecast error (pct. annual) | 1.36% | 2.04% | 1.36% | 1.36% | 1.22% | 1.50% | 1.23% | 1.49% |
| (pct. deviation from baseline) | (-39%) | (50%) | (-0%) | (0%) | (-10%) | (10%) | (-10%) | (10%) |
| Total precision | 8.762 | 3.232 | 5.089 | 5.085 | 5.087 | 5.087 | 6.358 | 4.239 |
| (pct. deviation from baseline) | (72%) | (-36%) | (0%) | (-0%) | (0%) | (0%) | (25%) | (-17%) |
| Prior precision (pct. of total precision) | 51% | 56% | 51% | 51% | 51% | 51% | 51% | 51% |
| Price precision (pct. of total precision) | 7% | 8% | 7% | 7% | 7% | 7% | 7% | 7% |
| Private precision (pct. of total precision) | 41% | 36% | 41% | 41% | 41% | 41% | 41% | 41% |
| Log TFP | 9% | 16% | 9% | 9% | 9% | 9% | 7% | 11% |
| (pct. deviation from baseline) | (-49%) | (73%) | (0%) | (0%) | (0%) | (0%) | (-20%) | (20%) |
| Income per capita | 494 | 680 | 395 | 593 | 479 | 510 | 443 | 552 |
| (pct. deviation from baseline) | (-15%) | (38%) | (-20%) | (20%) | (-3%) | (3%) | (-10%) | (12%) |
| Herfindhal index | 4.88 | 9.80 | 4.88 | 4.88 | 4.88 | 4.88 | 3.55 | 6.70 |
| (pct. deviation from baseline) | (-43%) | (101%) | (0%) | (0%) | (0%) | (0%) | (-27%) | (37%) |
| Stock return volatility (pct. annual) | 17.64% | 20.03% | 17.63% | 17.64% | 17.23% | 18.03% | 16.21% | 18.95% |
| (pct. deviation from baseline) | (-15%) | (14%) | (-0%) | (0%) | (-2%) | (2%) | (-8%) | (7%) |
| Variance of noise trading $\text{Var}(\theta)$ | 36.198 | 39.170 | 36.222 | 36.183 | 36.198 | 36.198 | 28.959 | 43.438 |
| Utility - σ | -0.250 | -0.250 | -0.250 | -0.250 | -0.250 | -0.250 | -0.250 | -0.250 |
| Utility - ω | 0.453 | 0.444 | 0.439 | 0.464 | 0.453 | 0.453 | 0.453 | 0.453 |
| Information cost - exponent | 2.121 | 10.689 | 2.117 | 2.124 | 2.424 | 1.874 | 1.397 | 3.146 |

This table presents results from a sensitivity analysis of the model calibrated to the U.S. economy assuming that the United States has reached a steady state. The calibrations all generate an investment elasticity of 0.723 and an R^2 of 12.6%, consistent with [Wurgler's \(2000\)](#) baseline estimates for the United States, a coefficient of relative risk aversion of one and an elasticity of substitution of 0.8. The following parameter values are used: $z = 1$, $\beta = 1/3$, $M = 443$, $\sigma_d^2 = 1.94$ /period, $\sigma_a^2 = 3.51$ /period, where a period in the model lasts 30 years, $q = 0.1$, and an information cost function of the form $C(x) = 0.001 \times x^{\text{exponent}}$. The first column displays the baseline estimate as in Column (2) of Table 1. The subsequent columns show how the economy reacts to a 20% change in each of the variables heading the columns, keeping fixed all the other parameters, including the elasticity and R^2 .

or accounting standards. These observations support the notion developed in Section 5 that the stock market, by aggregating and transmitting private information, enhances growth. [Carlin and Mayer \(2003\)](#) document further that the effect of the stock market is stronger for industries with high R&D investments and skilled labor. These findings suggest that the stock market is particularly useful for investing in new technologies about which opinions diverge widely.

Second, the model predicts that allocative efficiency and TFP grow with income. [Wurgler \(2000\)](#) reports that the elasticity of investments to value-added increases with a country's degree of financial development, and in particular with the informativeness of its stock market; in other words, k_α decreases with μ . [Levine and Zervos \(1998\)](#) show that stock markets promote growth in TFP.²¹

Third, the model predicts that the economy specializes as it grows. Empirically, [Imbs and Wacziarg \(2003\)](#) report that countries go through two stages of sectoral diversification. Diversification increases at first, but beyond a certain level of income, the process is reversed and economic activity starts concentrating. Although the focus of [Imbs and Wacziarg \(2003\)](#) is not on financial development, it is worth noting that the pattern this paper documents is consistent with our model to the extent that it applies to more advanced economies; Section 6.2 shows that more information is produced as incomes grows, only if income is above a threshold. In a similar vein, [Kalemli-Ozcan, Sorensen and Yosha \(2003\)](#) find that industrial specialization in a sample of developed countries is positively related to the share of the financial sector in GDP. Though the financial sector includes financial intermediaries, this study is supportive of the model to the extent that the share of the financial sector in GDP is positively related to information expenditures about public companies.

Fourth, the model predicts that wealth inequality conforms to a "Kuznets curve," widening at first and then narrowing. Recent evidence on this pattern is mixed (see [Acemoglu and Robinson 2002](#)) for a review). Recent studies also emphasize that the stock market plays a crucial role in magnifying wealth inequality even though it may not be its source (e.g., [Favilukis 2013](#); [Lusardi, Michaud, and Mitchell 2013](#)).

Fifth, according to the model, the trading activity and liquidity are inverted U-shape functions of income. Empirically, [Levine and Zervos \(1998\)](#) and [Rousseau and Wachtel \(2000\)](#) report that the share turnover on the stock market is positively related to output growth but do not document (or test for) a nonmonotonic pattern.

Finally, the model implies that stock prices are less correlated, idiosyncratic stock returns are less volatile, and market volatility remains constant as income

²¹ We stress that TFP grows in our model, though there is no technological progress (the distribution of productivity shocks and the cost of information are stationary), thanks to a more efficient allocation of capital. Empirically, variations in the allocation of resources account for a large fraction of the cross-country differences in TFP ([Restuccia and Rogerson 2008](#); [Hsieh and Klenow 2009](#)). Moreover, financial liberalizations are associated with increases in TFP (see, for example, [Bekaert, Harvey and Lundblad 2011](#) and the references therein).

grows. Empirically, [Morck, Yeung, and Yu \(2000\)](#) show that stock prices are less synchronous in richer economies. [Campbell et al. \(2001\)](#) document a strong increase in idiosyncratic return volatility in the United States from 1962 to 1997, whereas the volatility of the market return remained stable. Several studies argue that this phenomenon is caused by changes in the characteristics of listed firms which have become smaller and riskier ([Fama and French 2004](#); [Wei and Zhang 2006](#)). Such changes are absent from our model, which assumes a stationary distribution of technology shocks.²²

7. Irrational Behavior

We consider the implications for capital efficiency and growth of two common forms of irrationality: overconfidence and inattention.

7.1 Overconfidence

Starting with overconfidence, we assume in this section that agents overestimate the productivity of the information technology or, equivalently, that they underestimate the cost of learning: an agent who employs $C(x)z$ units of free time believes she acquires a signal of precision ϕx ($\phi \geq 1$) when in fact the signal's true precision is x . The parameter ϕ measures the degree of overconfidence, with $\phi = 1$ corresponding to the rational-agent case analyzed so far. We refer to x as the actual precision and to ϕx as the perceived precision. Investors know the average perceived precision ϕX and correctly assess the precision of the price signal. The equilibrium outcome does not depend on whether or not they realize that ϕX is an overestimate of the average actual precision.

Proposition 19. When agents are overconfident, firm m 's capital stock and share price have the same form as in the rational case, with the average perceived precision ϕX_t substituting for the average actual precision X_t :

$$K_t^m = P_t^m = \frac{Lw_t}{M} \exp(\Delta k_t^m z) \quad \text{and} \quad \Delta k_t^m \equiv k_\alpha(\mu_t)(\beta \Delta \tilde{\alpha}_t^m + \mu_t \Delta \tilde{\theta}_t^m) + o(1),$$

$$k_\alpha(\mu) \equiv \frac{1}{1-\beta} \left(1 - \frac{1}{\beta^2 \sigma_\alpha^2 (H(\mu) + \phi X(\mu))} \right) > 0 \quad \text{and} \quad \phi X(\mu) \equiv \frac{H(\mu)}{\frac{1-q}{q} \mu - 1}.$$

²² We conjecture that the model's prediction on idiosyncratic stock return volatility can be reconciled with the evidence reported in [Campbell et al. \(2001\)](#), without appealing to changes in the characteristics of listed firms, by increasing the number of trading rounds within each period—this would not affect the other predictions. With multiple trading rounds, when more information is produced, the current price tracks future dividends and prices more closely, thereby reducing the return volatility (as in the current model). But the volatility of future prices also increases because future prices track more closely dividends even further into the future.

The noisiness of stock prices, μ_t , is the unique solution to

$$\rho(\varphi(w_t))C' \left(\frac{1}{\phi} \frac{H(\mu_t)}{1-q} \mu_t - 1 \right) = \phi \tau(\varphi(w_t)) \frac{1-1/M}{2\beta^2\sigma_a^2} \left(\frac{1 - \frac{q}{(1-q)\mu_t}}{H(\mu_t)} \right)^2 + o(1). \tag{23}$$

Portfolio weights and stock prices are identical to those obtained in the rational case, except that actual precisions, x_t and X_t , are replaced with perceived precisions, ϕx_t and ϕX_t . The variable μ_t measures the actual noisiness of the price though it aggregates perceived private-signal precisions. Perceived precisions govern the actual noisiness because they determine how aggressively agents trade on their private signals and therefore how much of their information they impound in the price. Actual precisions do not matter in equilibrium because the economy is populated by a continuum of agents with independent signal errors that vanish when they are aggregated.²³ As expected, perceived precisions are higher for overconfident agents than for rational agents. But this need not be the case for actual precisions, which are shaped by two conflicting forces. On one hand, investors who overestimate the productivity of their information technology, allocate more free time to learning due to a substitution effect. On the other hand, they allocate less because they believe less time is sufficient to reach a given precision level (akin to an income effect). Which effect dominates depends on the elasticity of the right-hand side of Equation (23) with respect to the actual average precision. Because the capital allocation depends on perceived precisions that are inflated by overconfidence, it improves thanks to overconfidence: investments are more sensitive to technology shocks (k_α higher) and prices are more informative (μ lower) when investors are more overconfident (ϕ higher). The reason is that overconfidence encourages investors to rely less on priors and use private signals more aggressively. As a result, it increases income on average.

Several papers study the effects of overconfidence on stock market efficiency (e.g., Hirshleifer, Subrahmanyam, and Titman 1994; Kyle and Wang 1997; Odean 1998; Daniel, Hirshleifer, and Subrahmanyam 1998; Garcia, Sangiorgi, and Urosevic 2007; Ko and Huang 2007). Their findings vary depending, in particular, on the subject of overconfidence (precision vs. timing of private

²³ If the number of agents were finite, the equilibrium price would contain agents' signal errors through a term of the type $\frac{1}{L} \sum_{i=1}^L c_{st}^m \Delta \varepsilon_{i,t}^m$ where $c_{st}^m \equiv \frac{\phi X_t^m}{H(\mu_t^m) + \phi X_t^m}$ is the weight assigned by agents to private signals. The variance of this term for ex ante identical agents equals $\frac{1}{L} c_{st}^{m2} Var(\Delta \varepsilon_{i,t}^m)$, where $Var(\Delta \varepsilon_{i,t}^m)$, the actual variance of errors, equals $\frac{M-1}{M} \frac{1}{X_t^m}$. Thus, the precision of the price would be a function of both the actual precision (X_t^m in the denominator of $Var(\Delta \varepsilon_{i,t}^m)$) and the perceived precision (ϕX_t^m in the numerator of c_{st}^m). Note that the presence in the price of inflated errors is actually beneficial to income on average through a Jensen inequality effect (see the discussion following Proposition 15).

signals), on the structure of the market (oligopolistic vs. competitive), on the composition of the investor population (the presence, or not, of rational investors), and on the nature of information (endowed vs. endogenously acquired). The most closely related model is that of [Ko and Huang \(2007\)](#). In their setup, overconfident traders, on one hand, impound excessive errors into the price (they assume a finite number of investors but no noise trading) but, on the other hand, collect more precise private signals. As in our model, the authors find that overconfidence generally improves efficiency provided the level of overconfidence is not too high. Note that both models assume that excessive learning does not harm efficiency by crowding out resources. In our setup for instance, learning time comes out of free time rather than labor time. But because it reduces leisure, the impact on welfare is ambiguous.

We quantify the real effect of overconfidence in the calibrated model of Section 6.3. We use the parameter values derived in the baseline calibration (Column (2) in Table 1) and three values for the degree of overconfidence ϕ : 1.1, 1.5, and 2. Empirically, the consequences of overconfidence are sizable. Using gender as a proxy for overconfidence (men are more overconfident than women), [Barber and Odean \(2001\)](#) report that men trade 45% more than do women for 54% lower net returns. The results, displayed in Table 3, show a modest effect of overconfidence. For example, when agents overestimate by 50% their signals' precision ($\phi = 1.5$), the investment elasticity and log TFP are about a tenth higher than in the rational-agent economy, whereas income per capita is 2% higher.

7.2 Inattention

Turning to inattention, we assume here that investors are not aware of the existence of all M stocks but only of a subset $M - \phi$, where $\phi \geq 0$ denotes the number of ignored stocks. The full-attention case analyzed so far corresponds to $\phi = 0$. Ignored stocks are assumed to be evenly spread among the investor population so that the model's symmetry is preserved: Because there are $C_M^{M-\phi} \equiv \frac{M!}{(M-\phi)!\phi!}$ distinct attention sets (combinations of $M - \phi$ stocks out of M), of which $C_{M-1}^{M-\phi-1} \equiv \frac{(M-1)!}{(M-\phi-1)!\phi!}$ include any specific stock, each stock is recognized by a fraction $C_{M-1}^{M-\phi-1} / C_M^{M-\phi} = 1 - \frac{\phi}{M}$ of inattentive investors.

Proposition 20. When inattentive agents ignore ϕ stocks, firm m 's capital stock and share price are identical to those obtained in the rational case except that the noisiness of the stock price, μ_t , is higher (the average precision X_t is lower):

$$K_t^m = P_t^m = \frac{Lw_t}{M} \exp(\Delta k_t^m z) \quad \text{and} \quad \Delta k_t^m \equiv k_\alpha(\mu_t)(\beta \Delta \tilde{\alpha}_t^m + \mu_t \Delta \tilde{\theta}_t^m) + o(1),$$

$$k_\alpha(\mu) \equiv \frac{1}{1-\beta} \left(1 - \frac{1}{\beta^2 \sigma_\alpha^2 (H(\mu) + X(\mu))} \right) > 0 \quad \text{and} \quad X(\mu) \equiv \frac{H(\mu)}{1-q} \mu^{-1}.$$

Table 3
Effect of irrationality in the calibrated model

| Variable | Rational agents | Overconfident agents | | | Inattentive agents | | |
|---|-----------------|----------------------|--------------|------------|--------------------|---------------|----------------|
| | | $\phi = 1.1$ | $\phi = 1.5$ | $\phi = 2$ | $\phi = 0.5M$ | $\phi = 0.9M$ | $\phi = 0.99M$ |
| Elasticity | 0.723 | 0.742 | 0.802 | 0.854 | 0.723 | 0.719 | 0.677 |
| (pct. deviation from rational case) | | (3%) | (11%) | (18%) | (-0%) | (-1%) | (-6%) |
| R^2 | 0.126 | 0.132 | 0.151 | 0.168 | 0.126 | 0.125 | 0.112 |
| (pct. deviation from rational case) | | (5%) | (20%) | (34%) | (-0%) | (-1%) | (-11%) |
| Square forecast error (pct. annual) | 1.36% | 1.34% | 1.29% | 1.25% | 1.36% | 1.36% | 1.40% |
| (pct. deviation from rational case) | | (-1%) | (-5%) | (-8%) | (0%) | (0%) | (3%) |
| Total precision | 5.087 | 5.215 | 5.668 | 6.139 | 5.084 | 5.062 | 4.800 |
| Prior precision (pct. of total precision) | 51% | 50% | 46% | 43% | 51% | 52% | 55% |
| Price precision (pct. of total precision) | 7% | 8% | 8% | 9% | 7% | 7% | 7% |
| Private precision (pct. of total precision) | 41% | 42% | 46% | 49% | 41% | 41% | 39% |
| Log TFP | 9% | 9% | 10% | 11% | 9% | 9% | 9% |
| (pct. deviation from rational case) | | (3%) | (11%) | (18%) | (-0%) | (-1%) | (-6%) |
| Income per capita | 494 | 497 | 504 | 510 | 494 | 494 | 489 |
| (pct. deviation from rational case) | | (0%) | (2%) | (3%) | (-0%) | (-0%) | (-1%) |
| Herfindhal index | 4.88 | 4.934 | 5.101 | 5.243 | 4.880 | 4.870 | 4.749 |
| (pct. deviation from rational case) | | (1%) | (5%) | (7%) | (-0%) | (-0%) | (-3%) |
| Stock return volatility (pct. annual) | 17.64% | 17.59% | 17.44% | 17.30% | 17.64% | 17.64% | 17.74% |
| (pct. deviation from rational case) | | (-0%) | (-1%) | (-2%) | (0%) | (0%) | (1%) |

This table shows the effect of irrationality in the baseline calibrated model. The first column presents the rational-agent economy, and is identical to the baseline displayed in Column (2) of Table 1 and in Column (1) of Table 2. The next three columns present the overconfident-agent economy, where the parameter ϕ represents the degree of overconfidence (the higher ϕ , the more overconfident are agents). The last three columns display the inattentive-agent economy, where the parameter ϕ represents the proportion of stocks to which each agent is inattentive (the higher ϕ , the more inattentive are agents). The parameters used are obtained from the baseline estimation, which calibrates the model to the U.S. economy assuming that the United States has reached a steady state, and matches the investment elasticity and R^2 to Wurgler's (2000) estimates for the United States: $z = 1$, $\beta = 1/3$, $M = 443$, $\sigma_a^2 = 1.94/\text{period}$, $\sigma_\alpha^2 = 3.51/\text{period}$, where a period in the model lasts 30 years, $q = 0.1$, $C(x) = 0.01 \times x^{2.121}$, $q = 0.1$, $\sigma_\theta^2 = 36.20/\text{period}$, $\omega = 0.45$, $\sigma = -0.25$.

The noisiness of stock prices, μ_t , is the unique solution to

$$\rho(\varphi(w_t))C' \left(\frac{H(\mu_t)}{\frac{1-q}{q}\mu_t - 1} \right) = \tau(\varphi(w_t)) \frac{1 - 1/(M - \phi)}{2\beta^2\sigma_a^2} \left(\frac{1 - \frac{q}{(1-q)\mu_t}}{H(\mu_t)} \right)^2 + o(1). \tag{24}$$

As in the rational case, an agent l spreads her wealth equally among the $M - \phi$ stocks she knows of and then tilts the allocations according to her information: $f_{l,t}^m = \frac{1}{M - \phi} + \frac{\tau(\varphi(w_t))}{\varphi(w_t)\beta^2\sigma_a^2 z} E(\Delta_l \ln R_{t+1}^m | \mathcal{F}_{l,t}) + o(1)$, where $\Delta_l \ln \tilde{R}_{t+1}^m \equiv \ln \tilde{R}_{t+1}^m - \frac{1}{M - \phi} \sum_{m' \in \mathcal{A}_l} \ln \tilde{R}_{t+1}^{m'}$ represents the stock's excess performance relative to the $M - \phi$ stocks recognized by agent l ; and \mathcal{A}_l denotes the set of stocks agent l recognizes. Aggregating these demands leads to a capital stock and a stock price that are similar to those obtained in the full-attention case. In particular, if firms were identical, each would receive $\frac{w_t}{M - \phi}$ units of capital from each of the $(1 - \frac{\phi}{M})L$ investors aware of its existence, resulting in a capital stock of $\frac{Lw_t}{M}$. The only difference from the full-attention case is

that investors collect less private information so prices are more noisy. Indeed, information is less valuable when investors have fewer stocks to choose from. For example, an investor who ignores all but one stock ($\phi = M - 1$) does not produce any information because she will invest all her wealth in that single stock regardless of her signals. As a consequence, inattention is detrimental to capital efficiency and growth: the more inattentive investors are, the less closely their investments track firms' productivity, the less informative are prices, and the lower is income on average. Note that in the same way that there is no cost to overconfidence in terms of resources, there is no benefit to inattention, that is, inattention does not free up resources that can be redeployed in the productive sector because learning employs free time not labor time.

A large body of literature studies the implications of market segmentation. A subset focuses on inattention as a source of segmentation, appealing to the "Investor Recognition Hypothesis" (e.g., Merton 1987; Shapiro 2002; Van Nieuwerburgh and Veldkamp 2010). These papers usually assume that investors aware of an asset share the same information. The most closely related paper is that of Peress (2010b), who endogenizes information acquisition in a segmented market. In contrast to the current model, he finds that more information is produced about less widely held stocks. The reason is that he assumes a riskless bond can be invested in. When no riskfree asset is available as in the current model, each stock is evaluated in comparison to other stocks rather than relative to an exogenous benchmark, leading investors aware of fewer stocks to learn less.

Table 3 quantifies the real effect of inattention in the baseline calibration, assuming investors ignore 50%, 90%, and 99% of the 443 stocks in the economy. Note that empirically, U.S. households hold on average 4.3 stocks in their portfolio, out of a universe of several thousands (Barber and Odean 2000). The inattention effect is very modest. Even in the extreme scenario in which investors only follow 1% of available assets, TFP and income per capita are only, respectively, 6% and 1% lower than in the full-attention economy. The impact of inattention is limited by the large number of available stocks. It would also be bigger (regardless of its impact on precision choices) if inattention were unevenly distributed across stocks, such that some stocks were neglected by all investors.

8. Conclusion

This paper presents a fully integrated model of information acquisition and dissemination through stock prices, capital allocation, and economic growth. It does so by combining, under a small risk approximation, two standard frameworks, the neoclassical overlapping generations growth model and the noisy rational expectations stock market model. The stock market provides a mechanism for sharing information dispersed among many investors, a mechanism that is particularly useful when eliciting effort and truthful

disclosure from investors is difficult. At the heart of the model lies a tension between the benefit from sharing information and the incentive to collect it in the first place. Noise trading resolves this tension by ensuring that acquiring private information remains profitable even though it is partially revealed through prices.

The model yields several implications. First, the information sharing benefit outweighs the disincentive cost. Second, the learning process improves the long-run level of income and its transitory growth rate. But it has no bearing on long-run growth in the absence of technological progress; that is, it does not counter the diminishing returns to capital. Moreover, a calibration exercise indicates that whereas informational efficiency improves TFP considerably, income per capita is only moderately increased. This muted response is a consequence of the strongly decreasing returns to capital, which limit the benefit from a more efficient allocation. Third, the model delivers several predictions on the evolution of real and financial variables, including capital efficiency, TFP, industrial specialization, wealth inequality, stock trading intensity, liquidity, and return volatility. Finally, the paper analyzes the growth impact of two common forms of investor irrationality, overconfidence and inattention, and finds, somewhat surprisingly, that they yield opposite implications. Moreover, the calibrated model suggests that their growth impact is modest.

The paper leaves several questions pending. First, more work is needed to check whether its findings extend to large risks. Second, as the paper sheds light on a force that shapes TFP, namely, the ability to learn from stock prices about technologies, it leaves aside an essential ingredient of economic growth: technological progress. We can only speculate on what might happen in its presence. The primary consequence of technological progress is to raise the growth rate (and level) of income. We conjecture therefore that the allocational effects described here not only obtain but are even amplified. For example, if the scale effect of information dominates the substitution effect, then investors' information grows more precise at a faster rate than in an economy with no technological progress.

Technological progress may have additional implications depending on the form it comes in. Suppose, for example, that it leads to safer technologies, that is, productivity shocks have lower variance over time. Then investors will learn less. On the contrary, technological progress embodied in a larger variety of intermediate goods as in [Romer \(1990\)](#) will encourage learning by raising the opportunity cost of misinvesting, that is, of allocating capital to inefficient technologies, while missing out on successful ones. More work is needed to draw the implications of technological progress.

Conversely, the paper does not study what influence, if any, the stock market exerts on the determination of technologies, as in the endogenous growth literature. It is conceivable, for example, that the information contained in stock prices, or the mere expectation that future stock prices will be informative, impacts the distribution of productivity shocks or the number of available

technologies. In this case, the stock market may have a permanent growth effect through its information processing role. We leave these questions for future research.

Appendix

Proof of Lemma 1

We solve for the capital allocation $\{K_t^{mFB}\}$ chosen by a central planner who can perfectly infer the mean productivity shocks $\{\tilde{\alpha}_t^m\}$. We first note that, when $z=0$, there are no productivity shocks so firms are identical. In that case, given the diminishing marginal product of intermediate goods, the central planner distributes capital equally across the M firms: each firm is allocated $K_t^0 \equiv Lw_t/M$ units of capital, and consumption per capita equals $g_0 \equiv \beta \tilde{G}_{t+1}/L = M\beta L^{-\beta} K_t^{0\beta}$. When $z > 0$, firm m 's capital stock can be expressed as $K_t^{mFB} = K_t^0 \exp(\hat{k}_t^{mFB} z)$, where \hat{k}_t^{mFB} is determined by maximizing the Lagrangian:

$$E[U(\tilde{g}_{l,t+1}, 1) | \{\tilde{\alpha}_t^m\}] + \varsigma_t^{FB} (Lw_t - \sum_{m=1}^M K_t^{mFB}),$$

where ς_t^{FB} is the multiplier on the resource constraint and $\tilde{g}_{l,t+1} = \beta \tilde{G}_{t+1}/L = \sum_{m=1}^M \beta L^{-\beta} (\tilde{A}_t^m K_t^{mFB})^\beta$ denotes consumption per capita. The first-order condition with respect to K_t^{mFB} follows:

$$\varsigma_t^{FB} = E \left[\frac{\partial U}{\partial g}(\tilde{g}_{l,t+1}, 1) \cdot \beta^2 L^{-\beta} \tilde{A}_t^{m\beta} K_t^{mFB(\beta-1)} | \{\tilde{\alpha}_t^m\} \right].$$

It can be expressed as

$$\begin{aligned} \varsigma_t^{FB} K_t^{0(1-\beta)} L^\beta / \beta^2 &= E \left[\frac{\partial U}{\partial g}(\tilde{g}_{l,t+1}, 1) \cdot \exp(\beta \tilde{\alpha}_t^m z + (\beta-1) \hat{k}_t^{mFB} z) | \{\tilde{\alpha}_t^m\} \right] \\ &= E \left[\frac{\partial U}{\partial g}(\tilde{g}_{l,t+1}, 1) \cdot \left(1 + \beta \tilde{\alpha}_t^m z + (\beta-1) \hat{k}_t^{mFB} z + \frac{1}{2} \text{Var}(\beta \tilde{\alpha}_t^m z | \{\tilde{\alpha}_t^m\}) + o(z) \right) | \{\tilde{\alpha}_t^m\} \right] \\ &= E \left[\frac{\partial U}{\partial g}(\tilde{g}_{l,t+1}, 1) \cdot \left(1 + \beta \tilde{\alpha}_t^m z + (\beta-1) \hat{k}_t^{mFB} z + \beta^2 \sigma_a^2 z / 2 + o(z) \right) | \{\tilde{\alpha}_t^m\} \right]. \end{aligned}$$

We expand $\frac{\partial U}{\partial g}(\tilde{g}_{l,t+1}, 1)$ in a Taylor series in a neighborhood of $z=0$, that is, for $\tilde{g}_{l,t+1}$ around g_0 :

$$\frac{\partial U}{\partial g}(\tilde{g}_{l,t+1}, 1) = \frac{\partial U}{\partial g}(g_0, 1) + \frac{\partial^2 U}{\partial g^2}(g_0, 1)(\tilde{g}_{l,t+1} - g_0) + o(z),$$

$$\begin{aligned} \text{where } \tilde{g}_{l,t+1} - g_0 &= \sum_{m=1}^M \beta L^{-\beta} \left[(\tilde{A}_t^m K_t^{mFB})^\beta - K_t^{0\beta} \right] \\ &= \beta L^{-\beta} K_t^{0\beta} \sum_{m=1}^M [\exp(\beta \tilde{\alpha}_t^m z + \beta \hat{k}_t^{mFB} z) - 1] \\ &= \beta L^{-\beta} K_t^{0\beta} \sum_{m=1}^M (\beta \tilde{\alpha}_t^m z + \beta \hat{k}_t^{mFB} z + \frac{1}{2} \text{Var}(\beta \tilde{\alpha}_t^m z | \{\tilde{\alpha}_t^m\})) + o(z) \\ &= \beta L^{-\beta} K_t^{0\beta} \sum_{m=1}^M (\beta \tilde{\alpha}_t^m + \beta \hat{k}_t^{mFB} + \beta^2 \sigma_a^2 / 2) z + o(z). \end{aligned}$$

As a result, the first-order condition can be written as

$$\zeta_t^{FB} K_t^{0(1-\beta)} L^\beta / \beta^2 = E \left[\left(\frac{\partial U}{\partial g}(g_0, 1) + \frac{\partial^2 U}{\partial g^2}(g_0, 1) \beta L^{-\beta} K_t^{0\beta} \sum_{m=1}^M (\beta \tilde{\alpha}_t^m + \beta \widehat{k}_t^{mFB} + \frac{1}{2} \beta^2 \sigma_a^2) z \right) \cdot (1 + \beta \tilde{\alpha}_t^m z + (\beta - 1) \widehat{k}_t^{mFB} z + \frac{1}{2} \beta^2 \sigma_a^2 z) | \{\tilde{\alpha}_t^m\} \right] + o(z).$$

Isolating the order- z terms and denoting $\zeta_{1t}^{FB} z$ the order- z component of the Lagrange multiplier yields:

$$\begin{aligned} \zeta_{1t}^{FB} z K_t^{0(1-\beta)} L^\beta / \beta^2 &= E \left[\frac{\partial U}{\partial g}(g_0, 1) \left(\beta \tilde{\alpha}_t^m z + (\beta - 1) \widehat{k}_t^{mFB} z + \beta^2 \sigma_a^2 z / 2 \right) \right. \\ &\quad \left. + \frac{\partial^2 U}{\partial g^2}(g_0, 1) \beta L^{-\beta} K_t^{0\beta} \sum_{m=1}^M (\beta \tilde{\alpha}_t^m + \beta \widehat{k}_t^{mFB} + \beta^2 \sigma_a^2 / 2) z | \{\tilde{\alpha}_t^m\} \right] \\ &= \frac{\partial U}{\partial g}(g_0, 1) \left(\beta \tilde{\alpha}_t^m + (\beta - 1) \widehat{k}_t^{mFB} + \beta^2 \sigma_a^2 / 2 \right) \\ &\quad + \frac{\partial^2 U}{\partial g^2}(g_0, 1) \beta L^{-\beta} K_t^{0\beta} M (\beta \tilde{\alpha}_t + \beta \widehat{k}_t^{FB} + \beta^2 \sigma_a^2 / 2). \end{aligned}$$

Averaging this Equation across stocks yields

$$\begin{aligned} \zeta_{1t}^{FB} K_t^{0(1-\beta)} L^\beta / \beta^2 &= \frac{\partial U}{\partial g}(g_0, 1) \left(\beta \tilde{\alpha}_t + (\beta - 1) \widehat{k}_t^{FB} + \frac{1}{2} \beta^2 \sigma_a^2 \right) \\ &\quad + \frac{\partial^2 U}{\partial g^2}(g_0, 1) \beta L^{-\beta} K_t^{0\beta} M (\beta \tilde{\alpha}_t + \beta \widehat{k}_t^{FB} + \frac{1}{2} \beta^2 \sigma_a^2), \end{aligned}$$

and subtracting it from the previous one leads to

$$0 = \frac{\partial U}{\partial g}(g_0, 1) \left(\beta \tilde{\alpha}_t^m + (\beta - 1) \widehat{k}_t^{mFB} - \beta \tilde{\alpha}_t - (\beta - 1) \widehat{k}_t^{FB} \right).$$

A solution to this Equation is $\widehat{k}_t^{mFB} = \frac{1}{1-\beta} \Delta \beta \tilde{\alpha}_t^m + o(1)$ because $\overline{\Delta \beta \tilde{\alpha}_t} \equiv 0$. Therefore, $K_t^{mFB} = L w_t / M \exp(\Delta k_t^{mFB} z)$, where $\Delta k_t^{mFB} \equiv \frac{1}{1-\beta} \Delta \beta \tilde{\alpha}_t^m + o(1)$, as stated in Lemma 1.

Proof of Lemma 2

The number of final goods produced in the first best is

$$\tilde{G}_{t+1} = \sum_{m=1}^M L^{1-\beta} (\tilde{A}_t^m K_t^{mFB})^\beta = L w_t^\beta M^{1-\beta} \overline{\exp(\beta(\tilde{a}_t z + \Delta k_t^{FB} z))}.$$

Therefore, the wage and its average equal

$$\tilde{w}_{t+1} = (1 - \beta) \tilde{G}_{t+1} / L = (1 - \beta) w_t^\beta M^{1-\beta} \overline{\exp(\beta z(\tilde{a}_t + \Delta k_t^{FB}))},$$

$$\text{and } E(\tilde{w}_{t+1}) = (1 - \beta) w_t^\beta M^{1-\beta} E \left[\overline{\exp(\beta z(\tilde{a}_t + k_t^{FB}))} \right],$$

$$\begin{aligned} \text{where } E \left[\overline{\exp(\beta z(\tilde{a}_t + k_t^{FB}))} \right] &= E \left[\exp \left(\beta z \left(\tilde{\alpha}_t^m + \frac{1}{1-\beta} \Delta \beta \tilde{\alpha}_t^m \right) \right) \right] + o(z) \\ &= \exp \left[\frac{1}{2} \text{Var} \left(\beta z \left(\tilde{\alpha}_t^m + \frac{1}{1-\beta} \Delta \beta \tilde{\alpha}_t^m \right) \right) \right] + o(z) \\ &= \exp \left\{ \frac{1}{2} E \left[\text{Var} \left(\beta z \left(\tilde{\alpha}_t^m + \frac{1}{1-\beta} \Delta \beta \tilde{\alpha}_t^m \right) | \{\tilde{\alpha}_t^m\} \right) \right] \right\} \\ &\quad + \frac{1}{2} \text{Var} \left[E \left(\beta z \left(\tilde{\alpha}_t^m + \frac{1}{1-\beta} \Delta \beta \tilde{\alpha}_t^m \right) | \{\tilde{\alpha}_t^m\} \right) \right] \right\} + o(z). \end{aligned}$$

This expression reduces to

$$\begin{aligned}
 & E \left[\exp(\beta z (\tilde{\alpha}_t^m + k_t^{FB})) \right] \\
 &= \exp \left\{ \frac{1}{2} E [\text{Var}(\beta \tilde{\alpha}_t^m z | \{ \tilde{\alpha}_t^m \})] + \frac{1}{2} \text{Var} \left[\beta z \left(\tilde{\alpha}_t^m + \frac{1}{1-\beta} \Delta \beta \tilde{\alpha}_t^m \right) \right] \right\} + o(z) \\
 &= \exp \left\{ \frac{1}{2} E [\beta^2 \sigma_a^2 z^2] + \frac{1}{2} \text{Var} \left[\beta z \left(\tilde{\alpha}_t^m \left(1 + \frac{\beta}{1-\beta} \frac{M-1}{M} \right) - \frac{\beta}{1-\beta} \frac{1}{M} \sum_{\substack{m'=1 \\ m' \neq m}}^M \tilde{\alpha}_t^{m'} \right) \right] \right\} + o(z) \\
 &= \exp \left\{ \frac{1}{2} \beta^2 \sigma_a^2 z^2 + \frac{1}{2} \text{Var} \left[\beta z \left(\tilde{\alpha}_t^m \left(1 + \frac{\beta}{1-\beta} \frac{M-1}{M} \right) \right) + \frac{1}{2} \text{Var} \left[\frac{\beta}{1-\beta} \frac{1}{M} \sum_{\substack{m'=1 \\ m' \neq m}}^M \tilde{\alpha}_t^{m'} \right] \right] \right\} + o(z) \\
 &= \exp \left\{ \frac{1}{2} \beta^2 \sigma_a^2 z^2 + \frac{1}{2} \beta^2 \sigma_a^2 z^2 \left(1 + \frac{\beta}{1-\beta} \frac{M-1}{M} \right)^2 + \frac{1}{2} \sigma_a^2 z^2 \left(\frac{\beta}{1-\beta} \right)^2 \frac{M-1}{M^2} \right\} + o(z) \\
 &= \exp \left\{ \frac{1}{2} \beta^2 \sigma_a^2 z^2 + \frac{1}{2} \beta^2 \sigma_a^2 z^2 + \frac{1}{2} \beta^2 \sigma_a^2 \frac{M-1}{M} \frac{\beta(2-\beta)}{(1-\beta)^2} z^2 \right\} + o(z) \\
 &= \exp \left\{ \frac{1}{2} \beta^2 \sigma_a^2 z^2 + \frac{1}{2} \beta^2 \sigma_a^2 z^2 + \lambda^{FB} z^2 \right\} + o(z).
 \end{aligned}$$

Substituting this expression into the Equation for $E(\tilde{w}_{t+1})$ leads to the law of motion for average income presented in Lemma 2. steady state income increases with the number of intermediate goods M as the production possibility set expands, and with the variance of productivity shocks $\sigma_a^2 z + \sigma_a^2 z^2$ because output is a convex function of these shocks—a positive shock increases \tilde{G}_{t+1} more than a negative shock decreases it. It decreases with the factor share of intermediate goods β as the marginal product of labor is reduced.

Proof of Lemma 3

Given the conjectured capital allocation, observing the M stock prices (or the M capital stocks) is equivalent to observing $\Delta \xi_t^m$ for every firm m where $\xi_t^m \equiv \beta \alpha_t^m + \mu_t^m \theta_t^m$. Similarly, observing the private signals $\{s_{t,t}^m\}$ across the M stocks is equivalent, for an agent l , to observing $\Delta s_{t,t}^m$ for every firm m . The first step is to relate stock returns to productivity shocks and capital.

- Stock returns

Given its capital stock K_t^m , firm m sells $\tilde{Y}_{t+1}^m = \tilde{A}_t^m K_t^m$ intermediate goods for a profit $\tilde{\Pi}_{t+1}^m = \tilde{r}_{t+1}^m \tilde{Y}_{t+1}^m = \beta L^{1-\beta} \tilde{Y}_{t+1}^\beta = \beta L^{1-\beta} (\tilde{A}_t^m K_t^m)^\beta$. The gross return on stock m is then $\tilde{R}_{t+1}^m = \tilde{\Pi}_{t+1}^m / K_t^m = \beta L^{1-\beta} K_t^{0\beta-1} \exp[\beta \tilde{\alpha}_t^m z - (1-\beta) \Delta k_t^m z]$ where $K_t^0 \equiv L w_t / M$ denotes the firm's capital stock when $z=0$. The log return on stock m is $\ln \tilde{R}_{t+1}^m = \ln R_t^0 + r_{t+1}^m z$ where $R_t^0 = \beta L^{1-\beta} K_t^{0\beta-1} = \beta M^{1-\beta} w_t^{\beta-1} = \varphi(w_t) / w_t$ and $r_{t+1}^m z = \beta \tilde{\alpha}_t^m z - (1-\beta) \Delta k_t^m z$. We show below that investors' portfolio weights depend on expected relative returns $E(\Delta r_{t+1}^m z | \mathcal{F}_{l,t})$ and on the variance of returns $\text{Var}(r_{t+1}^m z | \mathcal{F}_{l,t})$. These are given by

$$E(\Delta r_{t+1}^m z | \mathcal{F}_{l,t}) = E(\beta \Delta \tilde{\alpha}_t^m z | \mathcal{F}_{l,t}) - (1-\beta) \Delta k_t^m z = E(\beta \Delta \tilde{\alpha}_t^m z | \mathcal{F}_{l,t}) - (1-\beta) \Delta k_t^m z, \tag{A1}$$

$$\text{and } \text{Var}(r_{t+1}^m z | \mathcal{F}_{l,t}) = \text{Var}(\beta \tilde{\alpha}_t^m z | \mathcal{F}_{l,t}) = \beta^2 \sigma_a^2 z + \text{Var}(\beta \tilde{\alpha}_t^m z | \mathcal{F}_{l,t}) = \beta^2 \sigma_a^2 z + o(z). \tag{A2}$$

The variance of returns is constant at the order z because $\text{Var}(\beta \tilde{\alpha}_t^m z | \mathcal{F}_{l,t})$ is of order z^2 . The next step is to estimate the expectation of $\Delta \tilde{\alpha}_t^m$ using the conjectured prices and private signals $s_{t,t}^m$.

- Signal extraction

For the capital allocation given in Equation (14), the conditional mean and variance of $\Delta \tilde{\alpha}_t^m$ are for agent l , whose private signal has precision $x_{l,t}^m$:

$$E(\beta \Delta \tilde{\alpha}_t^m z | \mathcal{F}_{l,t}) = c_{\xi_t}^m \Delta \xi_t^m z + c_{s_t}^m \Delta s_{l,t}^m z \tag{A3}$$

$$\text{where } \hat{h}_{l,t}^m \equiv H(\mu_t^m) + x_{l,t}^m, \quad c_{\xi_t}^m \hat{h}_{l,t}^m \equiv \frac{1}{\mu_t^m \sigma_\theta^2} \quad \text{and} \quad c_{s_t}^m \hat{h}_{l,t}^m \equiv x_{l,t}^m.$$

$E(\beta \Delta \tilde{\alpha}_t^m z | \mathcal{F}_{l,t})$ is a weighted average of priors (which equal 0), and public and private signals, where the weight on the private signal is increasing in $x_{l,t}^m$ and that on the public signal is decreasing in μ_t^m .

- Portfolio weights

We now solve for optimal portfolios. An agent with a wage w_t and precisions $\{x_{l,t}^m\}$ maximizes $E[U(\tilde{g}_{l,t+1}, j_t) | \mathcal{F}_{l,t}]$, where $\tilde{g}_{l,t+1} = w_t \tilde{R}_{l,t+1}$ and $j_t = 1 - \sum_{m=1}^M C(x_{l,t}^m)z$ are her consumption of final goods and leisure. Let $r_{l,t+1}z \equiv \ln \tilde{R}_{l,t+1} - \ln R_t^0$ capture terms of order z and smaller in her log portfolio return. Portfolio returns can be related to individual stock returns and portfolio weights as follows:

$$\begin{aligned} r_{l,t+1}z &= \ln \left(\sum_{m=1}^M f_{l,t}^m \tilde{R}_{t+1}^m / R_t^0 \right) = \ln \left(\sum_{m=1}^M f_{l,t}^m \exp(r_{t+1}^m z) \right) \\ &= \ln \left(\sum_{m=1}^M f_{l,t}^m (1 + r_{t+1}^m z + \text{Var}(r_{t+1}^m z | \mathcal{F}_{l,t})/2) + o(z) \right) \\ &= \sum_{m=1}^M \left(f_{l,t}^m r_{t+1}^m z + \frac{1}{2} f_{l,t}^m (1 - f_{l,t}^m) \text{Var}(r_{t+1}^m z | \mathcal{F}_{l,t}) \right) + o(z) \\ &= \sum_{m=1}^M \left(f_{l,t}^m r_{t+1}^m z + \frac{1}{2} f_{l,t}^m (1 - f_{l,t}^m) \beta^2 \sigma_a^2 z \right) + o(z) \end{aligned}$$

where we use $\sum_{m=1}^M f_{l,t}^m = 1$ and Equation (A2). Thus, the log portfolio return is approximately normal when z is small (e.g., Campbell and Viceira 2002) and its moments are given by

$$E(r_{l,t+1}z | \mathcal{F}_{l,t}) = \sum_{m=1}^M \left\{ f_{l,t}^m e_{l,t}^m z + \frac{1}{2} f_{l,t}^m (1 - f_{l,t}^m) \beta^2 \sigma_a^2 z \right\} + o(z) \quad \text{where } e_{l,t}^m \equiv E(r_{t+1}^m | \mathcal{F}_{l,t})$$

$$\text{and } \text{Var}(r_{l,t+1}z | \mathcal{F}_{l,t}) = \sum_{m=1}^M f_{l,t}^m \text{Var}(r_{t+1}^m z | \mathcal{F}_{l,t}) + o(z) = \sum_{m=1}^M f_{l,t}^m \beta^2 \sigma_a^2 z + o(z). \tag{A4}$$

The agent's utility can be expanded in a Taylor series in a neighborhood of $z=0$, that is, for $\tilde{g}_{l,t+1}$ and j_t respectively around $\varphi(w_t) = w_t R_t^0$ and 1. We denote the pair $(\varphi(w_t), 1)$ with a $*$:

$$U(\tilde{g}_{l,t+1}, j_t) = U(*) + \frac{\partial U}{\partial g} (*) \cdot (\tilde{g}_{l,t+1} - \varphi(w_t)) + \frac{\partial U}{\partial j} (*) \cdot (j_t - 1) + \frac{1}{2} \frac{\partial^2 U}{\partial g^2} (*) \cdot (\tilde{g}_{l,t+1} - \varphi(w_t))^2 + o(z).$$

Noting that $\tilde{g}_{l,t+1} - \varphi(w_t) = \varphi(w_t) (w_t \tilde{R}_{l,t+1} / \varphi(w_t) - 1) = \varphi(w_t) (\tilde{R}_{l,t+1} / R_t^0 - 1) = \varphi(w_t) (\exp(r_{l,t+1}z) - 1)$ and that $j_t - 1 = -\sum_{m=1}^M C(x_{l,t}^m)z$ allows to write the above expression as

$$\begin{aligned} U(\tilde{g}_{l,t+1}, j_t) &= U(*) + \frac{\partial U}{\partial g} (*) \cdot \varphi(w_t) (\exp(r_{l,t+1}z) - 1) - \frac{\partial U}{\partial j} (*) \cdot \sum_{m=1}^M C(x_{l,t}^m)z \\ &\quad + \frac{1}{2} \frac{\partial^2 U}{\partial g^2} (*) \cdot \varphi(w_t)^2 (\exp(r_{l,t+1}z) - 1)^2 + o(z). \end{aligned}$$

Taking expectations and noting that $E[(\exp(r_{l,t+1}z) - 1)^2 | \mathcal{F}_{l,t}] = E[(r_{l,t+1}z + \text{Var}(r_{l,t+1}z | \mathcal{F}_{l,t})/2 + o(z))^2 | \mathcal{F}_{l,t}] = \text{Var}(r_{l,t+1}z | \mathcal{F}_{l,t})/2 + o(z)$ yields:

$$E(U(\tilde{g}_{l,t+1}, j_t) | \mathcal{F}_{l,t}) = U(*) + \frac{\partial U}{\partial g} (*) \cdot \varphi(w_t) [E(r_{l,t+1}z | \mathcal{F}_{l,t}) + \text{Var}(r_{l,t+1}z | \mathcal{F}_{l,t})/2] - \frac{\partial U}{\partial j} (*) \cdot \sum_{m=1}^M C(x_{l,t}^m)z + \frac{1}{2} \frac{\partial^2 U}{\partial g^2} (*) \cdot \varphi(w_t)^2 \text{Var}(r_{l,t+1}z | \mathcal{F}_{l,t}) + o(z). \tag{A5}$$

Replacing $E(r_{l,t+1}z | \mathcal{F}_{l,t})$ and $\text{Var}(r_{l,t+1}z | \mathcal{F}_{l,t})$ using Equations (A4) and maximizing this expression with respect to $f_{l,t}^m$, subject to $\sum_{m=1}^M f_{l,t}^m = 1$, leads to the first-order conditions:

$$\frac{\partial U}{\partial g} (*) (e_{l,t}^m + \frac{1}{2} \beta^2 \sigma_a^2) + \beta^2 \sigma_a^2 \varphi(w_t) \frac{\partial^2 U}{\partial g^2} (*) f_{l,t}^m + o(1) = \varsigma_{l,t} \text{ for } m = 1, \dots, M, \tag{A6}$$

in which $\varsigma_{l,t}$ denotes the Lagrange multiplier on the constraint. Averaging Equation (A6) across stocks and noting that $\bar{f}_{l,t} \equiv \sum_{m=1}^M f_{l,t}^m / M = 1/M$ yields

$$\frac{\partial U}{\partial g} (*) (\bar{e}_{l,t} + \frac{1}{2} \beta^2 \sigma_a^2) + \beta^2 \sigma_a^2 \varphi(w_t) \frac{\partial^2 U}{\partial g^2} (*) \frac{1}{M} + o(1) = \varsigma_{l,t}. \tag{A7}$$

Subtracting Equation (A7) from the first-order condition (A6) leads to the formula for portfolio weights presented in Equation (9):

$$f_{l,t}^m = \frac{1}{M} + \frac{\tau(\varphi(w_t))}{\varphi(w_t) \beta^2 \sigma_a^2} \Delta e_{l,t}^m + o(1), \tag{A8}$$

where $\tau(\varphi(w_t))/\varphi(w_t) = -\frac{\partial U}{\partial g} (*) / \frac{\partial^2 U}{\partial g^2} (*) / \varphi(w_t)$. Substituting in the expression for $\Delta e_{l,t}^m \equiv E(\Delta r_{t+1}^m | \mathcal{F}_{l,t})$ and using Equations (A1) and (A3) leads to Equation (11) for the portfolio of a rational trader. Substituting in instead $\Delta e_{l,t}^m = \Delta \theta_t^m$ yields the portfolio of a noise trader displayed in Equation (13).

Proof of Proposition 4

We guess that the capital allocation is given by Equations (14) to (16), solve for the equilibrium and check that the guess is valid. Agents' portfolios under the conjectured capital allocation are described in Lemma 3. We multiply portfolio weights by income w_t and sum over agents. The aggregate demand for stock m emanating from rational traders equals

$$\int_{l \text{ rat.}} f_{l,t}^m w_t = \int_{l \text{ rational}} w_t \left\{ \frac{1}{M} + \frac{\tau(\varphi(w_t))}{\varphi(w_t) \beta^2 \sigma_a^2} \left[\frac{x_{l,t}^m}{\bar{h}_{l,t}^m} \Delta s_{l,t}^m + \left(\frac{1}{\bar{h}_{l,t}^m \mu_t^{m2} \sigma_\theta^2 k_{at}^m} - (1-\beta) \right) \Delta k_t^m \right] \right\} + o(1) = L w_t \left\{ \frac{1-q}{M} + \frac{\tau(\varphi(w_t))}{\varphi(w_t) \beta^2 \sigma_a^2} \left[T_t^m \beta \Delta \tilde{\alpha}_t^m + \left(\frac{U_t^m}{\mu_t^{m2} \sigma_\theta^2 k_{at}^m} - (1-q)(1-\beta) \right) \Delta k_t^m \right] \right\} + o(1), \tag{A9}$$

where $T_t^m \equiv \frac{1}{L} \int_{l \text{ rat.}} \frac{x_{l,t}^m}{\bar{h}_{l,t}^m}$ and $U_t^m \equiv \frac{1}{L} \int_{l \text{ rat.}} \frac{1}{\bar{h}_{l,t}^m}$. To derive this expression, we apply the law of large numbers to the sequence $\{ \frac{x_{l,t}^m}{\bar{h}_{l,t}^m} \Delta e_{l,t}^m \}$ of independent (across agents) random variables with the same mean 0 conditional on $\Delta \tilde{\alpha}_t^m$. It implies that $\int_l \frac{x_{l,t}^m}{\bar{h}_{l,t}^m} \Delta e_{l,t}^m = 0$ and hence that $\frac{1}{L} \int_{l \text{ rat.}} \frac{x_{l,t}^m}{\bar{h}_{l,t}^m} \Delta s_{l,t}^m = \frac{1}{L} \int_{l \text{ rat.}} \frac{x_{l,t}^m}{\bar{h}_{l,t}^m} (\beta \Delta \tilde{\alpha}_t^m + \Delta e_{l,t}^m) = \frac{1}{L} \int_{l \text{ rat.}} \frac{x_{l,t}^m}{\bar{h}_{l,t}^m} \beta \Delta \tilde{\alpha}_t^m = T_t^m \beta \Delta \tilde{\alpha}_t^m$.

The aggregate demand for stock m emanating from noise traders equals

$$\int_{l \text{ noise}} f_{l,t}^m w_t = q L w_t \left(\frac{1}{M} + \frac{\tau(\varphi(w_t))}{\varphi(w_t)\beta^2\sigma_a^2} \Delta\theta_t^m \right) + o(1).$$

Summing rational and noise traders' demand for stock m , $(\int_{l \text{ rat.}} f_{l,t}^m w_t + \int_{l \text{ noise}} f_{l,t}^m w_t) / P_t^m$, and equating it to the supply of shares (normalized to one) leads to:

$$L w_t \left\{ \frac{1}{M} + \frac{\tau(\varphi(w_t))}{\varphi(w_t)\beta^2\sigma_a^2} \left[q \Delta\theta_t^m + T_t^m \beta \Delta\tilde{\alpha}_t^m + \left(\frac{U_t^m}{\mu_t^{m2}\sigma_\theta^2 k_{\alpha t}^m} - (1-q)(1-\beta) \right) \Delta k_t^m \right] \right\} + o(1) = P_t^m.$$

Because the left-hand side of this Equation is deterministic at the order 0 in z (it equals $P_t^m = K_t^m = K_t^0 + o(1) \equiv L w_t / M + o(1)$), the expression in the square bracket must be equal to zero. As a consequence,

$$\Delta k_t^m = \frac{T_t^m}{(1-q)(1-\beta) - \frac{U_t^m}{\mu_t^{m2}\sigma_\theta^2 k_{\alpha t}^m}} (\beta \Delta\tilde{\alpha}_t^m + \frac{q}{T_t^m} \Delta\theta_t^m).$$

Comparing this expression to the conjectured capital allocation (Equation (14)) implies that

$$\mu_t^m T_t^m \equiv q \quad \text{and} \quad k_{\alpha t}^m = \frac{T_t^m}{(1-q)(1-\beta) - \frac{U_t^m}{\mu_t^{m2}\sigma_\theta^2 k_{\alpha t}^m}}, \tag{A10}$$

which in turn implies that $k_{\alpha t}^m \equiv \frac{1}{(1-\beta)(1-q)} (T_t^m + \frac{U_t^m}{\mu_t^{m2}\sigma_\theta^2})$. Equilibrium prices are linear in $\Delta\tilde{\alpha}_t^m$ and $\Delta\theta_t^m$, which confirms the initial guess. Moreover, if signal precisions are identical across agents ($x_{l,t}^m = X_t^m$ for all l), then T_t^m and U_t^m simplify to $T_t^m = (1-q)X_t^m / (H(\mu_t^m) + X_t^m)$ and $U_t^m = (1-q) / (H(\mu_t^m) + X_t^m)$. In this case, we obtain Equations (15) and (16) displayed in Proposition 2.

Proof of Lemma 5

The elasticity of investments to productivity shocks, $\partial(\ln K_t^m) / \partial\tilde{\alpha}_t^m$, equals $\beta(1-1/M)k_{\alpha t}$ which decreases with μ_t because

$$\frac{\partial k_{\alpha t}}{\partial \mu_t} = -\frac{1}{1-\beta} \frac{2\frac{1-q}{q}\mu_t + \frac{\mu_t^2\sigma_\theta^2}{\beta^2\sigma_a^2} - 1}{\beta^2\sigma_a^2\frac{1-q}{q}H(\mu_t)^2\mu_t^4\sigma_\theta^2} < 0. \tag{A11}$$

Given Equation (14), TFP equals $\exp(\text{cov}(\beta\tilde{\alpha}_t^m z, \beta\Delta k_t^m z)) = \exp(k_\alpha \beta^3 \sigma_a^2 z^2 (M-1)/M)$ and decreases with μ_t because

$$\frac{d \ln TFP}{d \mu_t} = \frac{M-1}{M} \beta^3 \frac{\partial k_{\alpha t}}{\partial \mu_t} \sigma_a^2 z^2 + o(z^2) = -\frac{M-1}{M} \frac{\beta}{1-\beta} \frac{2\frac{1-q}{q}\mu_t + \frac{\mu_t^2\sigma_\theta^2}{\beta^2\sigma_a^2} - 1}{H(\mu_t)^2\mu_t^4\sigma_\theta^2} z^2 + o(z^2) < 0,$$

The Herfindahl index for capital is given by:

$$Her(K_t^m) \equiv E(K_t^{m2}) / [E(K_t^m)]^2 = E[\exp(2\Delta k_t^m z)] / (E[\exp(\Delta k_t^m z)])^2 = \exp[\text{Var}(\Delta k_t^m z)],$$

which, from Equation (A13), decreases with noisiness μ_t . A similar calculation for profits implies that its Herfindahl index, $Her(\tilde{\Pi}_{t+1}^m)$, decreases too with noisiness.

Proof of Lemma 6

We start by computing $E(\tilde{w}_{t+1})$. Proceeding as in Lemma 2, the wage equals $\tilde{w}_{t+1} = (1 - \beta)\tilde{G}_{t+1}/L = (1 - \beta)w_t^\beta M^{1-\beta} \exp(\beta z(\tilde{\alpha}_t^m + \Delta k_t^m))$, and its average is given by

$$E(\tilde{w}_{t+1}) = (1 - \beta)w_t^\beta M^{1-\beta} E \left[\exp(\beta z(\tilde{\alpha}_t^m + \Delta k_t^m)) \right] = (1 - \beta)w_t^\beta M^{1-\beta} E \left[\exp(\beta z(\tilde{\alpha}_t^m + \Delta k_t^m)) \right] \\ = (1 - \beta)w_t^\beta M^{1-\beta} \exp \left[\frac{1}{2} \text{Var}(\beta z(\tilde{\alpha}_t^m + \Delta k_t^m)) \right],$$

as $E(\tilde{\alpha}_t^m) = 0$ and $E(\Delta k_t^m) = k_\alpha E((\beta \Delta \tilde{\alpha}_t^m + \mu_t \Delta \tilde{\theta}_t^m)) = 0$. Moreover,

$$\text{Var}[\beta(\tilde{\alpha}_t^m z + \Delta k_t^m z)] = \text{Var} \{ E[\beta(\tilde{\alpha}_t^m z + \Delta k_t^m z) | \{\tilde{\alpha}_t^m, \theta_t^m\}] \} + E \{ \text{Var}[\beta(\tilde{\alpha}_t^m z + \Delta k_t^m z) | \{\tilde{\alpha}_t^m, \theta_t^m\}] \} \\ = \text{Var} \{ \beta(\tilde{\alpha}_t^m z + \Delta k_t^m z) \} + E \{ \text{Var}[\beta(\tilde{\alpha}_t^m z) | \{\tilde{\alpha}_t^m, \theta_t^m\}] \} \\ = \text{Var} \{ \beta(\tilde{\alpha}_t^m z + \Delta k_t^m z) \} + \beta^2 \sigma_\alpha^2 z^2 \\ = \text{Var}(\beta \tilde{\alpha}_t^m z) + \text{Var}(\beta \Delta k_t^m z) + 2\text{cov}(\beta \tilde{\alpha}_t^m z, \beta \Delta k_t^m z) + \beta^2 \sigma_\alpha^2 z^2 \\ = \beta^2 \sigma_\alpha^2 z^2 + \text{Var}(\beta \Delta k_t^m z) + 2\text{cov}(\beta \tilde{\alpha}_t^m z, \beta \Delta k_t^m z) + \beta^2 \sigma_\alpha^2 z^2. \tag{A12}$$

The covariance and variance terms are given by

$$\text{cov}(\beta \tilde{\alpha}_t^m z, \beta \Delta k_t^m z) = \text{cov}(\beta \tilde{\alpha}_t^m z, \beta k_\alpha \beta \Delta \tilde{\alpha}_t^m z) + o(z^2) = \frac{M-1}{M} \beta^3 k_\alpha \sigma_\alpha^2 z^2 + o(z^2),$$

$$\text{Var}(\beta \tilde{\alpha}_t^m z) = \beta^2 \sigma_\alpha^2 z^2$$

$$\text{and } \text{Var}(\beta \Delta k_t^m z) = \text{Var}[\beta k_{\alpha t}(\beta \Delta \tilde{\alpha}_t^m + \mu_t \Delta \tilde{\theta}_t^m)z] + o(z^2) \\ = \beta^2 k_{\alpha t}^2 [\beta^2 \text{Var}(\Delta \tilde{\alpha}_t^m) + \mu_t^2 \text{Var}(\Delta \tilde{\theta}_t^m)]z^2 + o(z^2) \\ = \beta^2 k_{\alpha t}^2 \frac{M-1}{M} (\beta^2 \sigma_\alpha^2 + \mu_t^2 \sigma_\theta^2)z^2 + o(z^2).$$

Substituting these expressions into Equation (A12) yields

$$\text{Var}[\beta(\tilde{\alpha}_t^m z + \Delta k_t^m z)] = \beta^2 \sigma_\alpha^2 z^2 + \beta^2 \sigma_\alpha^2 z^2 + \beta^2 k_{\alpha t}^2 \frac{M-1}{M} (\beta^2 \sigma_\alpha^2 + \mu_t^2 \sigma_\theta^2)z^2 + 2 \frac{M-1}{M} \beta^3 k_{\alpha t} \sigma_\alpha^2 z^2 + o(z^2).$$

It follows that $E(\tilde{w}_{t+1}) = \Lambda \exp(\lambda(w_t)z^2)$ where λ and Λ are defined in Equations (21) and (6).

To evaluate $\partial E(\tilde{w}_{t+1})/\partial \mu_t$, we differentiate λ with respect to μ_t , holding current income w_t constant:

$$2 \frac{\partial \lambda}{\partial \mu_t} = \frac{\partial \text{Var}[\beta(\tilde{\alpha}_t^m z + \Delta k_t^m z)]}{\partial \mu_t} = \frac{\partial \text{cov}(\beta \tilde{\alpha}_t^m z, \beta \Delta k_t^m z)}{\partial \mu_t} + \frac{\partial \text{Var}(\beta \Delta k_t^m z)}{\partial \mu_t},$$

$$\text{where } \frac{\partial \text{cov}(\beta \tilde{\alpha}_t^m z, \beta \Delta k_t^m z)}{\partial \mu_t} = - \frac{M-1}{M} \frac{\beta}{1-\beta} \frac{2 \frac{1-q}{q} \mu_t + \frac{\mu_t^2 \sigma_\theta^2}{\beta^2 \sigma_\alpha^2} - 1}{\frac{1-q}{q} H(\mu_t)^2 \mu_t^4 \sigma_\theta^2} z^2 + o(z^2) < 0 \text{ from Equation (A11),}$$

$$\text{and } \frac{\partial \text{Var}(\beta \Delta k_t^m z)}{\partial \mu_t} = 2 \frac{M-1}{M} \beta^2 k_{\alpha t} \left[\frac{\partial k_{\alpha t}}{\partial \mu_t} (\beta^2 \sigma_\alpha^2 + \mu_t^2 \sigma_\theta^2)z^2 + k_{\alpha t} \mu_t \sigma_\theta^2 \right] + o(z^2), \tag{A13}$$

$$= -2 \frac{M-1}{M} \left(\frac{\beta}{1-\beta} \right)^2 \frac{(\frac{1-q}{q} \mu_t - 1)(\frac{1-q}{q} \mu_t + \frac{\mu_t^2 \sigma_\theta^2}{\beta^2 \sigma_\alpha^2})}{(\frac{1-q}{q})^2 H(\mu_t)^2 \mu_t^5 \sigma_\theta^2} z^2 + o(z^2) < 0.$$

It follows that $\frac{\partial E(\tilde{w}_{t+1})}{\partial \mu_t} = E(\tilde{w}_{t+1}) \frac{\partial \lambda}{\partial \mu_t} z^2 < 0$.

Proof of Lemma 7

The degree of inequality is measured as the variance of final wealth, $\tilde{g}_{l,t+1} = w_t \tilde{R}_{l,t+1} = w_t R_t^0 \exp(r_{l,t+1}z)$. Because final wealth is approximately log-normal when z is small, $Var(\tilde{g}_{l,t+1})$ is equivalent to a Gini index, which equals $2F(\sqrt{Var(r_{l,t+1}z)}) - 1$ where F is the cumulative distribution function for a standard normal and where $Var(r_{l,t+1}z) = Var[E(r_{l,t+1}z | \mathcal{F}_t)] + E[Var(r_{l,t+1}z | \mathcal{F}_t)] = E[\sum_{m=1}^M f_{l,t}^m \beta^2 \sigma_a^2 z] + o(z)$ given that $Var[E(r_{l,t+1}z | \mathcal{F}_t)]$ is of order z^2 and using Equation (A4). Substituting Equation (A8) into this expression leads to $Var(r_{l,t+1}z) = \frac{\beta^2 \sigma_a^2}{M} z + \frac{M}{\beta^2 \sigma_a^2} \frac{\tau(\varphi(w_t))^2}{\varphi(w_t)^2} E(\bar{e}_{l,t}^2 - \bar{e}_{l,t}^2) z + o(z)$ where $E(\bar{e}_{l,t}^2 - \bar{e}_{l,t}^2) = Var(\Delta e_{l,t}^m)$ from Equation (A17) below. Moreover, $Var(\Delta e_{l,t}^m) = Var(\Delta e_{l,t}^m) = \frac{M-1}{M} \frac{\varphi(w_t)}{\tau(\varphi(w_t))} \left(\frac{X_t}{h(X_t)^2} + \frac{q^2 \sigma_\theta^2}{(1-q)^2} \right)$ because in equilibrium μ_t^m is identical across stocks (see Proposition 11). As a result:

$$Var(r_{l,t+1}z) = \frac{\beta^2 \sigma_a^2}{M} z + \frac{M-1}{\beta^2 \sigma_a^2} \frac{\tau(\varphi(w_t))}{\varphi(w_t)} \left(\frac{X_t}{h(X_t)^2} + \frac{q^2 \sigma_\theta^2}{(1-q)^2} \right) z + o(z).$$

Differentiating this expression with respect to noisiness μ_t amounts to differentiating $X_t/h(X_t)^2 = q/[(1-q)h(X_t)\mu_t]$ (Equation (16)), where $h(X_t) \equiv H(\mu_t) + X(\mu_t)$:

$$\frac{\partial \ln(h(X_t)\mu_t)}{\partial \mu_t} = \frac{1-q}{q} \frac{\mu_t^2 - 2\mu_t q / (1-q) - \beta^2 \sigma_a^2 / \sigma_\theta^2}{H(\mu_t)\mu_t^2 (\mu_t(1-q)/q - 1)\beta^2 \sigma_a^2}.$$

The sign of this ratio is given by the sign of its numerator. It is positive for $\mu_t > \mu^+$ and negative for $\mu_t < \mu^+$, where

$$\mu^+ \equiv q/(1-q) + \sqrt{q^2/(1-q)^2 + \beta^2 \sigma_a^2 / \sigma_\theta^2}. \tag{A14}$$

Thus, $Var(r_{l,t+1}z)$ increases with noisiness μ_t over $(q/(1-q), \mu^+)$, and decreases over (μ^+, ∞) .

Proof of Lemma 8

Liquidity is given by $k_\theta(\mu_t) \equiv 1 / (\partial \ln K_t^m / \partial (\tilde{\theta}_t^m z)) = 1 / ((1-1/M)k_\alpha(\mu_t)\mu_t)$. Its derivative with respect to μ_t equals $\frac{\partial k_\theta}{\partial \mu_t} = k_\alpha + \mu_t \frac{\partial k_\alpha}{\partial \mu_t}$, where the first term reflects the usual beneficial effect of information on liquidity (improving information improves liquidity), and the second term is the offsetting effect of information on the sensitivity of the capital allocation to technology shocks. Using Equation (A11) leads to

$$\frac{\partial k_\theta}{\partial \mu_t} = -\frac{1}{1-\beta} \frac{q}{1-q} \frac{\frac{1}{\mu_t^2 \sigma_\theta^2} - \frac{1}{\beta^2 \sigma_a^2} + X_t}{X_t H(\mu_t) \mu_t^3 \sigma_\theta^2} = -\frac{1}{1-\beta} \frac{\mu_t^2 - 2\mu_t q / (1-q) - \beta^2 \sigma_a^2 / \sigma_\theta^2}{X_t H(\mu_t) \mu_t^4 \sigma_\theta^2 \beta^2 \sigma_a^2},$$

where the last expression uses Equations (16) and (12). This expression is negative for $\mu_t > \mu^+$ and positive for $\mu_t < \mu^+$, where μ^+ is defined in Equation (A14). Hence liquidity worsens (k_θ increases) when μ_t decreases over (μ^+, ∞) and improves (k_θ decreases) when μ_t decreases over $(q/(1-q), \mu^+)$.

The average value of shares traded equals $Vol = Vol_{Rat} + Vol_{Noise}$ where $Vol_{Rat} = E \left[\sum_{m=1}^M \left(\int_{Rqt} |f_{l,t}^m w_t| \right) / 2 \right]$ and $Vol_{noise} = E \left[\sum_{m=1}^M \left(\int_{Noise} |f_{l,t}^m w_t| / 2 \right) \right]$. $f_{l,t}^m$ is approximately normally distributed so $Vol_{Rat} = (1-q)MLw_t \sqrt{\frac{2}{\pi} Var(f_{Rat l,t}^m) / 2}$ and $Vol_{Noise} = qMLw_t \sqrt{\frac{2}{\pi} Var(f_{Noise,t}^m) / 2}$ (e.g., He and Wang 1995). Portfolio shares in equilibrium for a

rational agent are obtained by substituting Equation (14) into Equation (11), setting $x_{i,t}^m = X_t$ and denoting $h(X_t) \equiv H(\mu_t) + X_t(\mu_t)$:

$$\begin{aligned} f_{Rat\ l,t}^m &= \frac{1}{M} + \frac{\tau(\varphi(w_t))}{\varphi(w_t)\beta^2\sigma_a^2} \frac{X_t}{h(X_t)} (\Delta\varepsilon_{i,t}^m - \mu_t \Delta\theta_t^m) + o(1) \\ &= \frac{1}{M} + \frac{\tau(\varphi(w_t))}{\varphi(w_t)\beta^2\sigma_a^2} \left(\frac{X_t}{h(X_t)} \Delta\varepsilon_{i,t}^m - \frac{q}{1-q} \Delta\theta_t^m \right) + o(1) \text{ using Equation (16)}. \end{aligned}$$

Therefore, $\sqrt{Var(f_{Rat\ l,t}^m)} = \frac{\tau(\varphi(w_t))}{\varphi(w_t)\beta^2\sigma_a^2} \sqrt{\frac{M-1}{M}} \sqrt{\frac{X_t}{h(X_t)^2} + (\frac{q}{1-q})^2\sigma_\theta^2} + o(1)$, and

$$Vol_{Rat} = \frac{\tau(\varphi(w_t))w_t}{\varphi(w_t)} \frac{L\sqrt{M(M-1)}}{\sqrt{2\pi}\beta^2\sigma_a^2} \sqrt{\frac{(1-q)^2X_t}{h(X_t)^2} + q^2\sigma_\theta^2} + o(1).$$

For noise traders, $\sqrt{Var(f_{Noise,t}^m)} = \frac{\tau(\varphi(w_t))}{\varphi(w_t)\beta^2\sigma_a^2} \sqrt{\frac{M-1}{M}} \sqrt{\sigma_\theta^2} + o(1)$, and

$$Vol_{Noise} = \frac{\tau(\varphi(w_t))w_t}{\varphi(w_t)} \frac{L\sqrt{M(M-1)}}{\sqrt{2\pi}\beta^2\sigma_a^2} \sqrt{q^2\sigma_\theta^2} + o(1).$$

Summing both components of volume leads to:

$$Vol = \frac{\tau(\varphi(w_t))w_t}{\varphi(w_t)} \frac{L\sqrt{M(M-1)}}{\sqrt{2\pi}\beta^2\sigma_a^2} \left\{ \sqrt{\frac{(1-q)^2X_t}{h(X_t)^2} + q^2\sigma_\theta^2} + \sqrt{q^2\sigma_\theta^2} \right\} + o(1).$$

The share turnover is obtained by dividing by the total capitalization of the market, $\sum_{m=1}^M K_t^m = M(Lw_t/M) = Lw_t$. The trading intensity therefore equals:

$$Turn = \frac{\tau(\varphi(w_t))}{\varphi(w_t)} \frac{\sqrt{M(M-1)}}{\sqrt{2\pi}\beta^2\sigma_a^2} \left\{ \sqrt{\frac{(1-q)^2X_t}{h(X_t)^2} + q^2\sigma_\theta^2} + \sqrt{q^2\sigma_\theta^2} \right\} + o(1).$$

As with liquidity, the derivative of this expression with respect to μ_t is positive over $(q/(1-q), \mu^+)$, and negative over (μ^+, ∞) .

Proof of Lemma 9

From Proposition 4, the variance of stock prices equals $Var(P_t^m) = (Lw_t/M)^2 Var(\Delta k_t^m z)$. Equation (A13) implies that $\partial Var(P_t^m)/\partial \mu_t < 0$, that is, prices are more volatile as information improves.

Recall that stock returns are given by $r_{t+1}^m z = \beta \tilde{\alpha}_t^m z - (1-\beta)\Delta k_t^m z$. It follows that the equally weighted return on the market equals $\bar{r}_t z \equiv \frac{1}{M} \sum_{m=1}^M r_t^m z = \beta \tilde{\alpha}_t^m z$ and its volatility is a constant $\beta^2 \sigma_a^2 z/M$. Idiosyncratic return volatility is given by $Var(\Delta r_{t+1}^m z)$ because Δr_{t+1}^m is uncorrelated to the market ($Cov(\Delta r_{t+1}^m z, \bar{r}_t z) = 0$).

$$\begin{aligned} Var(\Delta r_{t+1}^m z) &= Var(\beta \Delta \tilde{\alpha}_t^m z - (1-\beta)\Delta k_t^m z) = Var(\beta \Delta \tilde{\alpha}_t^m z - (1-\beta)k_\alpha(\mu_t^m)(\beta \Delta \tilde{\alpha}_t^m + \mu_t^m \Delta \tilde{\theta}_t^m) z) \\ &= Var \left\{ E \left[\beta \Delta \tilde{\alpha}_t^m z - (1-\beta)k_\alpha(\mu_t^m)(\beta \Delta \tilde{\alpha}_t^m + \mu_t^m \Delta \tilde{\theta}_t^m) z \mid \{\tilde{\alpha}_t^m, \theta_t^m\} \right] \right\} \\ &\quad + E \left\{ Var \left[\beta \Delta \tilde{\alpha}_t^m z - (1-\beta)k_\alpha(\mu_t^m)(\beta \Delta \tilde{\alpha}_t^m + \mu_t^m \Delta \tilde{\theta}_t^m) z \mid \{\tilde{\alpha}_t^m, \theta_t^m\} \right] \right\} \\ &= Var \left\{ \beta \Delta \tilde{\alpha}_t^m z - (1-\beta)k_\alpha(\mu_t^m)(\beta \Delta \tilde{\alpha}_t^m + \mu_t^m \Delta \tilde{\theta}_t^m) z \right\} + E \left\{ Var \left[(\Delta \tilde{\alpha}_t^m z) \mid \{\tilde{\alpha}_t^m, \theta_t^m\} \right] \right\} \\ &= Var \left\{ \beta(1-(1-\beta)k_\alpha(\mu_t^m))\Delta \tilde{\alpha}_t^m z + (1-\beta)k_\alpha(\mu_t^m)\mu_t^m \Delta \tilde{\theta}_t^m z \right\} + \frac{M-1}{M} \beta^2 \sigma_a^2 z \\ &= Var \left\{ \frac{\beta \Delta \tilde{\alpha}_t^m z}{\beta^2 \sigma_a^2 h(X_t)} \right\} + Var \left\{ \left(\frac{1}{\mu_t^m \sigma_\theta^2 h(X_t)} + \frac{q}{1-q} \right) \Delta \tilde{\theta}_t^m z \right\} + \frac{M-1}{M} \beta^2 \sigma_a^2 z, \end{aligned}$$

where $h(X_t) \equiv H(\mu_t) + X(\mu_t)$. Rearranging leads to:

$$Var(\Delta r_{t+1}^m z) = \frac{M-1}{M} \left\{ \frac{h(X_t) + X_t}{h(X_t)^2} z^2 + \frac{q^2 \sigma_\theta^2}{(1-q)^2} z^2 + \beta^2 \sigma_a^2 z \right\}.$$

Noting that $\frac{\partial}{\partial \mu_t} \left(\frac{h(X_t) + X_t}{h(X_t)^2} \right) = \frac{2}{H(\mu_t)^2 \mu_t^3} \left(\frac{1}{\sigma_\theta^2} + \frac{q^2}{(1-q)^2 \beta^2 \sigma_a^2} \right) > 0$, implies that $\partial Var(\Delta r_{t+1}^m z) / \partial \mu_t > 0$.

Total volatility is given by $Var(r_{t+1}^m z) = Var(\Delta r_{t+1}^m z + \bar{r}_t z) = Var(\Delta r_{t+1}^m z) + Var(\bar{r}_t z)$ because $Cov(\Delta r_{t+1}^m z, \bar{r}_t z) = 0$. The market volatility is constant, so total volatility behaves in the same way as idiosyncratic volatility. Turning to the cross-correlation of returns, we observe that

$$\begin{aligned} Var(\bar{r}_t z) &= Var\left(\frac{1}{M} \sum_{m=1}^M r_t^m z\right) = \frac{1}{M^2} \left\{ \sum_{m=1}^M Var(r_t^m z) + \sum_{m=1}^M \sum_{m'=1}^M Cov(r_{t+1}^m z, r_{t+1}^{m'} z) \right\} \\ &= \frac{1}{M} \left\{ Var(r_t^m z) + (M-1)Cov(r_{t+1}^m z, r_{t+1}^{m'} z) \right\}, \end{aligned}$$

where we use the fact that μ_t^m is identical across stocks in equilibrium (see Proposition 11). So $Cov(r_{t+1}^m z, r_{t+1}^{m'} z) = (M Var(\bar{r}_t z) - Var(r_{t+1}^m z)) / (M-1)$ decreases when information improves.

Proof of Lemma 10

We solve for an investor’s optimal precision about stock m , $x_{l,t}^m$, given any noisiness μ_t^m . We first plug the formulas for the mean and variance of portfolio returns (Equations (A4)) into the expression for the expected utility (Equation (A5)). We note that $\sum_{m=1}^M \Delta e_{l,t}^m = 0$ and $\sum_{m=1}^M \Delta e_{l,t}^{m2} = M(e_{l,t}^2 - \bar{e}_{l,t}^2)$, so Equation (A8) implies that $\sum_{m=1}^M f_{l,t}^m e_{l,t}^m = \bar{e}_{l,t} + \frac{\tau(\varphi(w_t))M}{\varphi(w_t)\beta^2\sigma_a^2} (e_{l,t}^2 - \bar{e}_{l,t}^2)$ and $\sum_{m=1}^M f_{l,t}^{m2} = \frac{1}{M} + \left(\frac{\tau(\varphi(w_t))}{\varphi(w_t)\beta^2\sigma_a^2} \right)^2 M(e_{l,t}^2 - \bar{e}_{l,t}^2)$. After rearranging, we obtain:

$$E(U(\tilde{g}_{l,t+1}, j_t) | \mathcal{F}_{l,t}) = U(*) - \frac{\partial U}{\partial j} (*). \sum_{m=1}^M C(x_{l,t}^m)z + \frac{\partial U}{\partial g} (*). \varphi(w_t) Q_{l,t} z + o(z) \tag{A15}$$

$$\begin{aligned} \text{where } Q_{l,t} &\equiv E(r_{l,t+1} z | \mathcal{F}_l) + \frac{1}{2} Var(r_{l,t+1} z | \mathcal{F}_l) - \frac{\varphi(w_t)}{2\tau(\varphi(w_t))} Var(r_{l,t+1} z | \mathcal{F}_l) \\ &= \bar{e}_{l,t} + M \delta_t (\bar{e}_{l,t}^2 - \bar{e}_{l,t}^2) + d_t, \\ d_t &\equiv \frac{\beta^2 \sigma_a^2}{2} \left(1 - \frac{\varphi(w_t)}{M \tau(\varphi(w_t))} \right) \text{ and } \delta_t \equiv \frac{\tau(\varphi(w_t))}{2\varphi(w_t)\beta^2\sigma_a^2}. \end{aligned}$$

The agent’s unconditional expected utility, $E(U(\tilde{g}_{l,t+1}, j_t))$, follows:

$$E(U(\tilde{g}_{l,t+1}, j_t)) = U(*) - \frac{\partial U}{\partial j} (*). \sum_{m=1}^M C(x_{l,t}^m)z + \frac{\partial U}{\partial g} (*). \varphi(w_t) E(Q_{l,t})z + o(z) \tag{A16}$$

We evaluate next $E(Q_{l,t})$. The variable $e_{l,t}^m$ is a function of $\{\Delta s_{l,t}^m\}$ and $\{k_t^m\}$, which depend on $\{\Delta \tilde{\alpha}_t^m\}$, $\{\Delta \tilde{\theta}_t^m\}$, and $\{\Delta \tilde{\varepsilon}_{l,t+1}^m\}$ (see Equation (A18) below). Like all the random variables in the model, its unconditional mean $E(e_{l,t}^m)$ equals zero, so $E(\bar{e}_{l,t}) = 0$. Moreover:

$$E(\bar{e}_{l,t}^2 - \bar{e}_{l,t}^2) = E(\bar{e}_{l,t}^2 - 2\bar{e}_{l,t}^2 + \bar{e}_{l,t}^2) = E\left(\sum_{m=1}^M e_{l,t}^{m2} / M - 2\bar{e}_{l,t} \sum_{m=1}^M e_{l,t}^m / M + \bar{e}_{l,t}^2\right)$$

$$\begin{aligned}
 &= E\left(\sum_{m=1}^M (e_{1,t}^{m2} - 2\bar{e}_{1,t}e_{1,t}^m + \bar{e}_{1,t}^2) / M\right) = E\left(\sum_{m=1}^M (e_{1,t}^m - \bar{e}_{1,t})^2 / M\right) = E(\overline{(e_{1,t}^m - \bar{e}_{1,t})^2}) \\
 &= \overline{((e_{1,t}^m - \bar{e}_{1,t})^2)} = \overline{Var(e_{1,t}^m - \bar{e}_{1,t}) + (E(e_{1,t}^m - \bar{e}_{1,t}))^2} = \overline{Var(e_{1,t}^m - \bar{e}_{1,t})} + \overline{(E(e_{1,t}^m - \bar{e}_{1,t}))^2}
 \end{aligned}$$

so $E(\overline{(e_{1,t}^m - \bar{e}_{1,t})^2}) = \overline{Var(e_{1,t}^m - \bar{e}_{1,t})} = \overline{Var(\Delta e_{1,t}^m)}$ because $\overline{E(e_{1,t}^m)} = E(\bar{e}_{1,t}) = 0$. (A17)

The next step is to compute $\overline{Var(\Delta e_{1,t}^m)}$. We first note that from Equation (A1):

$$e_{1,t}^m = E(r_{t+1}^m | \mathcal{F}_{1,t}) = E(\beta \tilde{\alpha}_t^m z | \mathcal{F}_{1,t}) - (1 - \beta)k_t^m = c_{\xi t}^m \xi_t^m + c_{st}^m s_{1,t}^m - (1 - \beta)k_{\alpha t}^m \Delta \xi_t^m. \quad (A18)$$

It follows, because $\bar{k}_t \equiv 0$, that $\Delta e_{1,t}^m = \Delta(c_{\xi t}^m \xi_t^m) + \Delta(c_{st}^m s_{1,t}^m) - (1 - \beta)k_{\alpha t}^m \Delta \xi_t^m$. Substituting $\xi_t^m \equiv \beta \tilde{\alpha}_t^m + \mu_t^m \tilde{\theta}_t^m$, $s_{1,t}^m = \beta \tilde{\alpha}_t^m + \varepsilon_{1,t}^m$ and replacing $c_{\xi t}^m$ and c_{st}^m with their definitions (Equations (A3)) leads to:

$$\Delta e_{1,t}^m = \mathcal{A}_{1,t}^m \beta \tilde{\alpha}_t^m + (M - 1) \frac{x_{1,t}^m}{\widehat{h}_{1,t}^m} \tilde{\varepsilon}_{1,t}^m + \mathcal{B}_{1,t}^m \mu_t^m \theta_t^m + \sum_{\substack{n=1 \\ n \neq m}}^M \left(\mathcal{C}_{1,t}^{n,m} \beta \tilde{\alpha}_t^n - \frac{x_{1,t}^n}{\widehat{h}_{1,t}^n} \tilde{\varepsilon}_{1,t}^n + \mathcal{D}_{1,t}^{n,m} \mu_t^n \theta_t^n \right), \quad (A19)$$

where we recall that $\widehat{h}_{1,t}^m \equiv H(\mu_t^m) + x_{1,t}^m$ and define:

$$\mathcal{A}_{1,t}^m \equiv (M - 1) \left(1 - \frac{1}{\beta^2 \sigma_\alpha^2 \widehat{h}_{1,t}^m} - (1 - \beta)k_{\alpha t}^m \right)$$

$$\mathcal{B}_{1,t}^m \equiv (M - 1) \left(\frac{1}{\mu_t^{m2} \sigma_\theta^2 \widehat{h}_{1,t}^m} - (1 - \beta)k_{\alpha t}^m \right) = \mathcal{A}_{1,t}^m - (M - 1) \frac{x_{1,t}^m}{\widehat{h}_{1,t}^m}$$

$$\mathcal{C}_{1,t}^{n,m} \equiv -1 + \frac{1}{\beta^2 \sigma_\alpha^2 \widehat{h}_{1,t}^n} + (1 - \beta)k_{\alpha t}^m$$

$$\mathcal{D}_{1,t}^{n,m} \equiv -\frac{1}{\mu_t^{n2} \sigma_\theta^2 \widehat{h}_{1,t}^n} + (1 - \beta)k_{\alpha t}^m = \mathcal{C}_{1,t}^{n,m} + \frac{x_{1,t}^m}{\widehat{h}_{1,t}^m}.$$

Taking the variance of Equation (A19) yields

$$\begin{aligned}
 M^2 Var(\Delta e_{1,t}^m) &= (\beta^2 \sigma_\alpha^2 + \mu_t^{m2} \sigma_\theta^2) \mathcal{A}_{1,t}^{m2} - 2(M - 1) \mu_t^{m2} \sigma_\theta^2 \frac{x_{1,t}^m}{\widehat{h}_{1,t}^m} \mathcal{A}_{1,t}^m + (M - 1)^2 \mu_t^{m2} \sigma_\theta^2 \frac{x_{1,t}^{m2}}{\widehat{h}_{1,t}^{m2}} \\
 &+ (M - 1)^2 \frac{x_{1,t}^m}{\widehat{h}_{1,t}^{m2}} + \sum_{\substack{n=1 \\ n \neq m}}^M \left((\beta^2 \sigma_\alpha^2 + \mu_t^{n2} \sigma_\theta^2) \mathcal{C}_{1,t}^{n,m2} + \frac{x_{1,t}^n}{\widehat{h}_{1,t}^{n2}} + 2\mu_t^{n2} \sigma_\theta^2 \frac{x_{1,t}^n}{\widehat{h}_{1,t}^n} \mathcal{C}_{1,t}^{n,m} + \mu_t^{n2} \sigma_\theta^2 \frac{x_{1,t}^{n2}}{\widehat{h}_{1,t}^{n2}} \right).
 \end{aligned}$$

Completing the sum with the terms with the m superscript and rearranging yields

$$\begin{aligned}
 M^2 Var(\Delta e_{1,t}^m) &= (\beta^2 \sigma_\alpha^2 + \mu_t^{m2} \sigma_\theta^2) (\mathcal{A}_{1,t}^{m2} - \mathcal{C}_{1,t}^{m,m2}) - 2\mu_t^{m2} \sigma_\theta^2 \frac{x_{1,t}^m}{\widehat{h}_{1,t}^m} ((M - 1) \mathcal{A}_{1,t}^m + \mathcal{C}_{1,t}^{m,m}) \\
 &+ M(M - 2) \mu_t^{m2} \sigma_\theta^2 \frac{x_{1,t}^{m2}}{\widehat{h}_{1,t}^{m2}} + M(M - 2) \frac{x_{1,t}^m}{\widehat{h}_{1,t}^{m2}} \\
 &+ \sum_{n=1}^M \left((\beta^2 \sigma_\alpha^2 + \mu_t^{n2} \sigma_\theta^2) \mathcal{C}_{1,t}^{n,m2} + \frac{x_{1,t}^n}{\widehat{h}_{1,t}^{n2}} + 2\mu_t^{n2} \sigma_\theta^2 \frac{x_{1,t}^n}{\widehat{h}_{1,t}^n} \mathcal{C}_{1,t}^{n,m} + \mu_t^{n2} \sigma_\theta^2 \frac{x_{1,t}^{n2}}{\widehat{h}_{1,t}^{n2}} \right).
 \end{aligned}$$

Noting that $C_t^{m,m} = -A_t^m / (M - 1)$, replacing the A and C coefficients with their expressions and rearranging leads to

$$M^2 Var(\Delta e_{l,t}^m) = -M(M-2) \left(\frac{1}{\widehat{h}_{l,t}^m} + \mathcal{K}_t^{m,m} \right) + \sum_{n=1}^M \left(-\frac{1}{\widehat{h}_{l,t}^n} + \mathcal{K}_t^{n,m} \right)$$

where $\mathcal{K}_t^{n,m} \equiv (\beta^2 \sigma_a^2 + \mu_t^{n2} \sigma_\theta^2) (1 - (1 - \beta)k_{at}^m)^2 + \mu_t^{n2} \sigma_\theta^2 (2(1 - \beta)k_{at}^m - 1)$.

Note that $\mathcal{K}_t^{n,m}$ does not depend on the precisions chosen by agent l . Taking the average across all stocks yields:

$$\begin{aligned} M^2 \overline{Var(\Delta e_{l,t}^m)} &= -(M-2) \sum_{m=1}^M \left(\frac{1}{\widehat{h}_{l,t}^m} + \mathcal{K}_t^{m,m} \right) + \frac{1}{M} \sum_{m=1}^M \sum_{n=1}^M \left(-\frac{1}{\widehat{h}_{l,t}^n} + \mathcal{K}_t^{n,m} \right) \\ &= -(M-1) \sum_{m=1}^M \frac{1}{\widehat{h}_{l,t}^m} - (M-2) \sum_{m=1}^M \mathcal{K}_t^{m,m} + \frac{1}{M} \sum_{m=1}^M \sum_{n=1}^M \mathcal{K}_t^{n,m} \end{aligned}$$

because $\frac{1}{M} \sum_{m=1}^M \sum_{n=1}^M \frac{1}{\widehat{h}_{l,t}^n} = \frac{1}{M} M \sum_{n=1}^M \frac{1}{\widehat{h}_{l,t}^n} = \sum_{n=1}^M \frac{1}{\widehat{h}_{l,t}^n} = \sum_{m=1}^M \frac{1}{\widehat{h}_{l,t}^m}$. It follows that

$$\begin{aligned} E(Q_{l,t}) &= 0 + \frac{\delta_t}{M} \left(-(M-1) \sum_{m=1}^M \frac{1}{\widehat{h}_{l,t}^m} - (M-2) \sum_{m=1}^M \mathcal{K}_t^{m,m} + \frac{1}{M} \sum_{m=1}^M \sum_{n=1}^M \mathcal{K}_t^{n,m} \right) + d_t \\ &= -\frac{\delta_t(M-1)}{M} \sum_{m=1}^M \frac{1}{\widehat{h}_{l,t}^m} + Q_t, \end{aligned}$$

where $Q_t \equiv \frac{\delta_t}{M} \left(-(M-2) \sum_{m=1}^M \mathcal{K}_t^{m,m} + \frac{1}{M} \sum_{m=1}^M \sum_{n=1}^M \mathcal{K}_t^{n,m} \right) + d_t$ does not depend on the precisions chosen by agent l . We substitute this expression into Equation (A16) which characterizes agent l 's unconditional expected utility, and use $\widehat{h}_{l,t}^m \equiv H(\mu_t^m) + x_{l,t}^m$:

$$\begin{aligned} E(U(\widetilde{g}_{l,t+1}, j_t)) &= U(*) - \frac{\partial U}{\partial j} (*) \cdot \sum_{m=1}^M C(x_{l,t}^m) z + \frac{\partial U}{\partial g} (*) \cdot \varphi(w_t) \left(-\delta_t \left(1 - \frac{1}{M} \right) \sum_{m=1}^M \frac{1}{H(\mu_t^m) + x_{l,t}^m} + Q_t \right) z + o(z) \end{aligned} \tag{A20}$$

We maximize this expression with respect to $x_{l,t}^m$ taking as given the stocks' noisiness $\{\mu_t^m\}$. The first-order condition for this problem is, for every stock m and agent l :

$$\frac{\partial U}{\partial j} (*) C'(x_{l,t}^m) = \frac{\partial U}{\partial g} (*) \cdot \varphi(w_t) \delta_t \left(1 - \frac{1}{M} \right) \frac{1}{\left(H(\mu_t^m) + x_{l,t}^m \right)^2}. \tag{A21}$$

Substituting $\delta_t \equiv \frac{\tau(\varphi(w_t))}{2\varphi(w_t)\beta^2\sigma_a^2}$ and rearranging leads to Equation (18) in Lemma 5. Equation (A21) admits a unique solution because its left-hand side is monotonically increasing in $x_{l,t}^m$ starting from zero ($C'(0)=0$ by assumption), whereas its right-hand side is monotonically decreasing towards zero. Moreover, the Equation implies that signal precisions are identical across agents for any stock m ($x_{l,t}^m = X_t^m$ for all l).

The impact on the signal precision x_t^m of the factor share of intermediate goods, β , is complex. First, a lower β reduces investors' share of GDP and their consumption (the $\varphi(w_t)$ term), which enhances the marginal utility of final goods, so both ρ and τ increase. Second, a lower β implies that stocks are less sensitive to productivity shocks. These shocks have a component that can be

learnt ($\tilde{\alpha}_t^m$) and one that cannot ($\tilde{q}_t^m - \tilde{\alpha}_t^m$) so the implications are twofold. On the one hand, a lower β means that the shock $\tilde{\alpha}_t^m$ has a smaller impact on a firm's profit so learning about it is less valuable (the term $1/\beta^2\sigma_a^2$ embedded in $H(\mu_t^m)$ on the right-hand side of Equation (18)). On the other hand, it implies that stocks are less risky so investors trade them more aggressively, which makes information more valuable (the $\beta^2\sigma_a^2$ on the right-hand side of the Equation). The net effect of β on the signal precision depends on the relative magnitude of these effects.

Proof of Proposition 11

Equation (A21) implies that signal precisions are identical across agents for any stock m , that is, $x_{l,t}^m = X_t^m \equiv X(\mu_t^m)$ for all l . As a result, Equations (A10) which characterize stock prices for arbitrary precisions simplify to Equations (15) and (16). Replacing $x_{l,t}^m$ with $X(\mu_t^m)$ on both sides of Equation (A21) and noting that Equation (16) implies that $H(\mu_t^m) + X_t^m = H(\mu_t^m)/(1-q)/(1-q)/\mu_t^m$ leads to Equation (19). This Equation admits a unique solution μ_t^m for any level of income w_t , because its left-hand side is monotonically decreasing in μ_t^m toward zero, whereas its right-hand side is monotonically increasing from zero. Moreover, μ_t^m and therefore X_t^m are identical across stocks.

Proof of Lemma 12

Differentiating Equation (19) with respect to w_t yields

$$\left\{ \left(\frac{C''(X(\mu_t))}{C'(X(\mu_t))} + \frac{1}{H(\mu_t) + X(\mu_t)} \right) \frac{dX}{d\mu_t} + \frac{1}{H(\mu_t) + X(\mu_t)} \frac{dH}{d\mu_t} \right\} \frac{d\mu_t}{dw_t} = \frac{d}{dw_t} \left(\ln \frac{\tau(\varphi(w_t))}{\rho(\varphi(w_t))} \right).$$

The sign of $\frac{d\mu_t}{dw_t}$ is the opposite of that of $\frac{d}{dw_t} \left(\ln \frac{\tau(\varphi(w_t))}{\rho(\varphi(w_t))} \right)$ because $\frac{dX}{d\mu_t} < 0$ (Equation (16)) and $\frac{dH}{d\mu_t} < 0$ (Equation (12)). Moreover, φ is increasing (Equation (10)). So $\frac{d\mu_t}{dw_t} < 0$ (> 0) if τ/ρ is increasing (decreasing).

Proof of Lemma 13

We start by computing dX_t/dq , which captures the ex ante disincentive effect on the precision of private information. We write Equation (19) as $\rho(\varphi(w_t))C'(X_t)((1-q)/q)^2\mu_t^2X_t^2 = \tau(\varphi(w_t))(1-1/M)/(2\beta^2\sigma_a^2) + o(1)$ using Equation (16). We take logs, differentiate this Equation with respect to q , holding w_t constant, and obtain

$$\frac{C''(X_t)}{2C'(X_t)} \frac{dX_t}{dq} + \frac{1}{q(1-q)} + \frac{1}{\mu_t} \frac{d\mu_t}{dq} + \frac{1}{X_t} \frac{dX_t}{dq} = 0. \tag{A22}$$

We decompose $d\mu_t/dq$ into ex post and ex ante components, $\frac{d\mu_t}{dq} = \frac{\partial\mu_t}{\partial q} \Big|_{X_t \text{ fixed}} + \frac{\partial\mu_t}{\partial X_t} \Big|_{q \text{ fixed}} * \frac{dX_t}{dq}$.

We differentiate Equation (16) to evaluate $\frac{\partial\mu_t}{\partial q} \Big|_{X_t \text{ fixed}}$ and $\frac{\partial\mu_t}{\partial X_t} \Big|_{q \text{ fixed}}$ and substitute the results into the previous expression for $d\mu_t/dq$:

$$\frac{d\mu_t}{dq} = \frac{\mu_t}{H_t + X_t + \frac{2}{\mu_t^2\sigma_\theta^2}} \left\{ -\frac{H_t + X_t}{q(1-q)} + * \frac{dX_t}{dq} \frac{H_t}{X_t} \right\}.$$

Substituting back into Equation (A22) and rearranging leads to

$$\frac{dX_t}{dq} = \frac{2X_t}{q(1-q)\mu_t^2\sigma_\theta^2\mathcal{N}} > 0$$

where $\mathcal{N} \equiv X_t + \frac{2}{\mu_t^2\sigma_\theta^2} + \frac{\epsilon_{C'}(X_t)}{2} \left(H_t + X_t + \frac{2}{\mu_t^2\sigma_\theta^2} \right) > 0$ and $\epsilon_{C'}(X) \equiv \frac{XC''(X)}{C'(X)} > 0$.

The ex ante disincentive effect on the total precision is given by

$$\frac{d(H_t + X_t)}{dq} = -\frac{2}{\mu_t^3\sigma_\theta^2} \frac{d\mu_t}{dq} + \frac{dX_t}{dq} = -\frac{\epsilon_{C'}(X_t)(H_t + X_t)}{q(1-q)\mu_t^2\sigma_\theta^2\mathcal{N}} < 0.$$

Hence, less information is produced (X_t falls) but the total precision, $H_t + X_t$, rises nevertheless (because more information is shared through stock prices) when the fraction of noise traders q decreases.

Proof of Lemma 14

We evaluate Equation (19) when the fraction of noise traders is close to zero. When $q \approx 0$, Equation (16) can be approximated as $X(\mu_t) \approx \frac{H(\mu_t)}{\mu_t} \frac{q}{1-q} \approx \frac{H(\mu_t)}{\mu_t} q$ so $C'(X_t) \approx C''(0) \frac{H(\mu_t)}{\mu_t} q$ and $1/(H(\mu_t) + X(\mu_t))^2 \approx \frac{1}{H(\mu_t)^2} (1 - \frac{1}{\mu_t} \frac{2q}{1-q}) \approx \frac{1}{H(\mu_t)^2} (1 - \frac{2q}{\mu_t})$. Substituting these expressions into Equation (19) yields:

$$\frac{\mu_t}{H(\mu_t)} \approx 2 \left(C''(0) \frac{\beta^2 \sigma_a^2}{1 - 1/M} \frac{\rho(\varphi(w_t))}{\tau(\varphi(w_t))} + 1 \right) q + o(1). \tag{A23}$$

We guess that μ_t is close to zero so $H(\mu_t) \approx \frac{1}{\mu_t^2 \sigma_\theta^2}$. Substituting back into Equation (A23) and rearranging leads to:

$$\mu_t \approx \left\{ 2\sigma_\theta^2 \left(C''(0) \frac{\beta^2 \sigma_a^2}{1 - 1/M} \frac{\rho(\varphi(w_t))}{\tau(\varphi(w_t))} + 1 \right) \right\}^{1/3} q^{1/3} + o(1),$$

which confirms that μ_t is close to zero. This formula implies that $H(\mu_t)$ grows to infinity when q approaches zero: agents' information becomes perfect thanks to its revelation through stock prices even though the precision of their private signals goes to zero. As a result, the capital allocation and the income process converge to those of the first best.

Proof of Proposition 15

The first part of the proposition (Equations (20) and (21)) was established in the proof of Lemma 6. These Equations imply that the steady state level of income along the average path solves $w^* = \Lambda w^{*\beta} \exp(\lambda(w^*)z^2)$. To determine w^* at the order z^2 , we replace w^* in $\lambda(w^*)$ with its order-zero component, which is identical to the order-zero component of w^{FB} , that is, $(1 - \beta)^{1/(1-\beta)} M$ (see Equations (6) and (8)). This leads to Equation (22).

The last part of the proposition obtains by combining Lemma 6 with Lemma 12. Lemma 12 states that $\frac{d\mu_t}{dw_t} < 0$ (noisiness falls with income) if τ/ρ is an increasing function, while Lemma 6 shows that $\frac{\partial \lambda}{\partial \mu_t} < 0$ (income grows on average when noisiness is lower). Together, they imply that

$$\frac{d\lambda}{dw_t} > 0. \text{ If instead } \tau/\rho \text{ is a decreasing function, then } \frac{d\lambda}{dw_t} < 0.$$

If $\lim_{g \rightarrow u} \tau(g)/\rho(g) = \infty$, then Equation (19) implies that $\lim_{w_t \rightarrow u} \mu_t = q/(1-q)$, which corresponds to the perfect-information case. In that case, $\lim_{w_t \rightarrow u} \lambda(w_t) = \lambda^{FB} + \lambda^{Noise}$. Alternatively, if $\lim_{g \rightarrow u} \tau(g)/\rho(g) = 0$, then $\lim_{w_t \rightarrow u} \mu_t = \infty$ (no-information case) and $\lim_{w_t \rightarrow u} \lambda(w_t) = \lambda^{Noise}$.

Proof of Proposition 16

Proposition 16 follows directly from combining Lemmas 5 to 10 with Lemma 13.

Proof of Proposition 17

The proof of the proposition follows directly from the discussion following Proposition 15.

Proof of Lemma 18

Under CES preferences, $\tau(g)/\rho(g) = \omega g^\sigma (\varpi g^\sigma + 1 - \varpi) / (1 - \sigma) / (1 - \varpi)^2$. Substituting this expression into the condition in Proposition 15 leads to $w_t^\sigma > \frac{1 - \varpi}{2\varpi} \left(\sqrt{1 + \frac{8\varpi(1 - \sigma)\sigma_a^2(\sigma_a^2)^2}{(1 - \varpi)(1 - 1/M)\beta^2} C'(0) - 1} \right) \equiv \underline{w}^\sigma$ and to Lemma 18.

Proof of Proposition 19

- Capital allocation

Capital and stock prices have the same functional form as in the rational case and are only affected by overconfidence through the perceived precisions of private signals, \hat{x} and \hat{X} . Perceived precisions matter to equilibrium prices because they, not the actual precisions, determine how aggressively investors trade on their information and therefore how informative prices are. Equations (12), (14), (15), and (16) hold, substituting perceived precisions for the actual. The actual average precision is defined as $X \equiv \hat{X}/\phi$ (it also equals the average of actual individual precisions because \hat{X} equals the average of perceived individual precisions). Noisiness is related to the actual average precision through Equation (16) stated for the average perceived precision:

$$\hat{X}(\mu) \equiv \phi X(\mu) \equiv \frac{H(\mu)}{\frac{1-q}{q} \mu - 1}. \tag{A24}$$

- Information acquisition

The maximization problem of an overconfident agent is identical to that of a rational agent except that she wrongly believes that $C(x)z$ units of free time produce a signal of precision ϕx (the perceived precision) rather than x (the actual). Hence, her expected utility can be written as in Equation (A20) substituting ϕx for x in the terms reflecting the benefit of information:

$$\begin{aligned} E(U(\tilde{g}_{l,t+1}, j_t)) &= U(*) - \frac{\partial U}{\partial j} (*) \cdot \sum_{m=1}^M C(x_{l,t}^m) z \\ &\quad + \frac{\partial U}{\partial g} (*) \cdot \varphi(w_t) \left(-\delta_t \left(1 - \frac{1}{M} \right) \sum_{m=1}^M \frac{1}{H(\mu_t^m) + \phi x_{l,t}^m} + \mathcal{Q}_t \right) z + o(z). \end{aligned}$$

Maximizing this expression with respect to $x_{l,t}^m$, taking as given the stocks' noisiness $\{\mu_t^m\}$, and dropping for brevity the m superscripts and the l subscripts, yields the first-order condition:

$$\frac{\partial U}{\partial j} (*) C'(x_t) = \frac{\partial U}{\partial g} (*) \cdot \varphi(w_t) \delta_t \left(1 - \frac{1}{M} \right) \frac{\phi}{(H(\mu_t) + \phi x_t)^2},$$

which simplifies to:

$$\rho(\varphi(w_t)) C'(x_t) = \phi \tau(\varphi(w_t)) \frac{1 - 1/M}{2\beta^2 \sigma_a^2} \frac{1}{(H(\mu_t) + \phi x_t)^2} + o(1).$$

To obtain the equilibrium condition, we replace the individual (actual) precision with the average (actual) precision:

$$\rho(\varphi(w_t)) C'(X_t) = \phi \tau(\varphi(w_t)) \frac{1 - 1/M}{2\beta^2 \sigma_a^2} \frac{1}{(H(\mu_t) + \phi X_t)^2} + o(1).$$

Substituting out X_t thanks to Equation (A24) leads to Equation (23) characterizing the equilibrium noisiness under overconfidence.

Proof of Proposition 20

- Capital allocation

Inattentive agents form portfolios in a way similar to fully-attentive agents. An agent l spreads her wealth equally among the $M - \phi$ stocks she recognizes, represented by the set \mathcal{A}_l . She then tilts her portfolio shares according to her information. They are given by

$$f_{l,t}^m = \begin{cases} \frac{1}{M - \phi} + \frac{\tau(\varphi(w_t))}{\varphi(w_t)\beta^2\sigma_a^2z} E(\Delta_l \ln R_{t+1}^m | \mathcal{F}_{l,t}) + o(1) & \text{if stock } m \text{ belongs to the set } \mathcal{A}_l \\ 0 & \text{otherwise} \end{cases}$$

where $\Delta_l \ln \tilde{R}_{t+1}^m \equiv \ln \tilde{R}_{t+1}^m - \frac{1}{M - \phi} \sum_{m' \in \mathcal{A}_l} \ln \tilde{R}_{t+1}^{m'}$ represents the stock's excess performance relative to the $M - \phi$ stocks in set \mathcal{A}_l .

The aggregate demand for stock m equals

$$\int_l f_{l,t}^m w_t = w_t \int_{l \text{ aware of } m} \left[\frac{1}{M - \phi} + \frac{\tau(\varphi(w_t))}{\varphi(w_t)\beta^2\sigma_a^2z} E(\Delta_l \ln R_{t+1}^m | \mathcal{F}_{l,t}) \right].$$

The integral of the first term in the square bracket equals $\frac{1}{M - \phi}$ multiplied by the number of investors attentive to stock m , $L C_{M-1}^{M-\phi-1} / C_M^{M-\phi} = L \frac{M-\phi}{M}$, and simplifies to $\frac{L}{M}$.²⁴

Similar to the full-attention case, $E(\Delta_l \ln R_{t+1}^m | \mathcal{F}_{l,t}) = \Delta_l \tilde{\theta}_t^m$ for noise traders, and $E(\Delta_l \ln R_{t+1}^m | \mathcal{F}_{l,t}) = \frac{x_{l,t}^m}{h_{l,t}^m} \Delta_l \delta_{l,t}^m + \left(\frac{1}{\tilde{h}_{l,t}^m \mu_t^m \sigma_\theta^2 k_{at}^m} - (1 - \beta) \right) \Delta_l k_t^m$ for rational traders, guessing that stock prices are given in Proposition 20 and using the same notation as in the proof of Proposition 4. We break up $E(\Delta_l \ln R_{t+1}^m | \mathcal{F}_{l,t})$ for rational traders into two components, $\Delta_l \mathcal{G}_t^m z + \Delta_l \mathcal{L}_{l,t}^m z$. The first is common to all investors aware of the same stocks as agent l , and equals $\Delta_l \mathcal{G}_t^m \equiv \frac{x_{l,t}^m}{h_{l,t}^m} \Delta_l \beta \tilde{\alpha}_t^m + \left(\frac{1}{\tilde{h}_{l,t}^m \mu_t^m \sigma_\theta^2 k_{at}^m} - (1 - \beta) \right) \Delta_l k_t^m$. The second, $\Delta_l \mathcal{L}_{l,t}^m \equiv \frac{x_{l,t}^m}{h_{l,t}^m} \Delta_l \varepsilon_{l,t}^m$, is specific to agent l .

The integral of the second term can be evaluated as follows:

$$\int_{l \text{ aware of } m}^{l \text{ rat.}} E(\Delta_l \ln R_{t+1}^m | \mathcal{F}_{l,t}) = \int_{l \text{ aware of } m}^{l \text{ rat.}} (\Delta_l \mathcal{G}_t^m + \Delta_l \mathcal{L}_{l,t}^m) = \int_{l \text{ aware of } m}^{l \text{ rat.}} \Delta_l \mathcal{G}_t^m$$

because the law of large numbers applied to the sequences $\{\varepsilon_{l,t}^m\}$ implies $\int_{l \text{ aware of } m}^{l \text{ rat.}} \Delta_l \mathcal{L}_{l,t}^m = 0$.

$$\begin{aligned} \int_{l \text{ aware of } m}^{l \text{ rat.}} \Delta_l \mathcal{G}_t^m &= \int_{l \text{ aware of } m}^{l \text{ rat.}} \left(\mathcal{G}_t^m - \frac{1}{M - \phi} \sum_{m' \in \mathcal{A}_l} \mathcal{G}_t^{m'} \right) \\ &= \int_{l \text{ aware of } m}^{l \text{ rat.}} \left(\left(1 - \frac{1}{M - \phi}\right) \mathcal{G}_t^m - \frac{1}{M - \phi} \sum_{\substack{m' \in \mathcal{A}_l \\ m' \neq m}} \mathcal{G}_t^{m'} \right) \\ &= L(1 - q) \left(\left(1 - \frac{1}{M - \phi}\right) \frac{C_{M-1}^{M-\phi-1}}{C_M^{M-\phi}} \mathcal{G}_t^m - \frac{1}{M - \phi} \frac{C_{M-2}^{M-\phi-2}}{C_M^{M-\phi}} \sum_{m' \neq m} \mathcal{G}_t^{m'} \right), \end{aligned}$$

where $C_{M-1}^{M-\phi-1} / C_M^{M-\phi}$ is the proportion of investors aware of stock m , and $C_{M-2}^{M-\phi-2} / C_M^{M-\phi}$ is the proportion of investors aware of stock m and of a given stock m' . Adding \mathcal{G}_t^m to $\sum_{m' \neq m} \mathcal{G}_t^{m'}$

²⁴ C_M^i is the number of combinations of i elements (stocks) out of M , and equals $\frac{M!}{(M-i)!i!}$ for any integer i .

and subtracting it from the first term leads to

$$\begin{aligned}
 & \int_{\substack{l \text{ rat.} \\ \text{aware of } m}} \Delta_l \mathcal{G}_t^m \\
 &= L(1-q) \left(\left(1 - \frac{1}{M-\phi} + \frac{1}{M-\phi} \frac{C_{M-2}^{M-\phi-2}}{C_{M-1}^{M-\phi-1}} \right) \frac{C_{M-1}^{M-\phi-1}}{C_M^{M-\phi}} \mathcal{G}_t^m - \frac{1}{M-\phi} \frac{C_{M-2}^{M-\phi-2}}{C_M^{M-\phi}} \sum_{m'} \mathcal{G}_t^{m'} \right) \\
 &= L(1-q) \left(\frac{M}{M-\phi} \frac{C_{M-2}^{M-\phi-2}}{C_M^{M-\phi}} \mathcal{G}_t^m - \frac{M}{M-\phi} \frac{C_{M-2}^{M-\phi-2}}{C_M^{M-\phi}} \bar{\mathcal{G}}_{l,t} \right) \\
 &= L(1-q) \frac{M}{M-\phi} \frac{C_{M-2}^{M-\phi-2}}{C_M^{M-\phi}} \Delta \mathcal{G}_t^m \text{ where } \Delta \mathcal{G}_t^m \text{ is not agent specific,} \\
 &= L(1-q) \frac{M-\phi-1}{M-1} \Delta \mathcal{G}_t^m.
 \end{aligned}$$

It follows that the aggregate demand emanating from rational traders equals

$$\int_{l \text{ rat.}} f_{l,t}^m w_t = w_t L(1-q) \left[\frac{1}{M} + \frac{\tau(\varphi(w_t))}{\varphi(w_t) \beta^2 \sigma_a^2} \frac{M-\phi-1}{M-1} \Delta \mathcal{G}_t^m \right].$$

We proceed in a similar way for noise traders. Their aggregate demand equals

$$\int_{l \text{ noise}} f_{l,t}^m w_t = w_t Lq \left[\frac{1}{M} + \frac{\tau(\varphi(w_t))}{\varphi(w_t) \beta^2 \sigma_a^2} \frac{M-\phi-1}{M-1} \Delta \tilde{\theta}_t^m \right].$$

Equating the aggregate demand for stock m to its aggregate supplies leads to the price displayed in Proposition 20.

- Information acquisition

Proceeding as in the proof of Lemma 10, an inattentive investor sets the precision of her private signal about stock m , x_t^m , given $\frac{1}{\mu_t^m}$, the noisiness of its price, such that

$$\rho(\varphi(w_t)) C' (x_t^m) = \tau(\varphi(w_t)) \frac{1-1/(M-\phi)}{2\beta^2 \sigma_a^2} \frac{1}{(H(\mu_t^m) + x_t^m)^2} + o(1).$$

We then obtain the equilibrium Equation for noisiness (24) after replacing the individual precision with the average precision.

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