The tradeoff between risk sharing and information production in financial markets

Joel Peress [*]

INSEAD, Department of Finance, Boulevard de Constance, 77305 Fontainebleau Cedex, France

Abstract

The production of information in financial markets is limited by the extent of risk sharing. The wider a stock's investor base, the smaller the risk borne by each shareholder and the less valuable information. A firm which expands its investor base without raising capital affects its information environment through three channels: (i) it induces incumbent shareholders to reduce their research effort as a result of improved risk sharing, (ii) it attracts potentially informed investors, and (iii) it may modify the composition of the base in terms of risk tolerance or liquidity trading. Implications for individual firms and the market as a whole are derived.
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1. Introduction

Are stocks held by more investors more closely followed? This is an important question both for companies and policy makers who often take actions to promote a broad equity ownership.[1] The answer is clearly positive if each investor produces a fixed amount of information, indepen-

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[*] Fax: +33 1 6074 5500.
  E-mail address: joel.peress@insead.edu.

[1] For example, companies list on additional exchanges, split their stock, reduce the number of shares in a roundlot or advertise to expand their investor base (see the references below). Governments and regulators advertise shares in privatized companies and grant favorable tax treatment to equity to encourage households to become stockholders.

dent of the number of investors in the company. It is no longer clear-cut if this amount declines with the number of stockholders as we argue in this paper. We show that a tradeoff exists between the number of informed shareholders and their research effort. Everything else equal, a more widely-held stock is a stock in which each shareholder holds, on average, a smaller stake. Because the benefit of private information, unlike its cost, rises with the scale of investment, a more widely-held stock is less actively researched. Putting it differently, *information production is limited by the extent of risk sharing*: the wider the shareholder base, the smaller the risk borne by each shareholder in equilibrium and the less valuable information. This insight echoes the familiar concept from the corporate governance literature which argues that a firm's ownership structure affects corporate control (see Shleifer and Vishny [65] for a survey). However, the information we consider is not used for management monitoring but for portfolio selection. Therefore, and this is a crucial difference, it is acquired *ex ante*, i.e. before cash flows are observed, and is revealed through prices.

We develop a rational expectations model that formalizes this insight. The model builds on two independent strands of research. The first strand, the literature on limited stock market participation, examines markets in which only a restricted set of investors participate because of entry costs (e.g. Merton [51], Hirshleifer [36], Allen and Gale [4], Basak and Cuoco [8], Shapiro [63]). Merton [51], for example, assumes that investors are only aware of the existence of a subset of public companies, an assumption known as the "Investor Recognition Hypothesis" (IRH). This literature assumes that investors who trade an asset, share the same information. Second, we draw on the literature on information acquisition in competitive markets (e.g. Grossman and Stiglitz [28], Verrecchia [72]). In these papers, investors choose how much effort, if any, to put into research. Informed stockholders spend resources on predicting firms' cash flows. Uninformed stockholders do not research stocks but extract from prices part of the information uncovered by informed stockholders. The extraction is only partial because the presence of liquidity traders makes the supply of stocks noisy.[2] As in the first strand of literature but unlike the second, we endow a stock with a restricted set of shareholders, i.e. we limit exogenously the size of the investor base. As in the second but unlike the first, we allow these shareholders to research the stock. The result is a model in which shareholders' incentives to collect information depend on the size of the investor base. The model also incorporates other relevant features such as heterogeneity in shareholder risk tolerance, variations in liquidity trading and equity issues.[3]

Agents in the model have mean-variance preferences as is common in the aforementioned strands of literature but they maximize a non-expected utility of the Kreps–Porteus type. These preferences and the generalization of iso-elastic utility by Epstein and Zin [23] and Weil [77] are widely used in asset pricing and macroeconomics. They allow to depart from the traditional expected utility framework in a tractable way and to examine informational scale effects in a ra-

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[2] The presumption in this class of models (Grossman [27], Hellwig [34]) is that a greater number of traders makes the price more informative because the error from aggregating idiosyncratic private signals shrinks, holding fixed the precision of private signals. There is no aggregation error in our model because the number of traders is very large (Verrecchia [72], He and Wang [33]). We focus instead on the relation between the precision of private signals and the number of investors.

[3] Differences in risk tolerance capture differences in investment scale across shareholders, for example as a consequence of differences in wealth. More risk tolerant shareholders make, on average, larger trades and hence follow the stock more closely. We show that the entire distribution of ownership, in addition to the number of shareholders and their average risk tolerance, matters to a stock's informativeness (Proposition 8). Furthermore, we model the proportion of liquidity traders in the stock. This is a relevant parameter since private information becomes easier to conceal and therefore more valuable when liquidity trading accounts for a larger proportion of trades.

tional expectations equilibrium. Indeed, though portfolio choices are identical under both types of preferences, the demand for information under mean-variance expected utility is not influenced by the extent of risk sharing – it depends neither on the supply of shares nor on the number of investors. When we depart from mean-variance preferences remaining within the class of expected utilities, we find that these variables do matter.[4] Thus, it appears that the non-dependence of information production on risk sharing which we obtain under mean-variance expected utility is an exception rather than the rule.

When a company expands its investor base without raising capital, the amount of information revealed by its price in equilibrium, its informativeness, is affected through three channels. First, the expansion reduces incumbent shareholders' incentives to do research (the aforementioned risk sharing effect). Second, it adds potentially informed investors to the base. Finally, the composition of the base (i.e. the average risk tolerance of stockholders and their propensity to trade for liquidity reasons) may be modified. The net effect depends on the relative importance of the three channels, which we quantify in the model. We consider two scenarios that illustrate the range of outcomes. We start with homogeneous investors. There are then no composition effects, as new stockholders are identical to incumbents. Their contributions to informativeness and liquidity trading balance out, which leaves the depressing effect on incumbents' research effort. Hence, the stock is less actively researched as the investor base expands. In the second scenario, we add informed stockholders to an uninformed base. Since incumbent shareholders are uninformed, there is no loss of information on their part. They may actually become informed if more trades are liquidity motivated. In this case, the stock is more closely followed as investors join the base.

We also study the implications for the distribution of returns. When the base expands, returns are affected directly by the improvement in risk sharing and indirectly by the induced change in informativeness. Greater informativeness means that the price conveys more accurate information about the stock's future payoff so its risk is reduced. When risk sharing and informativeness both increase, the mean and variance of returns fall, as in the case of informed investors joining an uninformed base. The impact can be reversed when risk sharing improves while informativeness deteriorates. In the case of a homogeneous investor base for example, we show that expected returns fall but their variance rises.

Our results can have important implications for individual firms and the market as a whole. Public firms commonly take actions to expand their shareholder base. For example, they list on additional exchanges (e.g. Foerster and Karolyi [24] and Miller [53] for foreign companies listing on U.S. exchanges as American Depositary Receipts (ADRs) and Kadlec and McConnell [42] for U.S. over-the-counter companies listing on the NYSE), split their stock (Grinblatt, Masulis and Titman [26], Lamoureux and Poon [48], Mukherji, Kim and Walker [54] among others), reduce the number of shares in a roundlot (Amihud, Mendelson and Uno [5]) or simply advertise (Schonfeld and Boyd [60,61] and Grullon, Kanatas and Weston [29]). Their goal is to lower their cost of capital by improving risk sharing. Our analysis suggests that their stock's informativeness may deteriorate too. Empirical studies all document a fall in returns following these corporate actions but provide contradictory evidence on return volatility. In particular, listings on U.S. exchanges as ADRs and stock splits are associated with a permanent increase in return volatility (see Jayaraman, Shastri and Tandon [40] and Domowitz, Glen and Madhavan [21] on ADR listings and Ohlson and Penman [55], Lamoureux and Poon [48], Sheikh [64], Dubofsky [22], Lynch

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[4] We solve the model numerically under constant relative risk aversion expected utility (see Appendix B.2).

Koski [49] on stock splits).[5] In the absence of information effects, improvements in risk sharing should reduce expected returns without affecting their volatility. While several explanations have been put forward for the documented rise in volatility, our model shows that it is consistent with a deterioration in firms' information environment.[6] More work is needed to assess whether this hypothesis stands up to the data.

Our analysis can also be usefully applied to the market as a whole. Several trends have been observed over the past decades. Stock market participation by households has increased since the 50s and accelerated in the last decade (Vissing-Jørgensen [73]). For example, nearly half of U.S. households owned stocks at the end of 1999, up from a third ten years earlier. At the same time, passive investing has soared at the expense of active management. Bhattacharya and Galpin [9] document a steady decline in stock picking in the U.S. from a high of 60% in the 1960s to a low of 24% in the 2000s. Witness to this phenomenon is the remarkable growth in index funds. Their assets grew over a hundredfold over the last decade (though the first retail index fund was offered in 1976), ten times faster than equity funds overall (Investment Company Institute).[7] Idiosyncratic return variances have also been rising. Campbell, Lettau, Malkiel and Xu [16] find that the variance of individual stock returns has more than doubled from 1962 to 1997. Though caution is warranted when interpreting market-wide trends, our analysis suggests that these observations may be related. As participation in the equity market rises and risk sharing improves, investors reduce their collection of information to favor passive forms of investing. Return volatility rises if the information effect is strong enough as in the example of the homogeneous investor base.[8] We note that various other explanations have been put forward for the volatility increase so our interpretation again calls for a thorough empirical analysis.[9]

The remaining of the paper is organized as follows. Section 2 reviews the related literature. Section 3 describes the economy. Section 4 defines the equilibrium concept. Section 5 solves for the equilibrium. Section 6 analyzes the trade-off between information production and risk sharing in equilibrium. Section 7 concludes. Appendix A features proofs and Appendix B presents results under alternative preferences.

2. Related literature

Our results are based on scale effects in information that are well known to economists and have been applied in a variety of contexts (e.g. Wilson [79]).[10] In finance, Arrow [6], Peress [57] and Van Nieuwerburgh and Veldkamp [67] appeal to them to explain households' portfolio

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[5] Kadlec and McConnell [42], Amihud, Mendelson and Uno [5] and Grullon, Kanatas and Weston [29] do not explore any possible changes in return volatility.

[6] For example, it has been argued that firms listing as ADRs or splitting their stock attract investors who are more prone to liquidity shocks, i.e. small investors. However, this explanation is not fully consistent with the evidence (see footnote 23). Again, our setup allows but does not rely on a change in the extent of liquidity trading.

[7] A case in point is Vanguard's Index 500 fund which tracks the S&P 500 index. It became the largest U.S. mutual fund in April 2000, ahead of its active rival, Fidelity's Magellan fund.

[8] The model assumes for simplicity that stocks' payoffs are independent from one another so idiosyncratic return variance coincides with market return variance.

[9] Alternative explanations include an increase in institutional ownership (Malkiel and Xu [50]) and an intensification of competition in product markets (Irvine and Pontiff [39], Gaspar and Massa [25]). Brown and Kapadia [12] document that the rise in idiosyncratic volatility is the result of new listings by riskier companies, while Wei and Zhang [76] report that volatility increased also for existing firms.

[10] Unlike standard goods, information is non-rival, i.e. it is costly to generate but costless to replicate. This property, which applies to financial information (information about stock returns) as well as to technological knowledge (such as

choices. In particular, Van Nieuwerburgh and Veldkamp [67] show that investors specialize in a few assets when they are endowed with a fixed budget of signal precision to allocate across assets. In their setup, the benefits of diversification compete with those of specialization at the investor portfolio level, in the same way that in our model risk sharing limits the production of information at the stock level. They establish a connection from an asset's information structure to its ownership structure while we consider the reverse linkage (we do not impose any constraint on information acquisition but limit the set of stocks investors can trade). Calvo and Mendoza [14] examine the improvement in risk sharing generated by the globalization of capital markets. They show that less information is produced as the number of investable assets grows because of short-sales constraints. Our setting, in contrast, assumes no limit to shorting or borrowing. In addition, several authors model scale effects arising from the supply side, for example because information is a by-product of economic activity (Acemoglu and Zilibotti [1], Zeira [80], Veldkamp [69], Van Nieuwerburgh and Veldkamp [68]) or because its price declines with the quantity sold (Veldkamp [70,71]). Instead, the scale effects we discuss here are driven by the demand for information.

Other papers establish a link from an asset's ownership structure to its information structure but their mechanics are different. Miller [52] and Chen, Hong and Stein [18] show that, owing to short-sales constraints, optimistic investors exert a larger influence on stock prices than pessimistic investors do. As a result, companies that are more widely held are subject to a greater divergence of opinions and reflect more optimistic valuations. In Holmström and Tirole [38], managers expand their investor base to attract liquidity traders and stimulate the production of information. Our analysis does not assume short-sales constraints but allows for changes in liquidity trading.

The rational expectations models of Stein [66], Hau [32] and Holden and Subrahmanyam [37] are more closely related to our analysis. Stein [66] shows that opening a futures market can create noise if the new traders are badly informed. Hau [32] relates the number of traders in currency markets to the mistakes they make about expected return. In both models, the effect of information critically depends on its quality which is exogenous. Moreover, idiosyncratic, i.e. trader specific, noise prevails in the aggregate: Stein assumes that all futures traders have the same imperfect signal and Hau that expectational errors are correlated across traders. Consequently, as new traders enter, they bring some noise into the market, which, far from washing out, adds to the existing noise. Thus, a broader base means greater aggregate risk. Our model differs in that information may deteriorate even though new investors do not contribute to the equilibrium noise.[11] Instead, the information effect is a *rational* response to a broader investor base: new investors improve risk sharing and thus reduce incentives to acquire information. In Holden and Subrahmanyam [37], many informed traders compete against one another in the strategic setting of Kyle [46]. They compete more aggressively as their number increases. This leads private information to be revealed more rapidly and its value to decline. Though the implications share

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the design for a new good), leads to increasing returns: the cost of information is fixed while its benefit rises with the scale of its applications (the number of shares traded or the number of goods sold). This insight underlies much of the endogenous growth theory (see Jones [41] for a recent overview). The amount of information produced is only limited by the extent to which researchers can appropriate the rents generated. In financial markets, investors reveal their information to competitors as they trade. Incentives to collect information are preserved nevertheless because noise trading makes the revelation imperfect (Grossman and Stiglitz [28]). Similarly, technological innovations are easily copied and monopoly rights allow inventors to reap some benefit from their effort (Romer [58,59], Aghion and Howitt [3]).

[11] Our results obtain in particular when liquidity trading is not affected by the investor base expansion (Propositions 5 and 6). Furthermore, in our setup, private signals have independent disturbances which cancel out when aggregated.

common aspects, the mechanics are again different. Traders in their model are risk neutral and behave strategically while here they are risk averse and perfectly competitive. Moreover, the value of information in their setup is insensitive to the entry of uninformed traders. In our model, it is reduced by the entry of investors, whether they are informed or not. Thus in Holden and Subrahmanyam [37], information acquisition is not limited by risk sharing but by the intensity of competition.[12]

Finally, our paper relates to the literature that connects the volatility of stock returns to agents' participation in the stock market, abstracting from information production. Beyond the usual risk sharing effects, it emphasizes that broader participation can modify the composition of an asset's investor base if new investors are different from incumbents. For example, agents may differ in their access to cash as in Allen and Gale [4], or in their risk aversion as in Herrera [35]. When investors who are short of cash or more risk averse enter the market, fluctuations in stock prices are amplified and volatility rises.

3. The economy

We develop a model that blends Merton [51] with Verrecchia [72]. Investors trade stocks and a riskless bond competitively. As in Merton [51], they can only hold positions in a restricted set of stocks. As in Verrecchia [72], they may purchase private information about these stocks which is partially revealed through equilibrium prices.

3.1. Timing

The timing is depicted in Fig. 1. There are 3 periods. Period 0 is the planning period. Investors decide how much information, if any, to collect about the stocks they recognize. The second period ($\tau = 1$) is the trading period. Investors observe private signals of accuracy chosen in the previous period. At the same time, markets open and investors observe the equilibrium prices. Investors use public and private signals to estimate the risk and return characteristics of stocks and choose a position. In the third period ($\tau = 2$), they consume the proceeds from their investments.

3.2. Assets

A riskless asset (the bond) and a risky asset (the stocks) are traded competitively in the market. The riskless asset has a gross rate of return of R and is in perfectly elastic supply. The stock has Ω shares outstanding, a price P and a random payoff Π. The payoff Π is exogenous and normally distributed with mean $\overline{\Pi}$ and variance σ_Π^2. While the model applies to a multi-stock economy, we focus on a single risky asset to simplify the exposition. No insight is lost though, relative to a setting in which several stocks are traded when absolute risk aversion is constant (as assumed here, see below), the cost of information is separable across stocks and random variables are independent across stocks.[13]

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[12] In addition, Holden and Subrahmanyam [37] assume that traders have identical signals. Back, Cao and Willard [7] find opposite results under heterogeneous signals, i.e. that private information is revealed to the market *less* quickly when the number of informed traders increases. Investors have diverse signals in our setting.

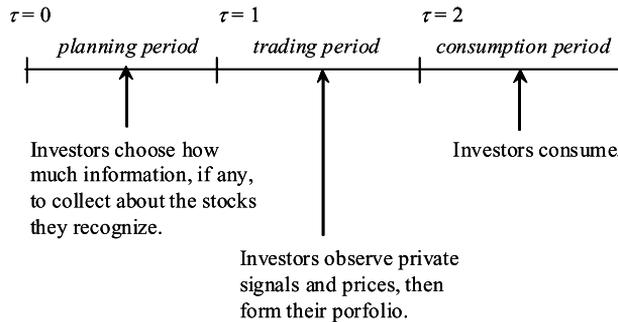[13] The working paper version of this article features multiple stocks and makes these assumptions.

Fig. 1. Timing.

3.3. Investors

The economy is populated by a large number of agents, modeled as a continuum. There are two groups of stockholders, rational investors (investors or stockholders for short) and liquidity traders.[14]

- Rational investors

There is a very large number, $\mathcal{J}$, of rational investors. In period 0, they maximize a mean-variance objective function of the Kreps–Porteus type, $E_0\{-t_j \ln E_1[\exp(-w'_j/t_j) \mid \mathcal{F}_j]\} = E_0\{E_1(w'_j \mid \mathcal{F}_j) - \frac{1}{2t_j} Var_1(w'_j \mid \mathcal{F}_j)\}$, where E_0 and E_1 denote respectively expectations formed in period 0 and 1, t_j is investor j's coefficient of absolute risk tolerance, w'_j is her final wealth and $\mathcal{F}_j$ is her information set in period 1. Kreps and Porteus [44,45] provide the axiomatic foundations for this class of non-expected utility. These preferences lead, in period 1, to the same mean-variance portfolio that obtains under CARA (constant absolute risk aversion or exponential) expected utility. In period 0, agents choose the precision of their information to maximize the expected certainty equivalent of their period-1 utility.[15]

We do not use exponential CARA expected utility as is typically assumed in rational expectations models with asymmetric information because it does not capture the scale effect of information we wish to investigate. Specifically, the demand for information under these prefer-

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[14] With an infinite number of traders, we can apply the Law of Large Numbers when we aggregate stock demands (e.g. Verrecchia [72], Admati [2], He and Wang [33]). The literature sometimes refers to rational investors as speculators and to liquidity traders as hedgers or noise traders.

[15] Kreps and Porteus [44,45] relax the expected utility axiom on the reduction of compound lotteries according to which agents faced with compound lotteries whose prizes are tickets to other lotteries care only about the compound probability of each prize. This axiom implies that agents are indifferent to the timing of the resolution of uncertainty. Relaxing this axiom allows, in particular, to disentangle risk aversion from the elasticity of intertemporal substitution. This property has been exploited in numerous papers in asset pricing and macroeconomics following the generalization of iso-elastic utility by Epstein and Zin [23] and Weil [77]. Weil [78], Hansen and Sargent [31], Hansen, Sargent and Tallarini [30] and Van Nieuwerburgh and Veldkamp [67] employ its exponential form. Kreps–Porteus exponential utility leads to a preference for early resolution. Hansen and Sargent [31] discuss its link to robust control theory. The fact that CRRA expected utility and CARA Kreps–Porteus utility both lead to scale effects in information suggests that it is a combination of the absence of wealth effects and the indifference to the timing of resolution of uncertainty under CARA expected utility that makes the demand for information independent of the supply of shares and the number of investors.

ences depends neither on the supply of shares nor on the number of investors (see the discussion following Theorem 2). We confirm that this non-dependence is the exception rather than the rule by solving the model numerically under constant relative risk aversion (CRRA) *expected* utility. The results, featured in Appendix B.2, reveal that investors' demand for information is not generally unrelated to the number of investors. Our preference specification allows to depart from the traditional CARA expected utility while maintaining a tractable framework. It enables us to model informational scale effects in a partially revealing rational expectations equilibrium.

We use risk tolerance to model exogenous differences in investment scale across investors. As we shall see, investors who are more risk tolerant, for example because they are wealthier, trade more aggressively and therefore collect more private information. Our goal is to relate the informativeness of prices to an exogenously given distribution of investor risk tolerance. Investors can borrow freely at the riskless rate so their initial wealth is not directly relevant, though it may matter indirectly through t_j, and is normalized to w. They are not subject to short-sales constraints but are restricted in the assets they can hold, as explained in the next subsection. Let e_j and b_j denote investor j's holdings of the stock and the bond.

- Liquidity traders

Liquidity traders are not modeled explicitly. They are subject to exogenous shocks that make their demand for assets random and price-inelastic. This assumption, standard in models in which prices aggregate and transmit information, makes the residual supply of stocks, i.e. the supply offered to investors once liquidity traders have placed their orders, random. It prevents equilibrium prices from fully revealing stocks' payoffs and preserves the incentives to purchase private information. Specifically, a liquidity trader demands $-\Theta$ shares where Θ is independent from the stock's payoff Π and normally distributed with mean 0 and variance σ_Θ^2. We assume that there are ρ liquidity traders for every rational investor. Therefore, a stock with J rational investors has ρJ liquidity traders demanding in aggregate $-\Theta \rho J$ shares. In the paper, we focus on the behavior of investors and abstract from the origin of the randomness in liquidity traders' demand.[16]

3.4. Information structure

Investors are limited in the set of assets that they can trade. We do not take a stand on the origin of this constraint which could stem from legal, financial or informational frictions. For expositional purposes, we adopt Merton's [51] Investor Recognition Hypothesis, which posits that investors are not aware of the existence of all assets. In the present single-stock framework, we define $G(t)$ (respectively $g(t)\,dt$) as the number of investors with risk tolerance below t (respectively between t and $t + dt$) who recognize the stock. The size of stock's investor base is denoted $J \equiv \lim_{t\infty} G(t)$.[17] G fully characterizes the stock's ownership structure. For example,

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[16] Other sources of noise used in the literature include random stock endowments (e.g. Verrecchia [72]), private risky investment opportunities (e.g. Wang [74]), liquidity needs, preference shocks (Campbell, Grossmand and Wang [15]) or the presence of irrational traders. Adding an idiosyncratic component to liquidity traders' stock demands would not affect the model. Indeed, if a liquidity trader l demands $-\Theta + \nu_l$ shares where ν_l is independent across liquidity traders, the law of large numbers implies that the average demand of liquidity traders reduces to $-\Theta$.

[17] There are no trading costs nor short-sales constraints so all investors aware of a stock trade it. Shareholders are broadly defined as agents with a position in the stock, be it long or short. The size of the investor base refers to the number of shareholders, thus broadly defined.

G is a single-step function for a stock owned by J identical investors. We denote the number of shares outstanding per investor as $\omega \equiv \Omega / J$.

Investors also differ in the quality of their knowledge. In contrast to the former, this source of heterogeneity is endogenous. Investors may devote time and resources to collecting information about the assets of which they are aware. If investor j knows about the stock, she may purchase a signal s_j about its payoff Π:

$$s_j = \Pi + \upsilon_j \tag{1}$$

where $Var(\upsilon_j) = 1/q_j$, υ_j is independent of Π, Θ and across agents. q_j is the precision or quality of agent j's signal. The signal costs $C(q_j)$. We assume C is continuous, increasing, strictly convex and $C(0) = 0$. For example, we illustrate the model with the specification $C(q) = cq^b + c'q$ where $c > 0$, $c' \geqslant 0$ and $b > 1$. In addition to their private signal, investors use the equilibrium price to forecast the payoff. Let $\mathcal{F}_j$ denote the information set of agent j, aware of the stock: $\mathcal{F}_j = \{s_j, P\}$. $E_1(. \mid \mathcal{F}_j)$ and $E_{0,j}(.)$ refer to conditional and unconditional expectations by investor j, i.e. where her private signals s_j have the precisions q_j.

4. Equilibrium concept

The equilibrium is defined by individual maximization and market clearing conditions.

4.1. Individual maximization

Obviously, an investor who does not recognize the stock allocates her entire wealth to the bond ($e_j = 0$ and $b_j = w$) and does not collect information ($q_j = 0$). To solve for the portfolio of an investor who does recognize the stock, we proceed in two stages, working from the end to the beginning of her horizon. In the trading period ($\tau = 1$), investor j observes the available signals and forms her portfolio, taking prices as given:

$$\max_{b_j, e_j} E_1\big(w'_j \mid \mathcal{F}_j\big) - \frac{1}{2t_j} Var_1\big(w'_j \mid \mathcal{F}_j\big) \quad \text{subject to} \quad \begin{cases} \Pi e_j + R b_j = w'_j, \\ P e_j + b_j + C(q_j) = w. \end{cases} \tag{2}$$

The precision (possibly equal to 0) of agent j's private signal, q_j, is inherited from the previous period. The top constraint states that the investor's final wealth consists of the payoffs of the shares, Πe_j, and bonds, $R b_j$, she acquires. The bottom constraint describes how she allocates her initial wealth w to the different assets of which she is aware (the bond and the stock) and to information collection ($C(q_j)$ measures her information expenditure). We denote $u(t_j, s_j, P, q_j)$ the value function for this problem.

In the planning period, the investor chooses the precision of her signal, q_j, in order to maximize her expected utility averaging over all the possible realizations of the signal and the price and taking $C(.)$ as given:

$$\max_{q_j \geqslant 0} E_{0,j}\big[u(t_j, s_j, P, q_j)\big]. \tag{3}$$

If $q_j > 0$ ($q_j = 0$) then stockholder j is informed (uninformed) about the stock. We solve for an investor's best precision choice given arbitrary precisions chosen by other investors, which we call her 'best information response function'. In equilibrium, precision choices are mutual best responses.

4.2. Market aggregation

The optimal behavior of a shareholder (portfolio and precision choices) depends on the behavior of other shareholders in the economy since the gains from trade and from private signals depend on how much risk shareholders are willing to bear and how much information is revealed through prices. Accordingly, we define the *average risk tolerance* for a stock as:

$$\bar{t} \equiv \frac{1}{J} \int_0^\infty t \, dG(t).$$

Again, G and J denote the cumulative distribution function and the number of rational investors aware of the stock. Similarly, we define the average weighted precision of private information about the stock as:

$$\overline{tq} \equiv \frac{1}{J} \int_0^\infty tq(t) \, dG(t). \tag{4}$$

$\overline{tq}/(\sigma_\Theta \rho)$ measures the stock's *informativeness*, implied by aggregating the precision choices of investors who recognize it and normalizing by the noise generated by liquidity traders (the proportion of liquidity traders times the standard deviation of their aggregate stock demand) (see Admati [2]). Consistent with this interpretation, we show in Theorem 1 that the signals provided by stock prices have a standard error $\sigma_\Theta \rho/\overline{tq}$. Finally, we define:

$$N \equiv \rho/\overline{tq} \tag{5}$$

to measure the stock's *noisiness*, a variable that will prove useful in the next sections. In the text, we refer indifferently to noisiness or informativeness, one being the scaled inverse of the other. In contrast to average risk tolerance, informativeness and noisiness are endogenous to the model. In equilibrium, they both depend on, and determine individual decisions. If risk tolerance, and therefore precisions, are identical across shareholders and denoted t and q, then $\bar{t} = t$, $\overline{tq} = tq$ and $N = \rho/tq$. We provide next the formal definition of an equilibrium and proceed to solving it in Section 5.

4.3. Definition of an equilibrium

A rational expectation equilibrium is given by investors' demand for the stock e_j and information q_j, prices P and noisiness N such that:

1. q_j and e_j solve the maximization problem of an investor taking P, N, J and $\bar{t}$ as given (Eqs. (2) and (3)).
2. The noisiness of the price N implied by aggregating individual precision choices equals the level assumed in the investor's maximization problem (Eqs. (4) and (5)):

$$\frac{1}{J} \int_0^\infty tq(t) \, dG(t) = \frac{\rho}{N}.$$

3. The stock price clears the market:

$$\int_0^\infty e \, dG(t) - \rho J\Theta = \Omega$$

where $\int_0^\infty e \, dG(t)$ is rational investors' aggregate demand for the stock, $-\rho J\Theta$ is liquidity traders' aggregate demand, and Ω is the number of shares outstanding.

5. Equilibrium description

Theorem 1 characterizes the equilibrium price and portfolio choice for arbitrary values of noisiness N and precisions q_j. Theorem 2 describes an investor's best information response function. Finally, Theorem 3 closes the model by determining the equilibrium value of N.

Theorem 1 (*Equilibrium price and stockholdings*). *Assume the information decisions have been made (i.e. N and q_j are given). There exists a unique linear rational expectations equilibrium.*

- *The equilibrium stock price is given by*

$$RP = P_0 + P_\Pi(\Pi - N\Theta) \tag{6}$$

where

$$h_0 \equiv \frac{1}{\sigma_\Pi^2} + \frac{1}{N^2\sigma_\Theta^2}, \qquad \bar{h} \equiv h_0 + \frac{\rho}{N\bar{t}}, \tag{7}$$

$$P_0 \equiv \frac{1}{\bar{h}}\left(\frac{\overline{\Pi}}{\sigma_\Pi^2} - \frac{\Omega}{J\bar{t}}\right) \quad and \quad P_\Pi \equiv \frac{1}{\bar{h}}\left(\frac{1}{N^2\sigma_\Theta^2} + \frac{\rho}{N\bar{t}}\right). \tag{8}$$

- *Shareholder j's holding of the stock equals*

$$e_j = t_j\big(e_0 + q_j(s_j - RP)\big) \quad where \quad e_0 \equiv \frac{\rho}{N\bar{t}}\left(\frac{P_0 - RP}{P_\Pi} + RP\right). \tag{9}$$

The characterization of the price is well known for this type of economy (e.g. Lemma 1 in Verrecchia [72]). It is identical for given values of N and q_j, whether agents with CARA preferences are expected utility maximizers or not. The equilibrium price depends on the payoff Π and liquidity traders' demand $\rho J\Theta$. Θ enters the price equation although it is independent of payoffs Π because it determines the number of stocks to be held and hence the total risk investors have to bear in equilibrium. Π appears directly in the price function though it is not known by any agent, because private signals s_j collapse to Π when aggregated. Observing P is equivalent to observing $\Pi - N\Theta$ which acts as noisy signals for Π. Its error $N\Theta$ has a standard deviation $N\sigma_\Theta$. Thus, $1/(\sigma_\Theta N) = \overline{tq}/(\sigma_\Theta \rho)$ *captures the informativeness of the price signal* for given σ_Π^2 and σ_Θ^2, consistent with the definition given above. h_0 measures the precision of public information. It equals the sum of the precisions from the prior, $1/\sigma_\Pi^2$, and from the price signal, $1/(N^2\sigma_\Theta^2)$. $h_0 + q$ measures the total precision for an investor using public information and a private signal of precision q. $\rho/(N\bar{t}) \equiv \overline{tq}/\bar{t}$ is a weighted average of private precisions, so $\bar{h} \equiv h_0 + \rho/(N\bar{t})$ is the weighted average total precision.

Two polar cases are instructive. If $N = 0$ then there is no noise and the price reveals the true payoff Π. There is no risk in this economy, the price function reduces to Π/R and the stock

has the same gross return as the bond, R. On the other hand, if $N = \infty$ then the price contains no information about the payoff. Π drops out from the price equation which becomes $[\overline{\Pi} - \sigma_\Pi^2(\Omega/J + \rho\Theta)/\bar{t}]/R$. The first term equals the present value of the expected payoff discounted at the risk free rate and the second term is the price discount that compensates investors for the risk incurred. This equilibrium corresponds to the one described in Merton [51].

Stockholders can be uninformed ($q_j = 0$) or informed ($q_j > 0$). Uninformed stockholders hold a number of shares equal to e_0 times their risk tolerance. Informed stockholders choose the same number of shares except that they scale it by their own risk tolerance and tilt it according to their private signal. Indeed, $s_j - RP$ is stockholder j's expectation of the excess return based on her private signal s_j. The more accurate the signal (the larger q_j), the greater the weight on this expectation.

Theorem 1 takes as given the information environment. In the next theorem, we solve for an investor's best information response function, i.e. her best precision choice given arbitrary precisions chosen by the other investors. These precisions aggregate to an arbitrary level of noisiness N.

Theorem 2 *(Best information response function). Assume the precisions chosen by all investors but investor j aggregate to an arbitrary level of noisiness N.*

- *There exist a threshold risk tolerance $t^*(N, \bar{t})$ such that investor j chooses to be informed if and only if her risk tolerance t_j is above t^*. The threshold is given by*

$$t^*(N, \bar{t}) \equiv \frac{2RC'(0)}{A(N, \bar{t})} \qquad (10)$$

where

$$A(N, \bar{t}) \equiv \frac{1}{\bar{h}^2}\left[\left(\frac{\Omega^2}{J^2} + \rho^2\sigma_\Theta^2\right)\frac{1}{\bar{t}^2} + \bar{h} + \frac{\rho}{N\bar{t}}\right] \qquad (11)$$

measures the marginal value of private information.
- *If investor j is informed ($t_j > t^*$), she chooses a signal of precision $q_j \equiv q(t_j, N, \bar{t})$ such that*

$$2RC'(q_j) = t_j A(N, \bar{t}). \qquad (12)$$

An investor collects information only if she is sufficiently risk tolerant ($t_j > t^*$). In this case, the first-order condition for her demand for information is given by Eq. (12) and is illustrated in Fig. 2. At the optimum, the marginal cost of private information equals its marginal benefit. The cost equals the cost of the signal, $C'(q_j)$, adjusted for the time value of money, R. The benefit is the product of a scalar, A, by the investor's risk tolerance t_j. A measures the marginal value of private information. To see this, consider the excess return in dollars of one share, $\Pi - RP$. The ratio of the mean excess return to its standard deviation is known as the stock's Sharpe ratio (it is investor-specific since it depends on the realization and precision of the private signal). Under CARA, an investor's certainty equivalent in the trading period is a linear function of the squared Sharpe ratio on her portfolio. As shown in Appendix A.2, the squared Sharpe ratio an investor expects to achieve on a stock in the planning period, if she acquires a signal of precision q_j, is
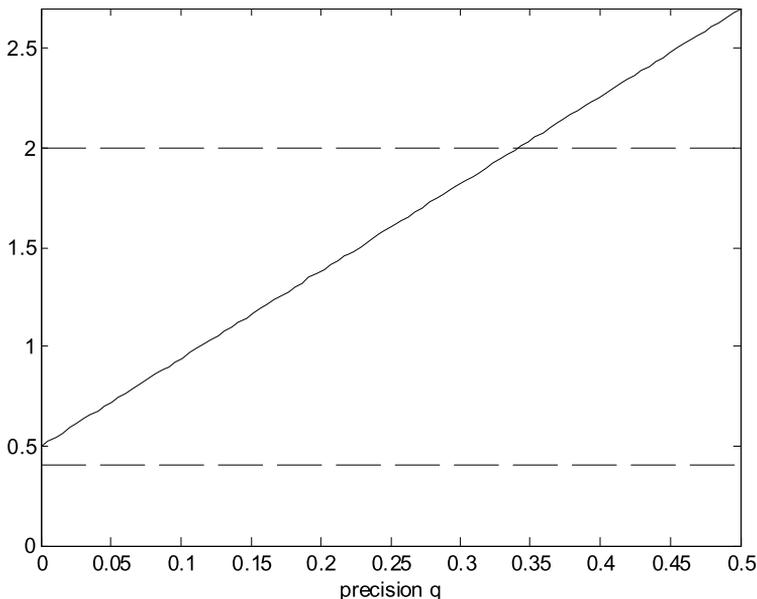
Fig. 2. The first order condition for the precision of information. The marginal cost of information $2RC'(q)$ (solid line) and its marginal benefit $t_j A$ (dashed lines, top line for a highly risk tolerant investor and bottom line for highly risk averse investor). Risk tolerant investors acquire information of precision at the intersection of $2RC'(q)$ and $t_j A$. Highly risk averse investors do not acquire information. The picture is drawn for $C(q) = q^2 + 0.5q$, t_j equals 2 for the highly risk tolerant investor and 0.4 for the highly risk averse investor, $R = 1.1$ and $A = 1$.

$(h_0 + q_j)A - 1$. Thus A captures the benefit to an investor of increasing by one unit the precision of her private signal.[18]

Eq. (12) shows that the benefit of information is scaled by risk tolerance. More risk tolerant investors trade, on average, a larger number of shares (see Eq. (9)) over which they can spread their informational advantage. Because the benefit of information increases with the scale of investment, captured by t_j (which in turn can be related to initial wealth), while its cost does not, the best information response function shifts upward for more risk tolerant investors. This property was established by Verrecchia [72] under CARA expected utility (see Corollary 1). For the same reason, only investors who are sufficiently risk tolerant ($t_j > t^*$) are informed. Those who are too risk averse ($t_j \leqslant t^*$) invest on too small a scale to make information profitable. Fig. 4 illustrates this property.

The best information response function plays an important role in the model. Its properties are tied to those of the marginal value of information A, and are summarized in Lemma 3. We introduce the following notation that will prove useful in the subsequent section. For any function $f(z_1, z_2, \dots)$, we denote ε_{f/z_1} the elasticity of f with respect to z_1 holding $z_2, \dots$ fixed. Formally, $\varepsilon_{f-z_1}(z_1, z_2, \dots) \equiv \frac{\partial \ln f}{\partial \ln z_1}$. In particular, $\varepsilon_{A-\bar{i}}$, ε_{A-N}, ε_{A-J} and $\varepsilon_{A-\rho}$ refer to

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[18] When $C'(0) = 0$, investors collect information about all the stocks of which they are aware because the benefit of information is linear in precision while its cost is increasing and convex. When investors face constraints on how much they can learn, Van Nieuwerburgh and Veldkamp [67] find that they limit their collection of information to a few stocks.
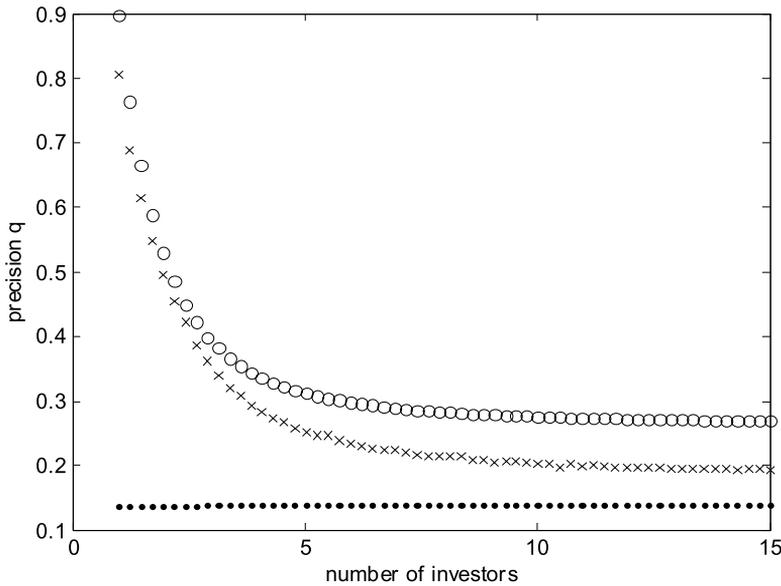
Fig. 3. The best information response and the number of shareholders under CRRA expected utility. The o-marked curve corresponds to a coefficient of relative risk aversion of 7 and the x-marked curve to a coefficient of 10. The dotted curve corresponds to CARA expected utility with a coefficient of absolute risk aversion of 7. The picture assumes the variance of liquidity trades does not increase with the number of investors. The other parameter values are $\sigma_\Pi^2 = \overline{\Pi} = 0.1$, $\sigma_\Theta^2 = 1$, $\rho = 1$, $N = 10$, $\Omega = 0.5$, $w = 10$, $R = 1.03$, $C(q) = 0.05q$ and $n_d = 3$. All three curves are computed numerically (see Appendix B for details).

the elasticities of A with respect to $\bar{t}$, N, J and ρ, and $\varepsilon_{C'-q}$ (> 0) refers to the elasticity of C' with respect to q. We emphasize that these derivatives are only partial, i.e. that they are taken with respect to one variable freezing all the others, including endogenous variables. In the next section, we consider the total impact of a change in an exogenous variable (we allow endogenous variables to adjust).

Lemma 3 (*Properties of the best information response function*).

- *The best information response function shifts down when the number of rational traders, J, or their average risk tolerance, $\bar{t}$ increase. Formally, $\varepsilon_{A-J} \leqslant 0$ and $\varepsilon_{A-\bar{t}} \leqslant 0$.*
- *The best information response function shifts up when noisiness, N, the number of shares outstanding, Ω, the proportion of liquidity traders, ρ, or the variance of their aggregate demand, σ_Θ^2 decrease. Formally, $\varepsilon_{A-N} \geqslant 0$, $\varepsilon_{A-\Omega} \geqslant 0$, $\varepsilon_{A-\rho} \geqslant 0$ and $\varepsilon_{A-\sigma_\Theta^2} \geqslant 0$.*
- *In addition, $\varepsilon_{A-\bar{t}} + \varepsilon_{A-\rho} = \varepsilon_{A-J}$.*

The marginal value of information A is a decreasing function of both the number of investors J and the average risk tolerance $\bar{t}$. An increase in J or $\bar{t}$ leads to an increase in price (on average) and to a reduction in agent's j stockholdings, for a given risk tolerance t_j. With less scope to gain from information, agent j's best response shifts downward. Putting it differently, *an improvement in risk sharing, ceteris paribus, leads to a reduction in the risk borne by stockholders, which*
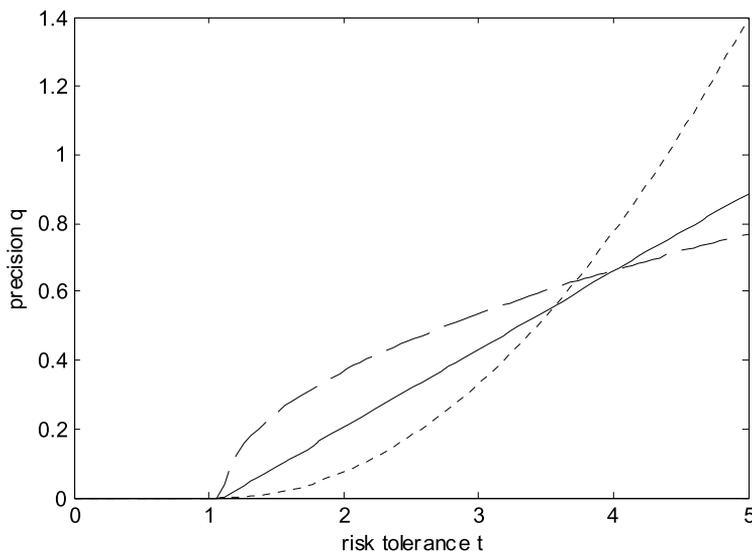
Fig. 4. The best information response as a function of risk tolerance. The solid curve corresponds to $C(q) = q^2 + 0.5q$, the dashed curve to $C(q) = q^3 + 0.5q$ and the dotted curve to $C(q) = q^{1.5} + 0.5q$. Investors with risk tolerance below $t^* \equiv 1.1$ do not acquire information. The picture is drawn for $R = 1.1$ and $A = 1$.

makes information less valuable. Conversely, A increases when the number of shares outstanding Ω grows, so the best response shifts up.[19]

It is important to note that this effect is absent from models in which investors have CARA expected utility as in Verrecchia [72]. As shown in Appendix B.1, the best information response function under CARA expected utility is given by the first order condition, $2RC'(q_j) = t_j/(h_0(N) + q_j)$ from which $\bar{t}$, J and Ω are absent.[20] There is no trade-off between risk sharing and information collection. We check that such a trade-off is not limited to non-expected utility maximizers. We show that it obtains under expected utility when one departs from the usual CARA structure by solving for the best response under CRRA expected utility. We resort to a numerical solution as no closed-form solution to this problem is known. The technical details are featured in Appendix B.2. Fig. 3 shows that, under CRRA expected utility, the best response shifts down when the number of shareholders grows, consistent with the preferences we use, but inconsistent with CARA expected utility. We examine the tradeoff between risk sharing and the value of information in equilibrium when we consider the implications of an investor base expansion which increases J in the next section.

The other properties of the best information response function are similar to those derived under CARA expected utility. It shifts upward when noisiness N rises. Indeed, larger noisiness implies that prices are less revealing, making private information easier to conceal (Corollary 2

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[19] In this exchange economy, each share is a claim to a fixed payoff Π.

[20] This condition corresponds to Eq. (17) in Appendix B.1 and to Eq. (8) in Verrecchia [72]. It implies in particular that, under CARA expected utility, investor j's best information response function depends on her own risk tolerance t_j and on the level of noise N but not on the average risk tolerance $\bar{t}$, the number of shares outstanding Ω or the number of investors J. This underlies the novel effects we explore in this paper: for a given level of noisiness N, an individual best information response function shifts downward when investors are on average more risk tolerant ($\bar{t}$ larger), or when their number is larger (J larger) or when their are more shares outstanding (Ω larger).

in Verrecchia [72]). For the same reason, it shifts upward when the proportion of liquidity traders ρ or the variance of their aggregate demand σ_Θ^2 increase (Corollary 3 in Verrecchia [72]). To summarize, an investor's best response shifts up when her own risk tolerance (t_j), noisiness (N), the number of shares outstanding (Ω), the proportion of liquidity trading (ρ) or the variance of noise trades (σ_Θ^2) increase or when the average risk tolerance $(\bar{t})$ or the number of investors (J) decrease. Similarly, the threshold for information collection t^* is decreasing in N, Ω, ρ and σ_Θ^2 and increasing in $\bar{t}$ and J.

Having characterized through $t^*(N, \bar{t})$ and $q(t_j, N, \bar{t})$ the best information response function for shareholders aware of the stock for any level of noisiness N, we can now close the model by solving for the equilibrium level of noisiness N.

Theorem 4 *(Stock informativeness). Noisiness N is the unique solution to*

$$\frac{1}{J} \int_{t^*(N,\bar{t})}^{\infty} t q(t, N, \bar{t}) \, dG(t) = \frac{\rho}{N} \tag{13}$$

where $t^*(N, \bar{t})$ and $q(t, N, \bar{t})$ are defined by Eqs. (10) *to* (12) *in Theorem* 2.

In equilibrium, precision choices are mutual best responses. Therefore, the equilibrium value of N is determined jointly by its definition (Eqs. (4) and (5)) and by the best information response function (Eqs. (10) to (12)). N is characterized implicitly and uniquely by Eq. (13). Theorems 2 and 4 together characterize the composition of the investor base. Only shareholders whose risk tolerance is large enough $(t_j > t^*)$ are informed. In general, informed and uninformed shareholders coexist in equilibrium. But all shareholders are informed if the least risk tolerant shareholder has a tolerance larger than t^*. This is the case when $C'(0)$ is small (for example equal to zero), when the investor base is narrow (J small) or when investors are on average very risk averse ($\bar{t}$ small).[21] Alternatively, all shareholders are uninformed if the most risk tolerant shareholder has a risk tolerance smaller than $t^*(\infty, \bar{t}) = 2RC'(0)/[(\Omega^2/J^2 + \rho^2\sigma_\Theta^2)\sigma_\Pi^2/\bar{t}^2 + 1]/\sigma_\Pi^2$. This occurs in particular when $C'(0)$ is large, when investors are numerous (J large) or highly risk tolerant on average ($\bar{t}$ large). In that case, no information is collected about the stock and the equilibrium collapses to the one described in Merton [51]. In the next section, we examine the determinants of a stock's informativeness in more detail.

6. Risk sharing vs. informativeness in equilibrium

This section focuses on the tradeoff between risk sharing and information production. We examine the equilibrium implications of an investor base expansion, i.e. we consider an increase in the number of investors aware of the stock. This experiment allows for a detailed discussion of the impact of risk sharing on informativeness in a realistic setting. As mentioned in the Introduction, companies often attempt to broaden their shareholder base by listing on additional exchanges, splitting their stock, reducing the number of shares in a roundlot or advertising. By improving risk sharing, they can benefit from a lower cost of capital. We argue that their information environment is altered in the process.

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[21] As we point out below, A is a decreasing function of J and $\bar{t}$.

An investor base expansion can affect a stock's informativeness through three channels. The first channel is a reduction in incumbent shareholders' research, as a result of enhanced risk sharing. New investors unload some of the risk from incumbent investors, and, in doing so, make information less valuable to them. If new investors are informed, then they also contribute to the stock's informativeness. This constitutes the second channel. Finally, the composition of the investor base may change, which will also affect incumbents' research effort. The net effect on the stock's informativeness depends on the relative importance of each channel, which we measure in terms of elasticities.

We analyze the addition to the base of $\alpha \geqslant 0$ identical shareholders of risk tolerance $t_\alpha > 0$. The expansion increases the number of shareholders from J to $J_\alpha = J + \alpha$. It can also modify the composition of the investor base if entrants differ from the average incumbent. To allow for such differences, we denote respectively the relative deviations in entrants' risk tolerance, tolerance-weighted precision and proportion of liquidity traders relative to the average incumbent as $\delta(t) \equiv (t_\alpha - \bar{t})/\bar{t}$, $\delta(tq) \equiv (t_\alpha q_\alpha - \overline{tq})/\overline{tq}$ and $\delta(\rho) \equiv (\rho_\alpha - \rho)/\rho$. Positive $\delta(t)$, $\delta(tq)$ and $\delta(\rho)$ indicate that the company has attracted agents who are more risk tolerant, better informed and more prone to liquidity trading than incumbents. We also allow for the issuance of shares at the time of the base expansion (for example as the company lists on an additional exchange) and denote $\delta(\Omega) \equiv (\Omega_\alpha - \Omega)/\Omega$, the relative change in the number of shares outstanding. We provide a general theorem followed by two applications.

Theorem 5 (*Impact of an investor base expansion on a stock's informativeness*). *Suppose a stock's investor base expands by α identical shareholders of risk tolerance t_α. The relative change in the stock's informativeness, $1/(N\sigma_\Theta) = \overline{tq}/(\rho\sigma_\Theta)$, equals*

$$-\frac{d \ln N}{d\alpha} = \frac{d \ln[\overline{tq}/(\rho\sigma_\Theta)]}{d\alpha}$$
$$= \frac{\beta\varepsilon_{A-\omega}(J\delta(\Omega) - 1) + \delta(tq) + \beta\varepsilon_{A-\bar{t}}\delta(t) + (\beta\varepsilon_{A-\rho} - 1)\delta(\rho)}{J(1 + \beta\varepsilon_{A-N})} \quad (14)$$

around $\alpha = 0$, where $\varepsilon_{A-\omega} \equiv \varepsilon_{A-\Omega} - \varepsilon_{A-J} > 0$, $\beta \equiv \frac{1}{J}[\int_{t^*}^\infty tq(t)/\varepsilon_{C'-q}(q(t)) \, dG(t)]/\overline{tq} \geqslant 0$ and q_α is the precision of new stockholders' signals (given by Eq. (12)). If $t_\alpha < t^*$, then new stockholders are uninformed ($q_\alpha = 0$). Otherwise, they are informed ($q_\alpha > 0$).

Eq. (14) identifies the three channels through which an investor base expansion can affect the stock's informativeness. (i) New investors unload some of the risk from incumbent investors, thereby reducing the value of information. This effect is represented by the term $-\beta\varepsilon_{A-\omega} < 0$ in the equation (recall that $\omega \equiv \Omega/J$ is the number of shares outstanding per investor). (ii) New investors contribute to informativeness if they collect more information than the average incumbent ($\delta(tq) > 0$). This happens for example if $t_\alpha = \bar{t}$ and they have access to a superior information technology (lower C) or if $t_\alpha > t^*$ where t^* is the threshold for information collection defined by Eq. (11). (iii) The composition of the investor base changes. New investors who are more risk tolerant than incumbents ($\delta(t) > 0$) are willing to bear a larger proportion of the firm's risk. They induce incumbents to reduce their research effort further ($\beta\varepsilon_{A-\bar{t}}\delta(t) < 0$). This is another reflection of the tradeoff between risk sharing and information production (the average risk tolerance $\bar{t}$ falls). Similarly, liquidity trading intensifies if new investors are more likely than incumbents to trade for liquidity motives ($\delta(\rho) > 0$). An increase in ρ affects informativeness directly (the term $-\delta(\rho) < 0$ in Eq. (11) which derives from ρ in the denominator of $\overline{tq}/(\rho\sigma_\Theta)$), and indirectly

through the incentives to collect information (the term $\beta\varepsilon_{A-\rho}\delta(\rho) > 0$ in the equation which derives from $\overline{tq}$ in the numerator of $\overline{tq}/(\rho\sigma_{\Theta})$). Indeed, incumbents intensify their research effort when prices are less revealing. Finally, a share issuance encourages information collection as investors load on the company's risk ($\beta\varepsilon_{A-\omega}J\delta(\Omega) > 0$). The net impact on informativeness is dampened by the response of investors since lower informativeness increases their incentives to collect information ($1 + \beta\varepsilon_{A-N} > 1$ in the denominator of Eq. (11)). Note that the coefficient β simplifies to $1/\varepsilon_{C'-q}$ when $\varepsilon_{C'-q}$ does not depend on q. For example if $C(q) = cq^b$ for $b > 1$, then $\beta = 1/(b-1)$.

Proposition 6 specializes Theorem 5 to the case of homogeneous investors, i.e. when new stockholders are (*ex ante*) identical to incumbents. It also examines how the stock's return distribution is affected. We consider the unconditional (period 0) mean and variance of the excess return in dollars of one share of stock, $\Pi - RP$.

Proposition 6 *(Homogeneous investor base). Consider a stock held by identical investors. Suppose the stock attracts new investors, identical to incumbents ($\delta(t) = \delta(tq) = \delta(\rho) = 0$). Then informativeness rises if and only if the number of shares outstanding per investor, $\omega \equiv \Omega/J$, rises.*

In particular, if the company does not issue shares ($\delta(\Omega) = 0$), then

- *informativeness falls,*
- *expected returns fall,*
- *the variance of returns rises.*

The base expansion's implications for informativeness follow from Theorem 5 by setting $\delta(t) = \delta(tq) = \delta(\rho) = \delta(\Omega) = 0$ in Eq. (14): new investors' risk tolerance, tolerance-weighted precision and proportion of liquidity trading is identical to that of incumbents. Thus, the risk sharing effect is left to operate alone, reducing incumbents' research effort. If no shares are issued ($\delta(\Omega) = 0$), then informativeness declines. It rises however if shares are issued more than proportionally to the number of shareholders ($\omega \equiv \Omega/J$ rises).

We discuss next the distribution of returns. A stock's expected excess return is determined by two factors, the extent of risk sharing and the amount of risk to bear. For a given level of risk, the wider the investor base or the more tolerant investors, the lower the risk premium. This is the risk sharing effect considered in the IRH. The level of risk is also endogenous in our model. It derives from the stock's informativeness, which in turn, depends on the extent of risk sharing. We refer to this effect as the "information effect". The two effects can be identified by noting that, in equilibrium, a stock's unconditional expected excess return equals $\Omega/(J\bar{h}\bar{t})$ which falls, on one hand, with the number of shareholders J and their average risk tolerance $\bar{t}$ (the risk sharing effect), and, on the other hand, with the average precision per investor $\bar{h}$ (the information effect). In the homogeneous base case we consider, these effects are in conflict. Improved risk sharing decreases expected returns while worsened informativeness increases them. We establish that the direct effect, i.e. the risk sharing effect, dominates and that expected returns decline.

The variance of returns is subject to the same risk sharing and information effects. It falls with the average risk tolerance though it is independent from the number of shareholders (holding fixed informativeness). It also falls when risk falls (holding fixed $\bar{t}$), i.e. when informativeness rises. Formally, $Var_0(\Pi - RP) = (1 - P_\Pi)^2\sigma_\Pi^2 + (NP_\Pi)^2\sigma_\Theta^2 = (\rho^2\sigma_\Theta^2/\bar{t}^2 + \rho/N/\bar{t} + \bar{h})/\bar{h}^2$. To appreciate the effect of informativeness on the variance of returns, start from perfect information ($N = 0$). The price tracks the future payoff perfectly. The return equals the riskless rate so its

variance is zero. As information worsens (N increases), the price is less sensitive to the payoff (P_Π decreases in Eq. (6)) whereas the return is more sensitive ($1 - P_\Pi$ increases). The effect on the noise term, NP_Π, is ambiguous (P_Π falls while N rises). We show that return variance increases, i.e. the sensitivity of returns to the payoff dominates. In the homogeneous base case, risk sharing does not affect the variance of returns ($\delta(t) = 0$) but the information effect does and pushes up volatility.

We conjecture that the implications for volatility obtain in a dynamic version of the model. A difficulty arises in a multiperiod setup because a stock's return variance not only involves next period's dividend payment but also the stock's resale price. When more information is acquired, the current price tracks future dividends and prices more closely, thereby reducing the return variance (as in the static model). But the variance of future prices also increases since future prices track more closely dividends even further into the future. However, because future prices are discounted, the former effect dominates the latter as Pagano [56] and West [75] show.[22]

The information effect could explain why firms that list as ADRs or split their stock experience a rise in return volatility. This is a puzzling observation, inconsistent with a pure risk sharing story such as the IRH. As we show, a rational response to an improvement in risk sharing is for investors to follow the stock less closely, which increases its risk and its return volatility. Note that this finding can be combined with an increase in liquidity trading (set $\delta(\rho) > 0$ in Eq. (14)), though such an increase is not required for the result to obtain.[23] Our findings can also be applied to the market as a whole to connect *a priori* independent trends in U.S. equity markets. They suggest that the growth in passive investing and the rise in idiosyncratic stock return volatility are consequences of increased stock market participation. While the mechanism we suggest rationalizes several observations, other explanations have been suggested. A detailed analysis of the data is therefore required.

Proposition 6 shows that informativeness falls while volatility rises with the number of shareholders when they are homogeneous. Proposition 7 demonstrates that the opposite occurs when informed investors join an uninformed base. We assume that informed and uninformed investors

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[22] The following example illustrates this point for risk neutral investors. Suppose investors know that a stock will pay three dividends, Π_1, Π_2 and Π_3 at dates τ_1, τ_2 and τ_3. Nothing is known about the dividends except that they are distributed with identical mean $\overline{\Pi}$ and variance σ_Π^2. Suppose further that they are revealed Δ periods ahead, i.e. Π_1 is revealed at $\tau_1 - \Delta \ldots$. The (cum dividend) stock price can easily be computed. For example at $t < \tau_1 - \Delta$, the price equals $\overline{\Pi}/R^{\tau_1-t} + \overline{\Pi}/R^{\tau_2-t} + \overline{\Pi}/R^{\tau_3-t}$, at $\tau_1 - \Delta \leqslant t \leqslant \tau_1$, it equals $\Pi_1/R^{\tau_1-t} + \overline{\Pi}/R^{\tau_2-t} + \overline{\Pi}/R^{\tau_3-t}$, at $\tau_1 < t \leqslant \tau_2 - \Delta$, it equals $\overline{\Pi}/R^{\tau_2-t} + \overline{\Pi}/R^{\tau_3-t} \ldots$. Importantly, early revelation of dividends does not simply shift prices forward, it also rescales them through the discount factor. Returns, $P_t + \Pi_t - RP_{t-1}$, equal 0 on every date except on the revelation dates when they equal $(\Pi_1 - \overline{\Pi})/R^\Delta$, $(\Pi_2 - \overline{\Pi})/R^\Delta$ and $(\Pi_3 - \overline{\Pi})/R^\Delta$. The variance of returns therefore equals $\sigma_\Pi^2/R^{2\Delta}$, a decreasing function of Δ: the earlier dividends are revealed, the lower the variance of returns.

[23] Some researchers have argued that these corporate events attract small investors. If these small shareholders are more prone to liquidity trading, they will enhance return variance. This explanation, however, is not entirely consistent with the facts. ADRs are relatively sophisticated instruments that most likely do not attract small investors (Karolyi [43]), while splits do not appear to increase the proportion of individual shareholders (Mukherji, Kim and Walker [54]). Moreover, the volume of trade does not seem to grow, as expected by an increase in liquidity trading (Black [11]). Domowitz, Glen and Madhavan [21] estimate the volatility change following ADR listings controlling for any possible volume effect and again find a very strong volatility change. (Jayaraman, Shastri and Tandon [40], Foerster and Karolyi [24] and Miller [53] do not examine volume.) Copeland [20], Lamoureux and Poon [48], Lakonishok and Lev [47], Conroy, Harris and Benet [19] and Lynch Koski [49] find a decline in daily trading volume after adjusting for the split factor, while Schultz [62] documents a large number of small buy orders.

can coexist in equilibrium (which requires $C'(0) > 0$) and that the company does not raise capital ($\delta(\Omega) = 0$).

Proposition 7 *(Addition of informed stockholders to an uninformed base). Consider a stock held exclusively by uninformed investors. Suppose informed investors join the stock's base while the number of shares outstanding is unchanged ($\delta(tq) = +\infty$ and $\delta(\Omega) = 0$). Then,*

- *informativeness rises,*
- *expected returns fall,*
- *the variance of returns falls.*

In this scenario, incumbent shareholders are already uninformed. Therefore, informativeness rises when informed shareholders join the base. These investors may be more risk tolerant or have access to a superior information technology. Risk sharing and informativeness both improve. The information and risk sharing effects work in the same direction, reducing both expected returns and volatility.

We have emphasized so far the tradeoff between risk sharing and informativeness through the role played by the number of shareholders J and their average risk tolerance $\bar{t}$. We conclude by showing that, beyond J and $\bar{t}$, the entire distribution of risk tolerance matters to a stock's informativeness. This point was first made by Verrecchia [72], who shows that a shift in the distribution of risk tolerance such that the new distribution first-order stochastically dominates the original one leads to an increase in informativeness.[24] We consider instead a new distribution that second-order stochastically dominates the original one, while keeping the same mean, i.e. the new distribution is a mean preserving spread of the original one.[25] We establish the following proposition.

Proposition 8 *(Informativeness and the distribution of risk tolerance). Suppose all stockholders are informed (for example, assume $C'(0) = 0$). Consider a mean preserving spread of the risk tolerance distribution, i.e. $\bar{t}$, J and ρ are kept constant.*

- *If $1/C'$ is convex, then informativeness increases.*
- *If $1/C'$ is concave, then informativeness decreases.*

Proposition 8 illustrates that the entire distribution of risk tolerance is important to the stock's informativeness. Focusing on a mean-preserving spread of the distribution of risk tolerance, i.e. a more unequal distribution with the same average $\bar{t}$, allows to isolate a pure distributional effect, devoid of changes in the extent of risk sharing. The implication of a mean-preserving spread depends on the curvature of $tq(t)$ (the integrand in the definition of $\overline{tq}$) as a function of t, which in turn depends on the curvature of $1/C'(q)$ as a function of q. If $1/C'$ is convex, then $tq(t)$ is convex. Shifting risk tolerance from a poorly tolerant to a highly tolerant stockholder increases the precision of the latter more than it decreases the precision of the former, and hence enhances

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[24] See Corollary 5 in Verrecchia [72]. Denoting G' the new distribution of risk tolerance, G' first-order stochastically dominates G if and only if $G'(t) \leqslant G(t)$ for all t.

[25] Denoting G' the new distribution of risk tolerance, G' second-order stochastically dominates G if and only if $\int_0^a G'(t)\, dt \leqslant \int_0^a G(t)\, dt$ for all $a \geqslant 0$.
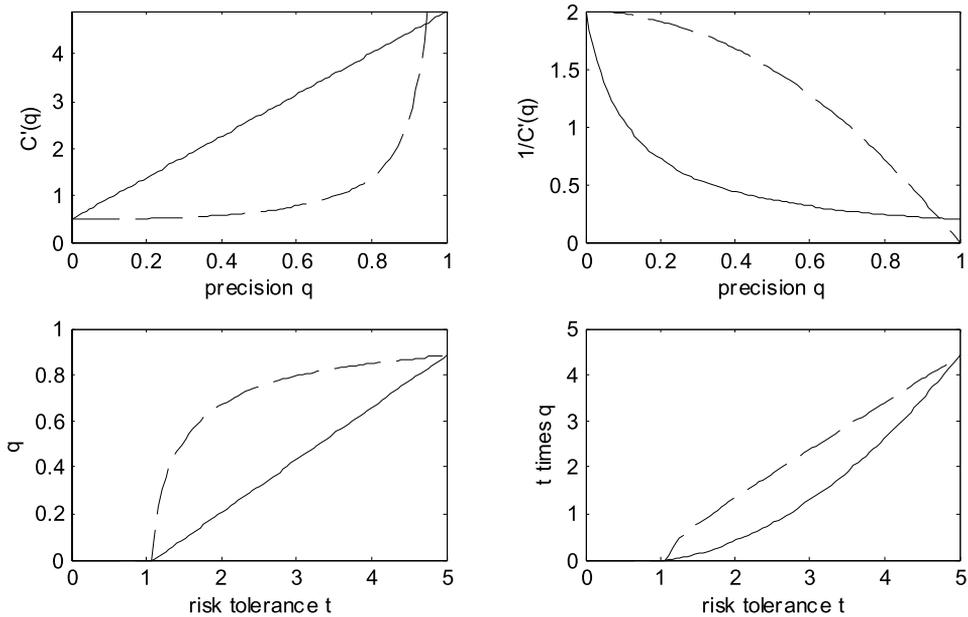
Fig. 5. The curvatures of the marginal information cost and the best information response. The top panels display the marginal cost of information $C'(q)$ (top left panel) and its inverse $1/C'(q)$ (top right panel) for different precision choices. The bottom panels display the best information response $q(t)$ (bottom left panel) and its contribution to informativeness $tq(t)$ (bottom right panel) as a function of risk tolerance. Solid curves are drawn for $C(q) = q^2 + 0.5q$ and dashed curves for $C(q) = 0.5\,\mathrm{atanh}(q)$. The other parameters are $R = 1.1$ and $A = 1$.

the stock's informativeness. If $1/C'$ is concave, then $tq(t)$ is concave and the opposite happens.[26] Fig. 5 illustrates the theorem thanks to two examples. $1/C'(q)$ and $tq(t)$ are convex for $C(q) = q^2 + 0.5q$ (solid curves) while they are concave for $C(q) = 0.5\,\mathrm{atanh}(q)$ (dashed curves).

The lemma assumes for simplicity that all stockholders are informed. The presence of uninformed stockholders can alter the effect of the spread because they cannot reduce their collection of information when risk tolerance is redistributed away from them towards informed investors. When $1/C'$ is convex, the increase in informativeness is enhanced. When $1/C'$ is concave, the decrease in informativeness is dampened or even reversed.

7. Conclusion

We show that a stock's ownership distribution determines how much information is produced about its payoff, i.e. its informativeness. This dependence is the consequence of a simple insight: shareholders research stock more actively when they can hold a larger number of shares. Indeed, the benefit from private information, unlike its cost, rises with the scale of investment. Everything else equal, a more widely held stock is a stock in which shareholders have, on average, a smaller stake and which they follow less closely. Putting is differently, information production is limited by the extent of risk sharing, i.e. information is less valuable when investors bear less risk.

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[26] C' convex is a necessary but not sufficient condition for $1/C'$ concave.

We analyze the implications of an investor base expansion. We identify three channels through which informativeness can be affected. First, an expansion leads to a redistribution of shares from incumbent to new shareholders (assuming no shares are issued). The redistribution reduces incumbents' incentives to collect information. Second, new stockholders may collect information and thus contribute to the stock's informativeness. Third, if liquidity trading intensifies, then informed trades become easier to conceal so more information is produced. The net effect on informativeness depends on the relative importance of the three channels. Moreover, the distribution of returns is affected by the changes in risk sharing and in informativeness. We consider two scenarios to illustrate these effects. When stockholders are homogeneous (entrants are *ex ante* identical to incumbents), informativeness and expected returns fall but volatility rises. When informed stockholders join an uninformed base, informativeness rises and the mean and variance of stock returns decline. Our results, as illustrated with a homogeneous investor base, may explain why the increase in stockmarket participation is accompanied by a rise in passive investing and in idiosyncratic stock return volatility. It may also explain why firms which expand their investor base through stock splits or ADR listings see their return variance increase, an observation at odds with a pure risk sharing story such as the IRH. Empirical work is needed to test these links.

The model derives a stock's information structure from an exogenous distribution of investor risk tolerance, which proxies for investor size. As we show, this distribution is important to a stock's informativeness as larger agents collect more information. These scale effects in information also encourage investors to form coalitions, such as mutual or pension funds. But their size is limited, on one hand, by organizational and agency costs, and on the other, by the price impact of trades (see Chen, Hong, Huang and Kubik [17] for evidence). These factors are not part of the present model which assumes in particular that investors behave competitively. It can be extended to include these factors, which would lead to a theory of fund size and its impact on the informativeness of stocks. This extension is of interest given that the growth in financial intermediation is one of the most remarkable trends of the post-war period. In addition, a model in which both the ownership and information decisions are endogenous can help assess the ownership implications of accounting standards and disclosure requirements. Such reforms are currently being debated in several countries.

Acknowledgments

Appendix A. Proofs

A.1. Proof of Theorem 1 (Equilibrium price and stockholding)

To prove Theorem 1, we guess that the equilibrium price is given by Eqs. (6) to (8) and solve for the optimal portfolio of an investor (recall that precisions are taken as given at this stage). We focus on a given investor j, aware of the stock. The first step is to estimate the mean and variance of the stock's payoff using the equilibrium price (or equivalently $\xi \equiv \Pi - N\Theta$) and the private signal s_j.

- Signal extraction

For the price function given in Eq. (6) (P is linear in Π and Θ), the conditional mean and variance of Π are:

$$Var_1(\Pi \mid \mathcal{F}_j) = \frac{1}{h_0 + q_j} \equiv \frac{1}{h_j} \quad \text{and} \quad E_1(\Pi \mid \mathcal{F}_j) = a_{0j} + a_{\xi j}\xi + a_{s_j}s_j$$

$$\text{where } a_{0j}h_j \equiv \frac{\overline{\Pi}}{\sigma_\Pi^2}, \ a_{\xi j}h_j \equiv \frac{1}{N^2\sigma_\Theta^2} \text{ and } a_{s_j}h_j \equiv q_j.$$

Intuitively, $Var_1(\Pi \mid \mathcal{F}_j)$ falls as the precision of the private signal, q_j or informativeness, $1/(\sigma_\Theta N)$, increase. Similarly, $E_1(\Pi \mid \mathcal{F}_j)$ is a weighted average of priors, public and private signals where the weight on the private signal (resp. the price) is increasing in q_j (resp. in $1/(\sigma_\Theta N)$). If the investor is uninformed, $q_j = 0$ and s_j vanishes from the equations.

- Individual portfolio choice

We can now compute investor j's optimal portfolio. Under mean-variance preferences, the optimal holding of a stock equals $t_j \frac{E_1(\Pi|\mathcal{F}_j) - RP}{Var_1(\Pi|\mathcal{F}_j)}$. Plugging in the above expression for $E_1(\Pi \mid \mathcal{F}_j)$ and $Var_1(\Pi \mid \mathcal{F}_j)$ leads to Eq. (9). Obviously, an investor unaware of a stock has a holding of zero.

- Market clearing

The equilibrium price clears the market for stocks. Aggregating Eq. (9) over all stockholders yields the aggregate stock demand:

$$\int_0^\infty e \, dG(t) - \rho J\Theta = \int_0^\infty t\big[e_0 + q_j(s_j - RP)\big] dG(t) - \rho J\Theta$$

$$= J\bar{t}e_0 + \frac{\rho J\Pi}{N} - \frac{\rho JRP}{N} - \rho J\Theta$$

where $N \int_0^\infty tq \, dG(t) \equiv \rho J$ and the term Π/N comes from applying the law of large numbers to the sequence $\{t_j q_j \upsilon_j\}$. Indeed, $\{t_j q_j \upsilon_j\}$ is a sequence of independent random variables with the same mean 0 (conditional on Π) and finite variances $t_j^2 q_j$ so $\int_0^\infty tq\upsilon \, dG(t) = 0$. Therefore, $\int_0^\infty tq s_j \, dG(t) = \int_0^\infty tq(\Pi + \upsilon) \, dG(t) = \Pi \int_0^\infty tq \, dG(t) + \int_0^\infty tq\upsilon \, dG(t) = \rho J\Pi/N$ (see Admati [2] and He and Wang [33] for more details). Finally, equating the aggregate demand to the aggregate supply Ω yields the equilibrium prices given by Eqs. (6)–(8).

A.2. Proof of Theorem 2 (Best information response function)

To prove Theorem 2, we need to solve the information acquisition problem of a stockholder given arbitrary precisions chosen by other investors, as summarized by the noisiness N. We plug Eq. (9) into the formula for Kreps–Porteus utility to obtain the utility of an investor with risk tolerance t_j and a signal of precision q_j:

$$E_0\big\{-t_j \ln E_1\big[\exp\big(-w_j'/t_j\big) \mid \mathcal{F}_j\big]\big\} = E_0\bigg\{E_1\big(w_j' \mid \mathcal{F}_j\big) - \frac{1}{2t_j} Var_1\big(w_j' \mid \mathcal{F}_j\big)\bigg\}.$$

We compute next the conditional expectation and variance of w'_j:

$$E_1\big(w'_j \mid \mathcal{F}_j\big) = R\big[w - C(q_j)\big] + e_j\big[E_1(\Pi \mid \mathcal{F}_j) - RP\big] = R\big[w - C(q_j)\big] + t_j z_j^2,$$
$$Var_1\big(w'_j \mid \mathcal{F}_j\big) = e_j^2 Var_1(\Pi \mid \mathcal{F}_j) = t_j^2 z_j^2,$$

where $z_j \equiv [E_1(\Pi \mid \mathcal{F}_j) - RP]/\sqrt{Var_1(\Pi \mid \mathcal{F}_j)}$ is investor j's Sharpe ratio, a function of s_j and P (and q_j). Substituting these expressions yields:

$$E_0\big\{-t_j \ln E_1\big[\exp\big(-w'_j/t_j\big) \mid \mathcal{F}_j\big]\big\} = R\big[w - C(q_j)\big] + \frac{t_j}{2} E_{0,j}\big(z_j^2\big). \tag{15}$$

$E_{0j}(z_j^2)$ is the squared Sharpe ratio stockholder j expects to achieve in the planning period. It no longer depends on s_j and P but it is still a function of q_j.

We integrate next z_j^2 over the distributions of s_j and P. To ease the computation, we define $h_j \equiv h_0 + q_j$ and $u_j \equiv z_j/\sqrt{Var_1(\Pi \mid \mathcal{F}_j)} = [E_1(\Pi \mid \mathcal{F}_j) - RP]h_j$. Substituting the expressions for $E_1(\Pi \mid \mathcal{F}_j)$ (see Appendix A.1) and for RP (Eqs. (6)–(8)) yields:

$$
\begin{aligned}
u_j &= \frac{\overline{\Pi}}{\sigma_\Pi^2} + \frac{1}{N^2\sigma_\Theta^2}\xi + q_j s_j - h_j(P_0 + P_\Pi \xi) \\
&= \frac{\overline{\Pi}}{\sigma_\Pi^2} + \frac{1}{N^2\sigma_\Theta^2}(\Pi - N\Theta) + q_j(\Pi + \upsilon_j) \\
&\quad - h_j\left[\frac{1}{\bar{h}}\left(\frac{\overline{\Pi}}{\sigma_\Pi^2} - \frac{\Omega}{J\bar{t}}\right) + \frac{1}{\bar{h}}\left(\frac{1}{N^2\sigma_\Theta^2} + \frac{\rho}{N\bar{t}}\right)(\Pi - N\Theta)\right] \\
&= \frac{\overline{\Pi}}{\sigma_\Pi^2}\left(1 - \frac{h_j}{\bar{h}}\right) + \frac{h_j}{\bar{h}}\frac{\Omega}{J\bar{t}} + \frac{1}{\sigma_\Pi^2}\left(\frac{h_j}{\bar{h}} - 1\right)\Pi + \left(h_j - \frac{h_j}{\bar{h}\sigma_\Pi^2} - \frac{1}{N^2\sigma_\Theta^2}\right)N\Theta + q_j\upsilon_j.
\end{aligned}
$$

The expectation of u_j equals $E_{0j}(u_j) = \frac{h_j}{\bar{h}}\frac{\Omega}{J\bar{t}}$ since $E_{0j}(\Theta) = E_{0j}(\upsilon_j) = 0$, and its variance equals $Var_{0j}(u_j) = \frac{1}{\sigma_\Pi^2}(\frac{h_j}{\bar{h}} - 1)^2 + (h_j - \frac{h_j}{\bar{h}\sigma_\Pi^2} - \frac{1}{N^2\sigma_\Theta^2})^2 N^2\sigma_\Theta^2 + q_j$ where the last term follows from the definition of q_j ($Var(\upsilon_j) \equiv 1/q_j$). We expand and rearrange the latter expression and obtain $Var_{0j}(u_j) = \frac{1}{\bar{h}^2}(\frac{\rho^2\sigma_\Theta^2}{\bar{t}^2} + \bar{h} + \frac{\rho}{N\bar{t}})h_j^2 - h_j$. It follows that $E_{0j}(u_j^2) = E_{0j}(u_j)^2 + Var_{0j}(u_j) = Ah_j^2 - h_j$ where $A(N, \bar{t})$ is defined in Eq. (11), and $E_{0,j}(z_j^2) = (h_0 + q_j)A - 1$.

To solve for the best information response function, we maximize stockholder j's utility with respect to q_j taking N and $\bar{t}$ (hence A) as given. If there is an interior solution, q_j, it must satisfy the first order condition $2RC'(q_j) = t_j A$. By assumption, C is convex in q_j so the right-hand side is increasing while the left-hand side is constant. This implies that a unique interior solution exists if and only if $2RC'(0) < t_j A$. Otherwise the solution will be the corner 0, i.e. the investor is uninformed. It follows that shareholders are informed if and only if their risk tolerance is above the threshold t^* defined in Eq. (11).

A.3. Proof of Lemma 3 (Properties of the best information response function)

The properties of the best information response function are governed by those of the marginal value of information A. Differentiating $\ln A$ and rearranging yields $\frac{\partial \ln A}{\partial \bar{t}} = -\frac{2}{A(\bar{t}\bar{h})^3}[\frac{\rho^2\sigma_\Theta^2}{\sigma_\Pi^2} + h_0\frac{\Omega^2}{J^2}] < 0$, $\frac{\partial \ln A}{\partial N} = \frac{2}{A\bar{h}^3 N^2}\{[(\frac{\Omega^2}{J^2} + \rho^2\sigma_\Theta^2)\frac{1}{\bar{t}^2} + \frac{\rho}{N\bar{t}}](\frac{\rho}{\bar{t}} + \frac{2}{N\sigma_\Theta^2}) + \frac{\bar{h}}{N\sigma_\Theta^2}\} > 0$, $\frac{\partial \ln A}{\partial J} = -\frac{2\Omega^2}{AJ^3(\bar{t}\bar{h})^2} <$

0, $\frac{\partial \ln A}{\partial \rho} = \frac{2}{A \bar{t}^2 \bar{h}^3 N^2} [\frac{\rho \sigma_\Theta^2}{\sigma_\Pi^2} - \frac{\Omega^2}{J^2 \bar{t} N}] > 0$ and $\frac{\partial \ln A}{\partial \omega} = \frac{2}{A \bar{t}^2 \bar{h}^2} \frac{\Omega}{J} > 0$. Therefore, an increase in the number of rational traders J or in their average risk tolerance $\bar{t}$, or a decrease in noisiness N, in the number of shares outstanding Ω, in the proportion of liquidity traders ρ or in the variance of their aggregate demand σ_Θ^2, all cause the best information response function to shift down. In addition, it can be checked from its expression that A is homogeneous of degree 0 in $\bar{t}$, $1/J$ and ρ. Therefore $\frac{\partial A}{\partial \bar{t}} \bar{t} - \frac{\partial A}{\partial J} J + \frac{\partial A}{\partial \rho} \rho = 0$ and $\varepsilon_{A-\bar{t}} + \varepsilon_{A-\rho} = \varepsilon_{A-J}$.

A.4. Proof of Theorem 4 (Informativeness)

Noisiness N is determined implicitly by Eq. (13), which is obtained by plugging the demand for information, $t^*(N, \bar{t})$ and $q(t, N, \bar{t})$, into its definition. To prove that the equilibrium is unique (within the class of linear equilibria), it suffices to show that N is uniquely defined. Let $\Sigma(N) \equiv N \int_{t^*(N,\bar{t})}^{\infty} t q(t, N, \bar{t}) \, dG(t)$. N is defined as the solution to $\Sigma(N) = \rho J$. Differentiating Σ yields:

$$\Sigma'(N) = \Sigma(N)/N - N t^* q^* g^* \frac{\partial t^*}{\partial N} + N \int\limits_{t^*(N,\bar{t})}^{\infty} t \frac{\partial q}{\partial N} \, dG(t),$$

where the second term comes from differentiating the lower bound in the integral and the third from differentiating the integrand. The second term drops out because $q^* \equiv q(t^*(N, \bar{t}), N, \bar{t}) = 0$ (this follows from the assumption that C is continuous at 0). Differentiating Eq. (12) yields $\frac{\partial q}{\partial N} = t \frac{\partial A}{\partial N}/(2RC''(q))$. C is strictly convex and $\frac{\partial A}{\partial N} > 0$ (Lemma 3) so $\frac{\partial q}{\partial N}$ is positive. Therefore $\Sigma'(N)$ is positive and Σ increases over the real positive line. Furthermore, $\Sigma(0) = 0$ and $\Sigma(\infty) = +\infty$. Hence, there exists a unique N satisfying Eq. (13).

A.5. Proof of Theorem 5 (Impact of an investor base expansion on a stock's informativeness)

We compute $\frac{d \ln N}{d \alpha}$ around $\alpha = 0$. We denote as N, J, $\bar{t}$, $\overline{tq}$, ρ and t^* the equilibrium values around $\alpha = 0$. Following the expansion, the distribution of risk tolerance is $G(t, \alpha) = G(t, 0) + \alpha 1_{\{t \geqslant t_\alpha\}}$ where $1_{\{t \geqslant t_\alpha\}}$ is the indicator function of the inequality $t \geqslant t_\alpha$ ($1_{\{t \geqslant t_\alpha\}} = 1$ if $t \geqslant t_\alpha$ and 0 otherwise). The number of investors is $J_\alpha = J + \alpha$. The average risk tolerance is $\bar{t}_\alpha = \frac{1}{J_\alpha} \int_{t_\alpha^*}^{\infty} t \, dG(t, \alpha) = \frac{\bar{t} J + \alpha t_\alpha}{J_\alpha}$ where t_α^* is the information collection threshold following the expansion, so $\frac{d \ln \bar{t}_\alpha}{d \alpha} = \frac{(t_\alpha - \bar{t})}{J \bar{t}}$ (around $\alpha = 0$). The proportion of liquidity traders is $\rho_\alpha = \frac{\rho J + \alpha \rho_\alpha}{J_\alpha}$ so $\frac{d \ln \rho_\alpha}{d \alpha} = \frac{(\rho_\alpha - \rho)}{J \rho}$. The number of shares per investor is $\omega_\alpha = \frac{\Omega_\alpha}{J_\alpha}$ so $\frac{d \ln \omega_\alpha}{d \alpha} = (\frac{1}{\Omega} \frac{d\Omega}{d\alpha} - \frac{1}{J}) = (\frac{\Omega_\alpha - \Omega}{\Omega} - \frac{1}{J})$ where $\Omega_\alpha - \Omega$ denotes the number of shares issued ($\Omega_\alpha - \Omega = 0$ if no shares are issued). Noisiness becomes $N_\alpha \equiv \rho_\alpha / \overline{tq}_\alpha = \rho_\alpha J_\alpha / I_\alpha$ where we define $I_\alpha \equiv J_\alpha \overline{tq}_\alpha = \int_{t_\alpha^*}^{\infty} q(t, \alpha) t \, dG(t, \alpha)$. We differentiate first the expression for N_α with respect to α, yielding $\frac{d \ln N_\alpha}{d \alpha} = \frac{d \ln \rho_\alpha}{d \alpha} + \frac{d \ln J_\alpha}{d \alpha} - \frac{d \ln I_\alpha}{d \alpha}$. We differentiate next the expression for I_α to obtain $\frac{d I_\alpha}{d \alpha} = \int_{t_\alpha^*}^{\infty} \frac{\partial q}{\partial \alpha} t \, dG(t, \alpha) + t_\alpha q_\alpha$ where the first (second) term comes from differentiating $q(t, \alpha)$ ($dG(t, \alpha)$) in the integral and $q_\alpha^* \equiv q(t_\alpha^*) = 0$ drops out. Therefore:

$$\frac{d \ln N_\alpha}{d \alpha} = \frac{(\rho_\alpha - \rho)}{J \rho} + \frac{1}{J} - \frac{N}{J \rho} \left[\int\limits_{t_\alpha^*}^{\infty} \frac{\partial q}{\partial \alpha} t \, dG(t, \alpha) + t_\alpha q_\alpha \right]. \tag{16}$$

Finally, we take the log of the first order condition (12) and differentiate it. We obtain $\frac{\partial q}{\partial \alpha} \frac{\varepsilon_{C'-q}}{q} = \varepsilon_{A-\bar{\imath}} \frac{d \ln \bar{\imath}_\alpha}{d\alpha} + \varepsilon_{A-N} \frac{d \ln N_\alpha}{d\alpha} + \varepsilon_{A-\rho} \frac{d \ln \rho_\alpha}{d\alpha} + \varepsilon_{A-\omega} \frac{d \ln \omega_\alpha}{d\alpha}$. Plugging back this expression into Eq. (16) yields:

$$\frac{d \ln N_\alpha}{d\alpha} = \frac{(\rho_\alpha - \rho)}{J\rho} + \frac{1}{J} - \frac{N}{J\rho}\left[\left(\varepsilon_{A-\bar{\imath}} \frac{d \ln \bar{\imath}_\alpha}{d\alpha} + \varepsilon_{A-N} \frac{d \ln N_\alpha}{d\alpha}\right.\right.$$
$$\left.\left. + \varepsilon_{A-\rho} \frac{d \ln \rho_\alpha}{d\alpha} + \varepsilon_{A-\omega} \frac{d \ln \omega_\alpha}{d\alpha}\right) H J + t_\alpha q_\alpha\right],$$

where $H \equiv \beta \bar{\imath} \bar{q}$. We substitute the expressions for $\frac{d \ln \bar{\imath}_\alpha}{d\alpha}$, $\frac{d \ln \rho_\alpha}{d\alpha}$ and $\frac{d \ln \omega_\alpha}{d\alpha}$, rearrange and obtain:

$$\left(1 + \frac{N}{\rho}\varepsilon_{A-N}H\right)\frac{d \ln N_\alpha}{d\alpha}$$
$$= \frac{(\rho_\alpha - \rho)}{J\rho} + \frac{1}{J} - \frac{N}{J\rho}\left(\varepsilon_{A-\bar{\imath}} \frac{d \ln \bar{\imath}_\alpha}{d\alpha} + \varepsilon_{A-\rho} \frac{d \ln \rho_\alpha}{d\alpha} + \varepsilon_{A-\omega} \frac{d \ln \omega_\alpha}{d\alpha}\right)HJ - \frac{N}{J\rho}t_\alpha q_\alpha$$
$$= \frac{(\rho_\alpha - \rho)}{J\rho} + \frac{1}{J} - \frac{NH}{J\rho}\left(\varepsilon_{A-\bar{\imath}} \frac{(t_\alpha - \bar{\imath})}{\bar{\imath}} + \varepsilon_{A-\rho} \frac{(\rho_\alpha - \rho)}{\rho} + \varepsilon_{A-\omega}\left(\frac{\Omega_\alpha - \Omega}{\Omega}J - 1\right)\right)$$
$$- \frac{N}{J\rho}t_\alpha q_\alpha.$$

Thanks to Eq. (5), this expression reduces to

$$J(1 + \varepsilon_{A-N}\beta)\frac{d \ln N_\alpha}{d\alpha}$$
$$= \frac{(\rho_\alpha - \rho)}{\rho} + 1 - \beta\left(\varepsilon_{A-\bar{\imath}} \frac{(t_\alpha - \bar{\imath})}{\bar{\imath}} + \varepsilon_{A-\rho} \frac{(\rho_\alpha - \rho)}{\rho} + \varepsilon_{A-\omega}\left(\frac{\Omega_\alpha - \Omega}{\Omega}J - 1\right)\right)$$
$$- \frac{1}{\bar{\imath}\bar{q}}t_\alpha q_\alpha.$$

Rearranging yields

$$\frac{d \ln N}{d\alpha} = \left[\bar{\imath}\bar{q} - t_\alpha q_\alpha - \varepsilon_{A-\bar{\imath}}(t_\alpha - \bar{\imath})/\bar{\imath}H - \left[J(\Omega_\alpha - \Omega)/\Omega - 1\right]\varepsilon_{A-\omega}H \right.$$
$$\left. + (\rho_\alpha - \rho)/\rho(-\varepsilon_{A-\rho}H + \bar{\imath}\bar{q})\right]\left[J(\bar{\imath}\bar{q} + \varepsilon_{A-N}H)\right]^{-1}. \qquad (17)$$

Finally, substituting $\delta(t) \equiv (t_\alpha - \bar{\imath})/\bar{\imath}$, $\delta(tq) \equiv (t_\alpha q_\alpha - \bar{\imath}\bar{q})/\bar{\imath}\bar{q}$, $\delta(\rho) \equiv (\rho_\alpha - \rho)/\rho$ and $\delta(\Omega) \equiv (\Omega_\alpha - \Omega)/\Omega$ into Eq. (17) yields Eq. (14).

A.6. Proof of Proposition 6 (Homogeneous investor base)

When investors (including new investors) are identical, $\delta(t) = \delta(tq) = \delta(\rho) = 0$. In addition, when no shares are issued, $\delta(\Omega) = 0$ so $\frac{d \ln \omega_\alpha}{d\alpha} = \frac{d\Omega}{d\alpha}/\Omega - 1/J = -1/J$. Eq. (14) becomes:

$$-\frac{d \ln N}{d\alpha} = \frac{-\beta\varepsilon_{A-\omega}}{J(1 + \beta\varepsilon_{A-N})} < 0.$$

Therefore noisiness rises and informativeness falls. We turn to the distribution of returns. Substituting Eqs. (6) to (8) into the definition for the unconditional expected excess return $M \equiv E_0(\Pi - RP)$, yields $M = \frac{\Omega}{J\bar{\imath}\bar{h}}$. Differentiating this expression yields $\frac{\partial M}{\partial J} = -\frac{M}{J} < 0$, $\frac{\partial M}{\partial \bar{\imath}} = -\frac{M h_0}{\bar{\imath}\bar{h}} < 0$

and $\frac{\partial M}{\partial N} = \frac{M}{\bar{t}\bar{h}N^2}(\rho + \frac{2\bar{t}}{N\sigma_{\Theta}^2}) > 0$. In words, M falls when the size of the base, average risk tolerance or informativeness rise. The net effect on M is ambiguous. We show that M falls, i.e. that the risk sharing effect dominates. We examine the sign of $\frac{dM}{d\alpha} = \frac{\partial M}{\partial J}\frac{dJ}{d\alpha} + \frac{\partial M}{\partial \bar{t}}\frac{d\bar{t}}{d\alpha} + \frac{\partial M}{\partial N}\frac{dN}{d\alpha} = \frac{\partial M}{\partial J} + \frac{\partial M}{\partial N}\frac{dN}{d\alpha}$ (recall that $\frac{dJ}{d\alpha} = 1$ and $\frac{d\bar{t}}{d\alpha} = 0$). M falls if and only if $-\frac{\partial M}{\partial J} > \frac{\partial M}{\partial N}\frac{dN}{d\alpha}$. Substituting each term with its expression given above and in Appendix A.3 shows that this inequality in indeed satisfied.

The procedure for V is simpler. Indeed, the unconditional variance of excess returns and its derivatives are $V \equiv Var_0(\Pi - RP) = A - M^2 = \frac{1}{\bar{h}}[\frac{\rho}{\bar{t}\bar{h}}(\frac{\rho\sigma_{\Theta}^2}{\bar{t}} + \frac{1}{N}) + 1]$, $\frac{\partial V}{\partial J} = 0$, $\frac{\partial V}{\partial \bar{t}} = -\frac{2}{(\bar{t}\bar{h})^3}\frac{\rho^2\sigma_{\Theta}^2}{\sigma_{\Pi}^2} < 0$ and $\frac{\partial V}{\partial N} = \frac{2\rho}{(\bar{t}\bar{h})^3N^3}(3\bar{t}\rho + \frac{h_0\bar{t}^3}{\rho\sigma_{\Theta}^2} + \frac{3\bar{t}^2}{N\sigma_{\Theta}^2} + N\rho^2\sigma_{\Theta}^2) > 0$. In particular, V rises when informativeness falls and is unaffected by the change in the size of the base so V grows following the expansion. Formally, $\frac{dV}{d\alpha} = \frac{\partial V}{\partial J}\frac{dJ}{d\alpha} + \frac{\partial V}{\partial \bar{t}}\frac{d\bar{t}}{d\alpha} + \frac{\partial V}{\partial N}\frac{dN}{d\alpha} = \frac{\partial V}{\partial N}\frac{dN}{d\alpha} < 0$.

A.7. Proof of Proposition 7 (Addition of informed stockholders to an uninformed base)

Risk sharing is enhanced by the entry of news investors (J increases). It is improved further (dampened) if average risk tolerance $\bar{t}$ rises (falls) following the expansion, i.e. $\frac{d\ln\bar{t}_\alpha}{d\alpha} = \frac{(t_\alpha - \bar{t})}{J\bar{t}} > 0 \ (< 0)$. Informativeness increases too. Indeed, Eq. (17) indicates that $\frac{d\ln N}{d\alpha} = -\infty$ (since all incumbents are uninformed, set $\overline{tq} = H = 0$; the numerator simplifies to $-t_\alpha q_\alpha < 0$ and the denominator to 0). The entry of informed investors increases informativeness from 0. Thus, risk sharing and informativeness improve with the expansion. We proceed as in Appendix A.6 for the distribution of returns. The information effect is so powerful ($\frac{dN}{d\alpha} = -\infty$) when the base consists of uninformed investors that it dominates any risk sharing effect (in case $\frac{d\ln\bar{t}_\alpha}{d\alpha}$ is not 0). Expected returns and their variance fall. Formally, $\frac{dM}{d\alpha} = \frac{\partial M}{\partial J}\frac{dJ}{d\alpha} + \frac{\partial M}{\partial \bar{t}}\frac{d\bar{t}}{d\alpha} + \frac{\partial M}{\partial N}\frac{dN}{d\alpha} = -\frac{M}{J\bar{t}\bar{h}}(h_0 t_\alpha + \rho/N) + \frac{\partial M}{\partial N}\frac{dN}{d\alpha} < 0$ and $\frac{dV}{d\alpha} = \frac{\partial V}{\partial J}\frac{dJ}{d\alpha} + \frac{\partial V}{\partial \bar{t}}\frac{d\bar{t}}{d\alpha} + \frac{\partial V}{\partial N}\frac{dN}{d\alpha} < 0$.

A.8. Proof of Proposition 8 (Informativeness and the distribution of risk tolerance)

Let χ be a parameter indexing a mean-preserving spread of $G(.)$, $G(., \chi)$. That is, the larger χ, the more unequal the distribution of risk tolerance, while its average, $\frac{1}{J}\int_0^\infty t\,dG(t, \chi)$, remains equal to $\bar{t}$. Formally, $\frac{1}{J}\int_0^\infty \varphi(t)\frac{\partial g(t,\chi)}{\partial \chi}\,dt > 0 \ (< 0)$ for any convex (concave) function φ, while $\frac{1}{J}\int_0^\infty t\frac{\partial g(t,\chi)}{\partial \chi}\,dt = 0$. The resulting values of N and t^* depend on χ but we do not write them explicitly as functions of χ to simplify the notation. Differentiating Eq. (13) with respect to χ yields $\frac{\rho J}{N}\frac{dN}{d\chi} - Nt^*q^*g^*(\frac{\partial t^*}{\partial \chi} + \frac{\partial t^*}{\partial N}\frac{dN}{d\chi}) + N\int_{t^*}^\infty t\frac{\partial q}{\partial N}\frac{dN}{d\chi}g(t, \chi)\,dt + N\int_{t^*}^\infty tq\frac{\partial g(t,\chi)}{\partial \chi}\,dt = 0$ where q^* and g^* are evaluated at t^*. The second term drops out because $q^* = 0$. Therefore,

$$\frac{dN}{d\chi} = -\left(\int_{t^*}^\infty tq\frac{\partial g(t, \chi)}{\partial \chi}\,dt\right)\Big/\left(\frac{\rho J}{N^2} + \int_{t^*}^\infty t\frac{\partial q}{\partial N}g(t, \chi)\,dt\right).$$

The numerator is positive because $\frac{\partial q}{\partial N} > 0$ ($\varepsilon_{A-N} > 0$ in Lemma 3). If tq is a convex function of t, then $\frac{dN}{d\chi} < 0$. This happens if $(tq(t))'' = 2q'(t) + tq''(t) > 0$ for all $t \geqslant t^*$. Differentiating the first order condition (12) twice and substituting yields $q''(t)/q'(t) = -C'(q(t))C'''(q(t))/[tC''(q(t))^2]$. Thus tq is convex if $2C''(q)^2 - C'(q)C'''(q) > 0$ for all positive q. This condition in turn is equivalent to $(1/C'(q))'' = [2C''(q)^2 - C'(q)C'''(q)]/C'(q)^3 > 0$. Intuitively, if $1/C'$ is convex, shifting risk tolerance from a low to a high risk tolerant investor

increases informativeness because the high tolerance investor increases her demand for information by more than the low tolerance investor reduces hers. Alternatively, if tq is a concave function of t, then $\frac{dN}{d\chi} > 0$. This happens if $1/C'$ is concave. In that case, the cost of information increases so steeply with the precision (C' is convex) that highly risk tolerant investors hardly increase their precision as they become more risk tolerant. Informativeness falls.

Appendix B. The tradeoff between risk sharing and information production under expected utility

B.1. CARA expected utility

We derive investors' demand for information under CARA *expected* utility as in Verrecchia [72]. We proceed backwards from the trading period to the planning period. In the trading period, investors maximize the same mean-variance objective as in the non-expected utility case. Hence, price and portfolios are identical to those obtained under CARA Kreps–Porteus utility (for any given values of noisiness N and precisions q_j) and are given by Eqs. (6) to (8). To determine the best information response function, we derive the utility of an investor with risk tolerance t_j and a signal of precision q_j. We plug Eq. (9) into the formula for expected utility and integrate over all possible values of Π and Θ:

$$
\begin{aligned}
E_0\{-\exp(-w'_j/t_j)\} &= E_0\{E_1[-\exp(-w'_j/t_j) \mid \mathcal{F}_j]\} \\
&= E_0\{-\exp[-z_j^2/2 + R(w - C(q_j))/t_j]\} \\
&= -\exp[R(C(q_j) - w)/t_j]E_{0j}\{\exp(-z_j^2/2)\},
\end{aligned}
$$

where z_j is investor j's Sharpe ratio as defined in Appendix A.2. In addition (see for example Brunnermeier [13, p. 64]), $E_{0j}\{\exp(-z_j^2/2)\} = [1 + Var_{0j}(z_j)]^{-\frac{1}{2}} \exp(-\frac{1}{2}\frac{E_{0j}(z_j)^2}{1+Var_{0j}(z_j)}) = [A_1(h_0 + q_j)e^{A_2}]^{-\frac{1}{2}}$ where $A_1 \equiv (\bar{h} + \frac{\rho}{N\bar{t}} + \frac{\rho^2\sigma_\Theta^2}{\bar{t}^2})/\bar{h}^2$ and $A_2 \equiv \frac{\Omega^2}{J^2\bar{t}^2}/(\bar{h} + \frac{\rho}{N\bar{t}} + \frac{\rho^2\sigma_\Theta^2}{\bar{t}^2})$.

Maximizing $E_0\{-\exp(-w'_j/t_j)\} = -\exp[R(C(q_j) - w)/t_j][A_1(h_0 + q_j)e^{A_2}]^{-\frac{1}{2}}$ with respect to q_j yields the first order condition

$$
2RC'(q_j) = t_j/(h_0 + q_j), \tag{18}
$$

which corresponds to Eq. (8) in Verrecchia [72]. This expression contrasts with Eq. (12) in that it does not feature $\bar{t}$, J or Ω. The number of shareholders and shares outstanding do not influence investors' best information responses, and the distribution of risk tolerance only matters indirectly through its impact on noisiness $N \equiv \rho/\overline{tq}$. Note that expected and non-expected utilities are related, for any given values of noisiness N and precisions q_j, through $E_{0j}(z_j^2) = E_{0j}(z_j)^2 + Var_{0j}(z_j)$ which implies $A \equiv A_1(A_2 + 1)$.

B.2. CRRA expected utility

In this appendix, we show that the main findings of the paper are not limited to non-expected utility maximizers. They also obtain under expected utility when one departs from the usual CARA structure. We solve for investors' demand for information under constant relative risk aversion (CRRA) expected utility. We show that investors reduce the precision of their signal when the base expands, holding noisiness fixed. Specifically, we assume investors have a coefficient of relative risk aversion a, i.e. they maximize $E_0\{E_1[w_j'^{1-a}/(1 - a) \mid \mathcal{F}_j]\}$, and are

endowed with identical initial wealth w. We show that their optimal precision q is a decreasing function of the number of shareholders. This is consistent with the Kreps–Porteus CARA preferences used in this paper but inconsistent with the commonly used CARA expected utility (see Appendix B.1).[27] We solve for q numerically using the approach suggested by Bernardo and Judd [10] because no closed-form solution is known under CRRA. We proceed in the same two steps as in the theoretical analysis. First, we find the equilibrium price function in the trading period for any number of investors and level of noisiness N. Second, we fix an investor and determine her best information response function, i.e. how much information she collects (q) in the planning period given the number of investors, the level of noisiness N and the price that is expected to prevail in the trading period. We then vary the number of investors holding fixed N to trace out q.

In the first step, we project the price and the demand for stocks on a basis of orthogonal polynomials:

$$\widehat{P}(\Pi, \Theta) \equiv \sum_{i=0}^{n_d} \sum_{k=0}^{n_d} a_{ik} H_i(\Pi) H_k(\Theta) \quad \text{and} \quad \widehat{e}(P, s) \equiv \sum_{l=0}^{n_d} \sum_{m=0}^{n_d} b_{lm} H_l(P) H_m(s),$$

where H_i is the degree i Hermite polynomial and n_d represents the highest degree polynomial used in the approximation. Hermite polynomials are used because they are mutually orthogonal with respect to the standard normal density: $\int_R H_i(x) H_k(x) \exp(-x^2)\, dx = 0$ for all $i \neq k$. We search for the unknown coefficients a_{ik} and b_{lm} that satisfy the first order and equilibrium conditions. As in Bernardo and Judd [10], we use the complete set of polynomials rather than the full tensor product to reduce the number of unknowns from $2(n_d + 1)^2$ to $(n_d + 1)(n_d + 2)$. The investor's first order condition $E[(\Pi - RP)w_j'^{-a} \mid \mathcal{F}_j] = 0$ implies that $E[(\Pi - RP)w_j'^{-a} \varphi(s_j) \psi(P)] = 0$ for all continuous bounded functions φ and ψ. We approximate it numerically with the new conditions:

$$E\big[(\Pi - R\widehat{P})\widehat{w}_j'^{-a} H_m(s_j) H_l(\widehat{P})\big] = 0 \quad \text{for } l = 0, \dots, n_d \text{ and } m = 0, \dots, n_d - l + 1,$$

where $\widehat{w}_j' = (\Pi - R\widehat{P})\widehat{e}(\widehat{P}, s_j) + R(w - C(q))$. A few projections are sufficient to obtain a useful approximation. As for the market clearing condition $\int_j e(P, s_j)\, dG(t_j) - \rho J\Theta = \Omega$, it cannot be imposed in each and every state so it is assumed that the deviations from market clearing are orthogonal to several of the basis polynomials:

$$E\left[\left[\int_0^\infty \widehat{e}(\widehat{P}, s_j)\, dG(t) - \rho J\Theta - \Omega\right] H_i(\Pi) H_k(\Theta)\right] = 0$$

for $i = 0, \dots, n_d$ and $k = 0, \dots, n_d - l + 1$.

In this fashion, the equilibrium problem has been reduced to a system of $(n_d + 1)(n_d + 2)$ equations in $(n_d + 1)(n_d + 2)$ unknowns.

In the second step, we fix an investor j and determine her information response function, i.e. the precision q that maximizes $E[\widehat{w}_j'^{1-a}/(1 - a)]$. This expectation, as well as the others, are computed using gaussian quadrature techniques (with 5 gridpoints). We then vary the number of investors keeping noisiness N fixed and repeat the two steps. As a measure of noisiness, we

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[27] As mentioned in the discussion following Theorem 2, the optimal precision is independent from aggregate risk tolerance and therefore from the number of shareholders under CARA expected utility.

use $E[Var(\Pi \mid P)]$, the variance of the payoff conditional on the price signal, averaged over all price realizations.[28] We start by projecting the conditional expectation $E(\Pi \mid P)$ on the basis of Hermite polynomials, $E(\Pi \mid P) = \sum_{l=0}^{n_d} c_l H_l(P)$. Since the conditional expectation is defined by $E[(\Pi - E(\Pi \mid P))\varphi(P)] = 0$ for any continuous bounded function φ, we numerically approximate it with the conditions:

$$E\big[\big(\Pi - E(\Pi \mid \widehat{P})\big)H_l(\widehat{P})\big] = 0 \quad \text{for } l = 0, \dots, n_d.$$

Then, we compute $E[Var(\Pi \mid P)] = Var(\Pi) - Var[E(\Pi \mid P)] = \sigma_\Pi^2 - \sum_{l=0}^{n_d} c_l^2$ where $Var[E(\Pi \mid P)] = \sum_{l=0}^{n_d} c_l^2$ follows from the fact that the H_l form a basis of orthogonal polynomials. As investors enter the market, we adjust the precision of their signal to keep noisiness constant.

The optimal q is plotted in Fig. 3 for CRRA expected utility with $a = 7$ and $a = 10$ and for CARA expected utility with a coefficient of absolute risk aversion of 7. The other parameter values are $\sigma_\Pi^2 = \overline{\Pi} = 0.1$, $\sigma_\Theta^2 = \rho = 1$, $N = 10$, $\Omega = 0.5$, $w = 10$, $R = 1.03$, $C(q) = 0.05q$ and $n_d = 3$. As depicted, q decreases with the number of investors under CRRA but remains constant under CARA expected utility, supporting the claim that the extent of risk sharing generally limits the production of information.

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[28] While this definition is reasonable, it may not perfectly capture noisiness. Indeed, other moments of the distribution of Π conditional on P may matter too because this distribution may not be normal. In their analysis of a Grossman–Stiglitz economy with CRRA expected utility and log-normal returns, Bernardo and Judd [10] argue that various definitions of noisiness lead to qualitatively identical behaviors (p. 29).

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