

Assortment Optimization under a Random Swap based Distribution over Permutations Model

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Assortment planning is an important problem that arises in many industries such as retailing and airlines where one of the key challenges is to identify the “right model” for the consumer preferences and substitution behavior. Distribution over preference lists or permutations is the most general framework for modeling preferences but is intractable in general. In this paper, we present a parsimonious distribution over permutations model that is induced by *random swaps* from a central preference list (which we also refer to as the *prototype list*). In particular, a random list from this distribution can be sampled as follows: sample N from a given distribution on the number of swaps and starting from the initial prototype list, perform N random swaps. A random swap operation consists of selecting a random pair of items in the current list and swapping their positions. We consider two types of random swaps: *i*) swapping an arbitrary pair of items, and *ii*) swapping an adjacent pair of items. More precisely, a pair is picked uniformly at random out of all $\binom{n+1}{2}$ possible pairs in *i*), and out all n pairs of adjacent items in *ii*). If the distribution over the number of swaps has sufficient large support, the distribution over permutations has non-zero probability on all preference lists. This model is motivated by practical applications where consumers preferences generally have many common items appear according to the same relative order and differ only in a small number of items. The prototype list used to generate a random preference list can intuitively be thought of as the mode of the distribution implied by the random swap model. This model also captures the well known Mallows distribution over permutation that is specified by a model permutation and a concentration parameter that determines how the probability of permutations decrease as a function of the distance from the modal permutation. Therefore, this is a fairly general model for consumer preferences.

We devise a polynomial-time approximation scheme (PTAS) for the assortment optimization problem under the random swap model for both types of random swaps. In other words, for any accuracy level $\epsilon > 0$, we show how to compute an assortment with expected revenue at least $(1 - \epsilon)$ times the optimal, in roughly $O(n^{O(1/\epsilon)})$ time. Our algorithm is based on establishing a surprising sparsity property, proving the existence of small-sized assortments that can be efficiently completed into near-optimal ones, crucially utilizing certain symmetries in the distribution over permutations. In particular, we show that among the set of items that do not exist in the prototype list, it is optimal to select a revenue ordered set of items, i.e., the top- k highest revenue ones (for some k). We then make use of a dynamic programming based algorithm to select the core set of items from the prototype.

For the case of arbitrary swaps, we present an improved PTAS, based on approximate dynamic programming. In particular, we propose an algorithm whose running time for any accuracy level $\epsilon > 0$ depends polynomially on n and $O((1/\epsilon)^{O((1/\epsilon) \cdot \log(1/\epsilon))})$. A result of this nature is also referred to as an EPTAS. We also show that our results can be extended to more general settings where we have capacity constraints on the assortment and when the distribution over permutations is generated from a mixture of several initial lists. Therefore, our model provides a tractable framework for capturing consumer preferences under fairly general settings.

Additional Key Words and Phrases: revenue optimization; assortment optimization; ranking models

The full version of this paper is available at http://www.columbia.edu/~vg2277/assortment_swap_distr.pdf. V. Goyal is supported by the NSF grants CMMI 1201116 and CMMI 1351838. Author’s addresses: A. Desir and V. Goyal, Columbia University, New York 10027. Email: {ad2918, vg2277}@columbia.edu. D. Segev, University of Haifa Haifa 31905, Israel. Email: segevd@stat.haifa.ac.il.

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