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On Equivalent Target-Oriented Formulations for Multiattribute Utility

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Targets are used quite often as a management tool, and it has been argued that thinking in terms of targets may be more natural than thinking in terms of utilities. The standard expected-utility framework with a single attribute (such as money) and nondecreasing, bounded utility is equivalent to a target-oriented setting. A utility function, properly scaled, can be expressed as a cumulative distribution function (cdf) and related to the probability of meeting a target value. We consider whether the equivalence of the two approaches extends to the case of multiattribute utility. Our analysis shows that a multiattribute utility function cannot always be expressed in the form of a cumulative distribution function and, furthermore, cannot always be expressed in the form of a target-oriented utility function. However, in each case equivalence does hold for certain well-known classes of utility functions. In general, our results imply that although interpreting utility as a cdf and thinking about achieving targets works fine in the case of a single attribute, this approach should be used with caution in the multiattribute case, with cdf representations requiring more caution than target-oriented representations.

Key words: target-oriented utility; utility and probability; multiattribute utility; multiattribute performance targets
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1. Introduction

The use of targets, even short-term targets involving daily or weekly sales, revenues, and costs, is widespread in business. Target-oriented utility involves interpreting an increasing, bounded utility function, properly scaled, as a cumulative distribution function (cdf) and relating it to the probability of meeting or exceeding a target value. This can always be done in the univariate case. In this paper, we focus on the multiattribute case and investigate whether we can find an equivalent target-oriented formulation corresponding to a multiattribute utility function, considering two types of equivalence. First, can any multiattribute utility function with n attributes be interpreted as an n-dimensional cdf and related to the probability of achieving all n targets? Second, can any multiattribute utility function be expressed in terms of a target-oriented setting with targets for each attribute, where utility takes on different values depending on which subset of targets is achieved? The answer to each of these questions is negative, although in each case equivalence does hold for certain classes of utility functions.

Borch (1968) may have been the first to present the concept of target-oriented utility, using terminology related to the probability of ruin. Berhold (1973) notes that "there are advantages to having the utility function represented by a distribution function" (p. 825), arguing that it permits the use of known properties of distribution functions to find analytical results. Manski (1988) calls this the "utility mass model." Castagnoli and Li Calzi (1996) note that the "expected utility model need not be based on the notion of a cardinal utility function over prizes and can in fact be entirely phrased in the language of probability" (p. 281).

More recent work pursues the notion of target-oriented utility in two directions, considering possible practical advantages of the target-oriented framework and developing the framework in greater depth.

Bordley and Li Calzi (2000) argue that a target-based language for decision analysis has some practical advantages over the equivalent traditional utility-based language. For example, they claim that the notion of a target is more natural and easier to understand than the more esoteric concept of a utility function. Bordley (2001) suggests using a target-oriented approach to teach decision theory. Abbas and Matheson (2004, 2005) explore the duality between probability and utility functions, taking standard properties and results from the primal problem and finding their counterparts in the dual problem. This leads to new concepts such as aspiration equivalents and utility dominance, which correspond to certainty equivalents and stochastic dominance.

In many decision-making situations, multiple attributes are of interest, so it is important to know whether the basic target-oriented results extend to the multiattribute case. Bordley and Kirkwood (2004) develop a target-oriented approach to assess a multiattribute preference function. Abbas and Howard (2005) introduce a class of multiattribute utility functions called attribute dominance utility functions, which can be manipulated like joint probability distributions and allow the use of probability assessment methods in utility elicitation. Taking a different tack, Tsetlin and Winkler (2006) consider decision making in a multiattribute target-oriented setting and study the impact on expected utility of changes in parameters of performance and target distributions (i.e., location, spread, and dependence).

As noted above, this paper considers two distinct types of equivalence. In §2, we investigate whether a multiattribute utility function with n attributes can always be interpreted as an n-dimensional cdf and therefore related to the probability of achieving all n targets. In §3, we ask whether a multiattribute utility function can be expressed in terms of a target-oriented setting with targets for each attribute, where utility takes on different values depending on which subset of targets is achieved. Note that the conditions in §2 are stronger than those in §3; any multiattribute utility function that can be interpreted as a cdf can be expressed in terms of a target-oriented setting, where the utility equals one if all targets are met and zero otherwise. A summary and a brief discussion are given in §4.

2. Multiattribute Utility as a Multivariate CDF

Suppose that the consequences of a decision can be represented by a single attribute, x (e.g., wealth), and that the decision maker's utility for x is given by a bounded, nondecreasing function $U(x)$, scaled such that $0 \le U(x) \le 1$. The expected utility for an action a is

$$EU(a) = \int_{-\infty}^{\infty} U(x) f(x \mid a)\, dx, \tag{1}$$

where $f(x \mid a)$ is the probability density function (pdf) of $\tilde{x}$ under a. Since U has the same properties as a cdf, we can write

$$\begin{aligned} EU(a) &= \int_{-\infty}^{\infty} P(\tilde{t} \le x) f(x \mid a)\, dx \\ &= \int_{-\infty}^{\infty} F_t(x) f(x \mid a)\, dx, \end{aligned} \tag{2}$$

where $\tilde{t}$ is independent of $\tilde{x}$ and has cdf $F_t = U$ (Castagnoli and Li Calzi 1996). Here $\tilde{t}$ can be interpreted as a target. In a natural target-oriented situation, the decision maker's utility is one if the target is met (i.e., $x \ge t$) and zero otherwise, and therefore the implied utility function for x is $F_t(x)$, which is the probability of meeting the target given performance x. Note that even if a real target $\tilde{t}$ does not exist, (1) and (2) are equivalent, and we can view $\tilde{t}$ as if it were a target with cdf $U(x)$.

The consequences in decision making often involve multiple attributes (Keeney and Raiffa 1976), and Bordley and Kirkwood (2004) give examples of situations with multiple targets. Suppose that the consequences of a decision are represented by a vector $\mathbf{x} = (x_1, x_2, \ldots, x_n)$ of n attributes and that the decision maker's utility function $U(\mathbf{x})$ is bounded and scaled such that $0 \le U(\mathbf{x}) \le 1$, with U nondecreasing in each of its arguments. The expected utility for an action a is

$$EU(a) = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} U(\mathbf{x}) f(\mathbf{x} \mid a)\, dx_1 \cdots dx_n. \tag{3}$$

However, unlike the single-attribute situation, there may not be an equivalent formulation involving a multivariate cdf F_t of a vector of n targets $\tilde{\mathbf{t}} = (\tilde{t}_1, \tilde{t}_2, \ldots, \tilde{t}_n)$ for the n attributes comprising $\mathbf{x}$,

$$EU(a) = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} F_t(\mathbf{x}) f(\mathbf{x} \mid a)\, dx_1 \cdots dx_n, \tag{4}$$

with $\tilde{\mathbf{t}}$ independent of $\tilde{\mathbf{x}}$. Note that for a given $\mathbf{x}$, $F_t(\mathbf{x})$ is the probability of achieving all n targets.

Proposition 1. *The expected utility in* (3) *does not necessarily have an equivalent formulation in which the utility function can be expressed in terms of a cdf, as in* (4).

Proof. Consider the additive utility function $U(\mathbf{x}) = \sum_{i=1}^{n} k_i U_i(x_i)$, with U_i nondecreasing and differentiable, $0 \le U_i(x_i) \le 1$, $k_i > 0$ for $i = 1, 2, \ldots, n$, and $\sum_{i=1}^{n} k_i = 1$. If U can be represented as a cdf $F_t(\mathbf{x})$, then the corresponding pdf is $\partial^n U(\mathbf{x})/\partial x_1 \cdots \partial x_n$. But $\partial^n U(\mathbf{x})/\partial x_1 \cdots \partial x_n = 0$ for all $\mathbf{x}$, which is not a proper pdf. □

Proposition 1 shows that a key result in Castagnoli and Li Calzi (1996), that an expected utility in standard form (1) can always be expressed as in (2), with the utility function represented as a cdf, does not generalize to the multiattribute case. Although every nondecreasing and bounded U in the single-attribute case corresponds to a proper cdf when scaled appropriately, this is not true in the multiattribute case.

Proposition 2. *A multiplicative utility function*, $U(\mathbf{x}) = \prod_{i=1}^{n} U_i(x_i)$, *with* U_i *nondecreasing and* $0 \le U_i(x_i) \le 1$ *for* $i = 1, \ldots, n$, *has an equivalent formulation in which the utility function can be expressed in terms of a cdf, as in* (4).

Proof. For $i = 1, \ldots, n$, define $F_{t_i}(x_i) = U_i(x_i)$ and observe that $F_{t_i}(x_i)$ is a cdf. Then $U(\mathbf{x}) = \prod_{i=1}^{n} F_{t_i}(x_i) = F_t(\mathbf{x})$ is a cdf with independent targets $\tilde{t}_1, \ldots, \tilde{t}_n$. □

The example in the proof of Proposition 1 shows that the commonly used additive utility is not consistent with the interpretation of utility in terms of a cdf. In contrast, Proposition 2 provides a more positive result, showing that multiplicative utility is consistent with such a probability-based formulation. In general, $U(\mathbf{x})$, scaled between 0 and 1 and nondecreasing in each of its arguments, can be represented as a cdf if (1) $U(\mathbf{x}) = 0$ whenever at least one attribute is set at its minimum, and (2) $\partial^n U(\mathbf{x})/\partial x_1 \cdots \partial x_n \ge 0$ for all $\mathbf{x}$. The first condition restricts $U(\mathbf{x})$ to the class of attribute dominance utility (Abbas and Howard 2005), which is necessary but not sufficient for $U(\mathbf{x})$ to be a cdf.

3. Equivalent Target-Oriented Formulations for Multiattribute Utility

The previous section shows that quite often multiattribute utility cannot be expressed as a multivariate cdf. This section considers whether every multiattribute utility that is bounded and nondecreasing in each of its arguments can be thought of as coming from a target-oriented setting where utility takes different values depending on which subset of targets is achieved. First, we introduce target-oriented utility.

Following Bordley and Kirkwood (2004), we say that a decision maker is target oriented if her utility depends only on the subset of attributes for which the targets are met (i.e., for which $x_i \ge t_i$). Let $\mathbf{I} = (I_1, \ldots, I_n)$, where $I_i = 1$ if $x_i \ge t_i$ and 0 otherwise. A target-oriented decision maker has a utility function $U_{\mathbf{I}}(\mathbf{I})$ assigning utilities to the 2^n possible values of $\mathbf{I}$. Let $U_{\mathbf{I}}(\mathbf{I}) = u_A$, where A is the set of indices $\{i \mid I_i = 1\}$ corresponding to the attributes in $\mathbf{I}$ for which the target is met. For example, $U_{\mathbf{I}}(1, 0, \ldots, 0) = u_1$, $U_{\mathbf{I}}(0, 1, 1, 0, \ldots, 0) = u_{2,3}$, and so on. If $A_1 \subseteq A_2$, then $u_{A_1} \le u_{A_2}$; utility can never be reduced by meeting additional targets. Also, $0 \le u_A \le 1$ for all A, with $u_\varnothing = U_{\mathbf{I}}(0, \ldots, 0) = 0$ and $u_{1,2,\ldots,n} = 1$, leaving $2^n - 2$ utilities u_A to be assessed.

From $U_{\mathbf{I}}$, the induced $U(\mathbf{x})$ can be found for any distribution of $\tilde{\mathbf{t}}$. If $n = 2$,

$$U_{\mathbf{I}}(\mathbf{I}) = u_1 I_1 + u_2 I_2 + (1 - u_1 - u_2) I_1 I_2. \tag{5}$$

Recall that I_i depends on whether $x_i \ge t_i$ and that $\tilde{\mathbf{t}}$ is independent of $\tilde{\mathbf{x}}$. Integrating out the uncertainty about $\tilde{\mathbf{t}}$, we get

$$U(\mathbf{x}) = u_1 F_{t_1}(x_1) + u_2 F_{t_2}(x_2) + (1 - u_1 - u_2) F_{t_1, t_2}(x_1, x_2), \tag{6}$$

where F_{t_i} is the cdf of $\tilde{t}_i$ and F_{t_1, t_2} is the joint cdf of $\tilde{\mathbf{t}}$. This looks almost multilinear, and it is if $F_{t_1, t_2}(x_1, x_2) = F_{t_1}(x_1) F_{t_2}(x_2)$, i.e., if $\tilde{t}_1$ and $\tilde{t}_2$ are independent.

Can we extend this approach to the case of n targets? As in (5), $U_{\mathbf{I}}$ can be expressed as a weighted average of the products of I_i terms for the $2^n - 1$ combinations of such terms, not including the empty set. Then, as in (6), $U(\mathbf{x})$ is a weighted average of corresponding $F_{\{t_i \mid i \in A\}}(\{x_i \mid i \in A\})$ terms:

$$U(\mathbf{x}) = \sum_A w_A F_{\{t_i \mid i \in A\}}(\{x_i \mid i \in A\}), \tag{7}$$

with $\sum_A w_A = 1$. The weight w_A is a linear combination of u_B terms, with $w_A = u_A$ as a special case. In (6), for example, $w_i = u_i$ for $i = 1, 2$, but $w_{1,2} = u_{1,2} -$

$u_1 - u_2$. Note that the setting of §2 is a special case of (7) with $w_{1,1,\ldots,1} = 1$ and $w_A = 0$ for $A \neq \{1, 1, \ldots, 1\}$. However, just as $U(\mathbf{x})$ cannot always be represented as a cdf, it cannot always be represented as in (7). Proposition 3 specifies a class of utility functions that cannot be expressed as target-oriented utility.

Proposition 3. *If*

$$\frac{\partial^n U(\mathbf{x}')}{\partial x_1 \cdots \partial x_n} < 0 \quad \text{and} \quad \frac{\partial^n U(\mathbf{x}'')}{\partial x_1 \cdots \partial x_n} > 0$$

for some $\mathbf{x}'$ *and* $\mathbf{x}''$, *then* $U(\mathbf{x})$ *cannot be presented in the form* (7).

Proof. Assume the contrary, i.e., that there exists a cdf $F_t(\mathbf{x})$ such that $U(\mathbf{x})$ has the form (7). Then $\partial^n U(\mathbf{x})/\partial x_1 \cdots \partial x_n = w_{1,\ldots,n} \partial^n F_t(\mathbf{x})/\partial x_1 \cdots \partial x_n$. For $\partial^n F_t(\mathbf{x})/\partial x_1 \cdots \partial x_n$ to be a pdf, it should be nonnegative for all $\mathbf{x}$, and therefore $\partial^n U(\mathbf{x})/\partial x_1 \cdots \partial x_n$ should have the same sign for all $\mathbf{x}$, a contradiction. □

For $n = 2$, $U(\mathbf{x}) = x_1 x_2[1 + (1 - x_1)x_2]$, $0 \leq x_1 \leq 1$, $0 \leq x_2 \leq 1$, illustrates Proposition 3. It is increasing in x_1 and x_2 but, per Proposition 3, cannot be presented in the target-oriented form (7) since $\partial^2 U(\mathbf{x})/\partial x_1 \partial x_2 = 1 + 2x_2 - 4x_1 x_2$ is positive for some $\mathbf{x}$ [e.g., $(0, 0)$] and negative for other $\mathbf{x}$ [e.g., $(1, 1)$]. Also, note that this U satisfies the conditions of Abbas and Howard (2005) for attribute dominance utility, but it is not a cdf. Moreover, it is a member of the class of generalized multiplicative utility functions (Bell 1979). To extend the example to $n > 2$, let $U(\mathbf{x}) = x_1 x_2[1 + (1 - x_1)x_2]x_3 x_4 \ldots x_n$, with $0 \leq x_i \leq 1$ for $i = 1, \ldots, n$.

Proposition 3 says that a necessary condition for a utility function $U(\mathbf{x})$ to have a target-oriented formulation is that $\partial^n U(\mathbf{x})/\partial x_1 \cdots \partial x_n$ has the same sign for all $\mathbf{x}$. As the next proposition shows, this is not a sufficient condition.

Proposition 4. *Multiattribute utility* $U(\mathbf{x})$ *with*

$$\frac{\partial^n U(\mathbf{x})}{\partial x_1 \cdots \partial x_n} \geq 0$$

does not necessarily have an equivalent target-oriented representation (7).

Proof. Consider $U(\mathbf{x}) = (x_1^2 + x_1 x_2)/2$, with $0 \leq x_1 \leq 1$, $0 \leq x_2 \leq 1$. If $U(\mathbf{x})$ could be represented in the form (7), then either $F_{t_1, t_2}(x_1, x_2) = (x_1^2 + x_1 x_2)/2$, which corresponds to $w_1 = w_2 = 0$ and $w_{1,2} = 1$, or $F_{t_1, t_2}(x_1, x_2) = x_1 x_2$ and $F_{t_1}(x_1) = x_1^2$, which corresponds to $w_1 = w_{1,2} = 1/2$ and $w_2 = 0$. The first case is ruled out because $(x_1^2 + x_1 x_2)/2$ is not a proper cdf, and the second is ruled out because $F_{t_1, t_2}(x_1, x_2) = x_1 x_2$ implies $F_{t_1}(x_1) = x_1$, a contradiction. To demonstrate the result for general n, take $U(\mathbf{x}) = (x_1^2 + x_1 x_2 + x_3 + x_4 + \cdots + x_n)/n$, with $0 \leq x_i \leq 1$ for $i = 1, \ldots, n$. □

The utility function in the proof of Proposition 4 is a member of the class of two-attribute utility functions exhibiting one-way utility independence (Bell 1979). Note that it satisfies the second condition given in §2 for $U(\mathbf{x})$ to be represented as a cdf but not the first condition (attribute dominance utility). On a more positive note, Proposition 5 shows two important classes of utility functions that can be expressed in target-oriented form.

Proposition 5. *Additive and multilinear utility functions have equivalent target-oriented representations.*

Proof. An additive utility function

$$U(\mathbf{x}) = \sum_{i=1}^{n} k_i U_i(x_i),$$

with U_i nondecreasing and differentiable, $0 \leq U_i(x_i) \leq 1$, $k_i > 0$ for $i = 1, 2, \ldots, n$, and $\sum_{i=1}^{n} k_i = 1$ has an equivalent target-oriented representation with marginal cdfs $F_{t_i}(x_i) = U_i(x_i)$, $w_i = k_i$ for $i = 1, \ldots, n$, and $w_A = 0$ otherwise. A multilinear utility function

$$\begin{aligned} U(\mathbf{x}) = &\sum_{i=1}^{n} k_i U_i(x_i) + \sum_{i=1}^{n} \sum_{j>i} k_{ij} U_i(x_i) U_j(x_j) \\ &+ \sum_{i=1}^{n} \sum_{j>i} \sum_{l>j} k_{ijl} U_i(x_i) U_j(x_j) U_k(x_k) \\ &+ \cdots + k_{123\ldots n} \prod_{i=1}^{n} U_i(x_i), \end{aligned}$$

with U_i nondecreasing and differentiable and $0 \leq U_i(x_i) \leq 1$ for $i = 1, \ldots, n$, and $\sum_A k_A = 1$ can be expressed in the form (7) with $w_A = k_A$ and $F_{t_i}(x_i) = U_i(x_i)$ when $\tilde{t}_1, \tilde{t}_2, \ldots, \tilde{t}_n$ are independent. □

Proposition 5 illustrates the fact that the conditions for expressing $U(\mathbf{x})$ as a cdf are more stringent than the conditions for $U(\mathbf{x})$ to have an equivalent target-oriented representation. Additive and multilinear $U(\mathbf{x})$ cannot be expressed as cdfs, but each has an equivalent target-oriented representation. Bell

(1979, Table I) discusses six different classes of utility functions, from which only additive and multilinear utility (those satisfying additive and joint utility independence, respectively) admit target-oriented representations. Note that additive utility specifies only marginal distributions $F_{t_i}(x_i)$, so that the dependence structure in $F_t(\mathbf{x})$ is not restricted. Multilinear utility with $k_{123\ldots n} \neq 0$, however, uniquely specifies a full joint distribution, since $F_t(\mathbf{x}) = \prod_{i=1}^{n} U_i(x_i) = \prod_{i=1}^{n} F_i(x_i)$, implying independent targets.

4. Conclusions

Castagnoli and Li Calzi (1996) and others have shown that the standard expected utility framework with a single attribute and increasing, bounded utility is equivalent to a target-oriented setting, where the utility function, properly scaled, can be expressed as a cdf. We investigate whether these results extend to the multiattribute case, focusing on multiattribute utility functions that are bounded and nondecreasing in each of their arguments.

First, we ask whether a multiattribute utility function can always be interpreted as a multivariate cdf and therefore be related to the probability of achieving all targets. Proposition 1 shows that the answer to this question is no. For example, the commonly used additive utility function cannot be represented in this manner. However, Proposition 2 shows that a multiplicative utility function can be so represented. In general, U can be represented as a cdf if (1) $U(\mathbf{x}) = 0$ if any of the attributes is set at its minimum (i.e., U is an attribute dominance utility function), and (2) its nth-order cross-partial derivative is nonnegative. Therefore, duality between utility functions and probability distributions (Abbas and Matheson 2004) is limited in the multivariate case to utility functions satisfying these restrictive conditions.

Next, we ask whether a multiattribute utility function can always be expressed in target-oriented utility form. The answer to this question is also no. Proposition 3 shows that if the nth-order cross-partial derivative of U changes sign, then U cannot be expressed in target-oriented utility form. And even if this cross-partial derivative is nonnegative everywhere, Proposition 4 shows that U might not have an equivalent target-oriented representation. As Proposition 4 illustrates, it is hard to come up with sufficient conditions for U to be expressed in target-oriented form. However, Proposition 5 indicates that widely used additive utility is consistent with the target-oriented formulation (7), as is multilinear utility if the targets are independent (see also Bordley and Kirkwood 2004, Tsetlin and Winkler 2006).

Some researchers suggest that working with cdfs is easier (or more natural) than working with utilities, that utility-probability duality leads to new interpretations and insights, and that thinking in terms of achieving targets is intuitively appealing. For example, see Berhold (1973), Bordley and Li Calzi (2000), Bordley (2001), Abbas and Matheson (2004, 2005), and Abbas and Howard (2005). Apart from mathematics, then, why can't all utility functions be represented by cdfs and expressed in target-oriented form? There are circumstances where such representations work mathematically and may lead to new interpretations. For example, multiplicative utility mimics the product form generated for a cdf exhibiting independence, but we feel that utility (measuring preferences) and probability (representing uncertainty) are, at heart, two entirely different animals. The fact that a joint probability is zero when any of the individual events is impossible seems intuitively reasonable in a universal sense, in addition to being technically correct. The use of a multiplicative utility function that is zero whenever any of its attributes is at its minimum is a matter of one's personal preferences, although it is quite limiting in that it means that no tradeoffs are possible that would cause *any* attribute to reach its minimum.

We have noted that the conditions for expressing multiattribute utility in target-oriented form are not as stringent as those for expressing multiattribute utility as a cdf. At a basic level, the target-oriented form involves the utilities of achieving different combinations of targets and therefore amounts to comparing utilities with utilities instead of trying to think of utilities in terms of probabilities. Perhaps that helps to explain why some forms, such as additive utility, can be expressed in target-oriented form but not in cdf form. In general, our results imply that although interpreting utility as a cdf (as in §2) and thinking about achieving targets (as in §3) work fine in the case of a single attribute, this approach should be used with caution in the multiattribute case, with cdf representations requiring more caution than target-oriented representations.

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