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Manufacturer's control of retail price

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In the supply chain, decision on retail price can be made by the manufacturer or retailer. We study the scenario where it is the manufacturer who controls retail price. The manufacturer announces wholesale and retail prices, anticipating that the retailer will act as a newsvendor. We characterize the equilibrium for this general scenario and provide closed form solutions to commonly used stochastic demand models with multiplicative uncertainty.

Key words: Resale price maintenance, newsvendor, supply chain, uncertain demand.

1. Introduction

Within the flourishing research in Supply Chain Management over the past two decades, one of the essential issues arising among the supply chain entities is pricing, including retail pricing (price paid by the final customer) and pricing between two supply chain entities. The typical assignment of decision of retail price in the academic literature is that it is done by the retailer. However, in many cases the manufacturer has the major influence on this decision. In this paper, we formulate and analyze a parsimonious model of this setting with the newsvendor model as a building block.

The manufacturer's suggested retail price (also called "sticker price" in auto industry), list price, or recommended retail price are quite common, as well as minimum advertised price and, more generally, resale price maintenance (RPM).[1] As a few examples, Oakley in N. America sanctions retailers who sell their sunglasses below a suggested retail price. The European Court of Justice in 2001 banned Tesco from discounting Levi's jeans and upheld the manufacturer's right to determine the distribution strategy for their products. More recently, L'Oreal prevailed against Nille in Norway

[1] A general description is at http://en.wikipedia.org/wiki/Resale_price_maintenance; for RPM examples see Verge (2008).

on similar grounds.[2]

In general, RPM specifies price floor and/or price ceiling. Price ceiling helps to avoid double marginalization in the case of a monopolistic retailer, and is not an issue when several retailers compete on price. Also, in all examples mentioned above, the retailers were sanctioned for selling below a suggested retail price, and not above it. This suggests that under RPM the price floor is essentially binding, and setting a fixed retail price or price floor are equivalent.

We investigate manufacturer's control of retail price from the operations and supply chain perspective, focusing on price-only contracts and demand uncertainty. In the setting considered, the manufacturer's contract specifies the wholesale price and retail price, and the retailer (newsvendor) applies its best response, ordering the quantity prior to the resolution of demand uncertainty. Our model directly extends to the case of multiple retailers competing on price — as mentioned, in this case a fixed retail price is equivalent to price floor, and fixed retail price alleviates retailers' competition, allowing us to focus on a single retailer.

Our work is related to two streams of literature – operations literature on supply chain and economics literature on vertical restraints and RPM. Much of operations management literature builds on the newsvendor model and, to the best of our knowledge, the setting where the manufacturer sets both the wholesale and retail price and the retailer operates as a newsvendor has not been studied.[3] Lariviere and Porteus (2001) consider a manufacturer selling to a single newsvendor, where the manufacturer sets the wholesale price acting as a Stackelberg leader. This setting also corresponds to "push" price-only contract from Cachon (2004). We adopt a similar notion of setting wholesale price. When the manufacturer controls retail price (in addition to setting the wholesale price), the retailer becomes a newsvendor. This is different from Petruzzi and Dada (1999), Bernstein and Federgruen (2005), Yao et al. (2006), and Kocabiyikoglu and Popescu (2011), where it is

[2] Court judgments for Levi's and L'Oreal cases are provided at
http://curia.europa.eu/en/actu/communiques/cp01/aff/cp0158en.htm and
http://www.eftacourt.int/images/uploads/9_10_07_Judgment_E.pdf, correspondingly.

[3] Ha (2001) considers a setting with buyback contract and supplier's control of retail price, but the set of contracts and overall model there is very different from our price-only contract setting.

the newsvendor who sets the retail price in anticipation of uncertain market conditions and facing an exogenous wholesale price.

The economics papers on RPM under demand uncertainty and retail competition do not consider retailers as newsvendors, and the focus in these papers is on contracts that coordinate the whole supply chain, without restriction of price-only contracts. Krishnan and Winter (2007) and Vergé (2008) are two recent papers in this domain, that also provide a comprehensive literature review. Deneckere et al. (1996, 1997) consider a single manufacturer selling to a continuum of retailers.

2. Manufacturer sets retail price

We consider a single manufacturer selling to a single (risk-neutral) retailer. Market demand is given by $\tilde{D} = e^{\mu+\sigma\tilde{\varepsilon}}g(p)$, where p is retail price, $g(p)$ is deterministic demand curve, and $\tilde{\varepsilon}$ is a random shock.[4] This specification allows us to explore how the solution depends on the mean and standard deviation of the random shocks, captured by μ and σ. Note that this formulation of multiplicative demand shock is without loss of generality: If $\tilde{D} = \tilde{\xi}g(p)$, $\tilde{\xi} > 0$, then $\mu + \sigma\tilde{\varepsilon} = \ln(\tilde{\xi})$.

The manufacturer announces wholesale price w and retail price p, and then the retailer acts as a newsvendor and orders quantity q. The manufacturer's unit cost is c, and the retailer's salvage value is zero. [5]

The retailer's best response order quantity to the contract (w, p) is given by the well known newsvendor equation $\Pr\left(\tilde{D} < q\right) = \Pr\left(e^{\mu+\sigma\tilde{\varepsilon}}g(p) < q\right) = \frac{p-w}{p}$. The manufacturer's profit is $\Pi = (w-c)q$, and the challenge is to find w and p that would maximize it.

First consider the situation where $\sigma = 0$, i.e., demand is certain. In this case it is optimal to set $p = w$, so that the retailer's margin is zero, and the order quantity is $e^{\mu}g(w)$. Then w is set to maximize $(w-c)g(w)$. This is the price that the integrated channel would charge when demand is given by $g(w)$, and its first-order condition is $w + \frac{g(w)}{g'(w)} = c$. For $\sigma > 0$, the manufacturer has to set $p > w$, because otherwise the retailer will order nothing. Overall, the manufacturer faces the

[4] We use tilde to highlight random variables.

[5] This is without loss of generality, since if the salvage value is s, the solution is equivalent to our model with the manufacturer's unit cost $c_s = c - s$ and deterministic demand curve $g_s(p) = g(p+s)$. We provide the details in Section 3.

following tradeoffs: Increasing wholesale price w increases her margin, but decreases the retailer's quantity. Increasing retail price p stimulates retailer's incentive to order more by increasing the retailer's margin, but at the same time reduces demand since demand curve $g(p)$ is decreasing.

To ensure that the solution of the above problem is well-behaved, we need to make some assumptions about demand and the distribution of random shocks, stated below.

Assumption 1.

(i) Market demand is $\tilde{D} = e^{\mu+\sigma\tilde{\varepsilon}} g(p)$, *with* $g'(p) < 0$, *and manufacturer's marginal cost is* c, $c \geq 0$.

(ii) Random shock $\tilde{\varepsilon}$ *has cumulative distribution function (cdf)* F, $\overline{F} = 1 - F$, *probability density function (pdf)* f, *and increasing failure rate, i.e.,* $f(z)/\overline{F}(z)$ *is increasing in* z.

(iii) Demand becomes weakly more elastic with price, i.e., $-pg'(p)/g(p)$ *is nondecreasing with* p, *and* $-pg'(p)/g(p) > 1$ *for large enough* p.

(iv) If $c = 0$, *there is exactly one point* p_0, $p_0 > 0$, *at which* $-p_0 g'(p_0)/g(p_0) = 1$.

Assumption 1(i) summarizes our setting. Assumption 1(ii) assumes increasing failure rate of the shock $\tilde{\varepsilon}$, which holds for many distributions, including uniform and normal. This assumption is standard and guarantees that price-inventory optimization problem is well-behaved (Yao et al. 2006, Kocabiyikoglu and Popescu, 2011), and it is equivalent to assuming that the distribution of $e^{\mu+\sigma\tilde{\varepsilon}}$ has increasing generalized failure rate (Lariviere, 2006, Theorem 1). Assumption 1(iii) assumes that price elasticity is increasing, which is a standard assumption in economics and is satisfied by most forms of demand (Yao et al. 2006); assuming price elasticity greater than one for large enough p ensures that charging infinite price is suboptimal. Assumption 1(iv) is minor and technical, to avoid the situations where charging zero price is optimal. Theorem 1 below provides a closed-form solution for the optimal w and p to be set by the manufacturer, with all second-order conditions being satisfied.

Theorem 1. *Suppose Assumption 1 holds. Define* $p^*(z)$ *as the solution of the equation*

$$p^*(z) + \frac{g(p^*(z))}{g'(p^*(z))} = \frac{c}{\overline{F}(z)}, \tag{1}$$

and z^* as the solution of the equation

$$\frac{f(z^*)}{\overline{F}(z^*)}\left(-p^*(z^*)\frac{g'(p^*(z^*))}{g(p^*(z^*))}\right)=\sigma. \tag{2}$$

Then $p^*(z)$ is increasing in z for $c>0$, and z^* is increasing in σ. The manufacturer's profit is maximized at retail price $p^*=p^*(z^*)$ and wholesale price $w^*=p^*(z^*)\overline{F}(z^*)$. Retailer's order quantity is $q^*=e^{\mu+\sigma z^*}g(p^*(z^*))$. Both retail price and wholesale price do not depend on μ. Furthermore:

- *If $c>0$, retail price is strictly increasing with σ, and wholesale price is weakly decreasing in σ.*
- *If $c=0$, retail price is given by equation $-p^*g'(p^*)/g(p^*)=1$ and thus does not depend on σ; wholesale price is strictly decreasing in σ.*

Proof: Denote $z=F^{-1}\left(\frac{p-w}{p}\right)$. From $\Pr\left(e^{\mu+\sigma\tilde{\varepsilon}}g(p)<q\right)=\frac{p-w}{p}$, the retailer's order quantity is $q=e^{\mu+\sigma z}g(p)$. Then $w=p(1-F(z))=p\overline{F}(z)$ and the manufacturer's profit, as a function of p and z, is $\Pi(p,z)=(w-c)q=\left[p\overline{F}(z)-c\right]e^{\mu+\sigma z}g(p)$. Differentiating with respect to p yields

$$\begin{aligned}\frac{\partial\Pi(p,z)}{\partial p}&=\overline{F}(z)e^{\mu+\sigma z}g(p)+\left[p\overline{F}(z)-c\right]e^{\mu+\sigma z}g'(p)=\left[\overline{F}(z)(g(p)+pg'(p))-cg'(p)\right]e^{\mu+\sigma z}\\&=e^{\mu+\sigma z}\left(-g'(p)\right)\left[c-\overline{F}(z)\left(p+\frac{g(p)}{g'(p)}\right)\right].\end{aligned}$$

First-order condition is $p+\frac{g(p)}{g'(p)}=\frac{c}{\overline{F}(z)}$, which is equivalent to (1) with $p=p^*(z)$. To check that (1) indeed gives the unique maximum of $\Pi(p,z)$ for a fixed z as a function of p, consider two cases, $c>0$ and $c=0$. For $c>0$, observe that if $-pg'(p)/g(p)\le 1$ then $\frac{\partial\Pi(p,z)}{\partial p}>0$. If $-pg'(p)/g(p)>1$ then $p+\frac{g(p)}{g'(p)}=p\left(1-\frac{1}{-pg'(p)/g(p)}\right)$ is strictly increasing by Assumption 1.(iii), and therefore $\frac{\partial\Pi(p,z)}{\partial p}>(<)0$ for $p<(>)p^*(z^*)$. For $c=0$, by Assumption 1.(iv) there exists p_0, $-p_0g'(p_0)/g(p_0)=1$, such that $p+\frac{g(p)}{g'(p)}>(<)0$ for $p<(>)p_0$. By (1), $p_0=p^*(z^*)$ when $c=0$. Therefore, $\frac{\partial\Pi(p,z)}{\partial p}>(<)0$ for $p<(>)p^*(z^*)$.

Now define $p^*(z)$ as the solution of (1), and consider manufacturer's profit as a function of z:

$$\Pi(p^*(z),z)=\left[p^*(z)\overline{F}(z)-c\right]e^{\mu+\sigma z}g(p^*(z)).$$

By the envelope theorem,

$$\frac{d\Pi(p^*(z),z)}{dz}=\left[\sigma(p^*(z)\overline{F}(z)-c)-f(z)p^*(z)\right]e^{\mu+\sigma z}g(p^*(z)).$$

From (1), $p^*(z)\overline{F}(z) - c = -\overline{F}(z)\frac{g(p^*(z))}{g'(p^*(z))}$, and therefore

$$\begin{aligned}\frac{d\Pi(p^*(z),z)}{dz} &= \left[-\sigma\overline{F}(z)\frac{g(p^*(z))}{g'(p^*(z))} - f(z)p^*(z)\right]e^{\mu+\sigma z}g(p^*(z)) \\ &= \left[\sigma - \frac{f(z)}{\overline{F}(z)}\left(-p^*(z)\frac{g'(p^*(z))}{g(p^*(z))}\right)\right]e^{\mu+\sigma z}g(p^*(z))\overline{F}(z)\left(-\frac{g(p^*(z))}{g'(p^*(z))}\right).\end{aligned}$$

By (1), $p^*(z)$ is increasing, and thus by Assumption 1.(iii) $\left(-p^*(z)\frac{g'(p^*(z))}{g(p^*(z))}\right)$ is increasing in z; $\frac{f(z)}{\overline{F}(z)}$ is increasing by Assumption 1.(ii), and therefore $\frac{f(z)}{\overline{F}(z)}\left(-p^*(z)\frac{g'(p^*(z))}{g(p^*(z))}\right)$ is increasing in z. That implies that the second-order condition holds, profit is maximized at z^* given by (2), and that z^* is increasing with σ.

To conclude, the solution is the following: 1) Find $p^*(z)$ from (1) and find z^* from (2). 2) Retail price is $p^*(z^*)$, wholesale price is $w^* = p^*(z^*)\overline{F}(z^*)$, and order quantity is $q = e^{\mu+\sigma z^*}g(p^*(z^*))$. It remains to consider the effect of changing σ. If $c = 0$, $p^*(z^*) = constant$ by (1) and $w^* = p^*(z^*)\overline{F}(z^*)$ is decreasing in σ because, by (2), z^* is increasing in σ. If $c > 0$, $p^*(z^*)$ is increasing in σ (since z^* is increasing in σ) and from (1) the wholesale price $w^* = p^*(z^*)\overline{F}(z^*)$ can be expressed as $w^* = c - w^*\frac{g(p^*)}{p^*g'(p^*)}$. From here $w^* = \frac{c}{1-\left(-\frac{g(p^*)}{p^*g'(p^*)}\right)}$ which is decreasing in σ by Assumption 1.(iii) and because p^* is increasing in σ. Note that if price elasticity is constant, i.e., $-pg'(p)/g(p) = constant$, then w^* does not change with σ. $\square$

By Theorem 1, if manufacturer's marginal cost is zero (i.e., $c = 0$), retail price is the same as for the seller facing deterministic demand $g(p)$, regardless of σ. However, as uncertainty about demand increases, manufacturer lowers the wholesale price in order to increase the retailer's margin. If marginal cost is positive (i.e., $c > 0$), then retail price is also increasing with σ. Indeed, the wholesale price cannot go below the marginal cost, and thus the retail price has to increase in order to make retailer's margin large enough. As follows from the proof of Theorem 1, if demand elasticity is constant (i.e., demand is isoelastic), the wholesale price does not change with σ.

3. Examples

Theorem 1 holds for general demand $g(p)$. We now illustrate it showing closed form solutions for three forms of demand often studied in the literature, namely linear, exponential (log-linear), and isoelastic. (Wu et al. (2011) and references therein discuss the use and empirical support of these demand models.) Price elasticity is strictly increasing for linear and exponential demand, and is constant for isoelastic demand. Therefore, wholesale price is strictly decreasing with σ for linear and exponential demand, and is independent of σ for isoelastic demand. We also accommodate a nonzero (retailer's) salvage value s. For $s > 0$, equations in Theorem 1 become

$$p^*(z) - s + \frac{g(p^*(z))}{g'(p^*(z))} = \frac{c-s}{\overline{F}(z)}, \tag{3}$$

$$\frac{f(z^*)}{\overline{F}(z^*)}\left(-(p^*(z^*) - s)\frac{g'(p^*(z^*))}{g(p^*(z^*))}\right) = \sigma, \tag{4}$$

and the equation for w^* is

$$w^* = s + (p^*(z^*) - s)\overline{F}(z^*). \tag{5}$$

Note that if $\sigma = 0$ then $\overline{F}(z^*) = 1$ and, by (3), $p^*(z^*)$ does not depend on s.

Example 1. *Linear demand:* $g(p) = 1 - \gamma p$, $\gamma > 0$.

From (3), $p^*(z) = \frac{1}{2}\left(s + \frac{1}{\gamma} + \frac{c-s}{\overline{F}(z)}\right)$, and equation (4) for z^* is $\frac{f(z^*)}{\overline{F}(z^*)}\left(\frac{2-2\gamma s}{1-\gamma s-\gamma(c-s)/\overline{F}(z^*)} - 1\right) = \sigma$.
By (5), wholesale price $w^* = s + (p^*(z^*) - s)\,\overline{F}(z^*) = s + \frac{1}{2}\left(\frac{1}{\gamma} + \frac{c-s}{\overline{F}(z)} - s\right)\overline{F}(z^*) = \frac{1}{2}\left(c + s + \left(\frac{1}{\gamma} - s\right)\overline{F}(z^*)\right)$ is decreasing with z^* and thus with σ.

Example 2. *Exponential demand:* $g(p) = e^{-\gamma p}$, $\gamma > 0$.

From (3), $p^*(z) = s + \frac{1}{\gamma} + \frac{c-s}{\overline{F}(z)}$, and equation (4) for z^* is $\frac{f(z^*)}{\overline{F}(z^*)}\left(1 + \frac{\gamma(c-s)}{\overline{F}(z)}\right) = \sigma$.
By (5), wholesale price $w^* = s + (p^*(z^*) - s)\,\overline{F}(z^*) = s + \left(\frac{1}{\gamma} + \frac{c-s}{\overline{F}(z)}\right)\overline{F}(z^*) = c + \frac{\overline{F}(z^*)}{\gamma}$ is decreasing with z^* and thus with σ.

Example 3. *Isoelastic demand:* $g(p) = p^{-\gamma}$, $\gamma > 1$, $c > 0$.

From (3), $p^*(z) = \frac{\gamma}{\gamma-1}\left(s + \frac{c-s}{\overline{F}(z)}\right)$, and equation (4) for z^* is $\frac{f(z^*)}{\overline{F}(z^*)}\left(1 - \frac{\gamma-1}{\gamma}\frac{s}{s+(c-s)/\overline{F}(z^*)}\right) = \frac{\sigma}{\gamma}$.
By (5), wholesale price $w^* = s + (p^*(z^*) - s)\,\overline{F}(z^*) = \frac{\gamma}{\gamma-1}(c-s) + s\left(1 + \frac{1}{\gamma-1}\overline{F}(z^*)\right)$, and for $s = 0$ w^* does not depend on z^* and thus on σ either.

4. Conclusions

We contribute to the supply chain literature by formulating and analyzing a parsimonious model where the manufacturer sets retail price via price-only contract for the retailer operating as a newsvendor. Theorem 1 analyzes this problem under general demand curve with multiplicative random shocks. If demand is known (i.e., standard deviation of the random shocks is zero), the channel is coordinated with the retail price equal to the wholesale price and equal to the price that the monopolistic seller would set, in the absence of retail intermediary. As uncertainty about demand increases, wholesale price goes down (stays constant if demand is isoelastic) and retail price goes up (stays constant if the manufacturer's marginal cost is zero). This is because the manufacturer has to provide higher retail margin to compensate the retailer for holding the inventory.

As mentioned in the introduction, much of operations management literature builds on newsvendor model. Our model of manufacturer's controlling retail price in a newsvendor setting provides a building block that can be used and extended in many directions, such as different types of price-only contracts (Cachon, 2004), settings with asymmetric information, competing manufacturers and retailers, and multiperiod models.

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