

# Fund Transfer Pricing for Bank Deposits: The Case of Products with Undefined Maturity

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HIGHLY TECHNICAL

## **Abstract**

The paper presents a pedagogical yet rigorous analysis of fund transfer pricing for deposits with undefined maturity. The objective is to identify the conditions needed to convert the case of deposits with undefined maturity into one with a single effective maturity. This in turn allows us to identify the many circumstances under which the practice of conversion into a single effective maturity is not warranted. Attention is called to the context in which the choice of a maturity is made: pricing, evaluation of performance and hedging of interest rate risk on deposits with undefined maturity.

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## INTRODUCTION

Fund transfer pricing (FTP) is used by bankers to evaluate the profitability of deposits and loans and for pricing. It is used by academics and antitrust authorities to evaluate the degree of competition in banking markets. The challenge, as far as on-balance sheet banking is concerned, is as follows. When one evaluates the profitability of deposits, one knows the cost – the interest paid on deposits and the operating expenses associated with deposits collection, such as employee time and IT. However, determining the return on deposits is more problematic because they can be used to finance various types of assets: consumer loans, corporate loans, interbank assets, bonds, and fixed assets. Revenue – known as the fund transfer price – must be identified to remunerate deposits.

For loans, the problem is symmetrical: the return on loans is known (that is, the interest income net of expected bad debt expense), but not the cost of funding loans. The reason for this is that banks use several sources of funds to finance assets: demand deposits, savings deposits, time deposits, corporate deposits, interbank deposits, subordinated debt, and equity. Again, there will be a need for a specific fund transfer price to evaluate the cost of funding loans. Appropriate identification of the FTP, in particular its maturity, is fundamental for the pricing of commercial products, performance evaluation, bank strategy design and hedging of interest-rate risk.

In three publications [Dermine (2007, 2013, and 2015)], I present foundation and advanced approaches to fund transfer pricing. The foundation approach, used throughout the banking world, covers two cases: products with fixed and undefined maturities. I argued that as a result of the global financial crisis, attention should be given to five potential issues: rationing on the interbank market, the funding of a Basel III liquidity coverage ratio, the necessity to adjust FTP to the credit-riskiness of specific assets, the need to include a liquidity premium in the case of long-term funding and, finally, the choice of a consistent methodology to incorporate the credit spread on the bank's own debt due to the perceived risk of bank default. I concluded that an advanced approach to fund transfer pricing must be adopted by banks.

Having observed the heated debate that the choice of a specific maturity for the FTP applicable to deposits with undefined maturity – such as demand and savings deposits – can generate, I propose a pedagogical yet rigorous discussion of the issues involved. More specifically, I have observed on several occasions an attempt to identify a single effective maturity which makes it possible to convert the complex case of products with undefined maturity into one with a fixed effective maturity. Deposits are divided in two (or several) buckets: (1) volatile “transient” deposits with a short-maturity

and (2) loyal “core” deposits with a long-maturity. The effective maturity is then a weighted average of short- and long-maturity buckets. Indeed, the notion of “behavioral” maturity is referred to by the European Banking Authority (2015) in a report on the measurement of interest-rate risk on the banking book. My purpose here is to identify the conditions that are necessary to convert the case of deposits with undefined maturity into one with a single effective maturity in order to shed light upon the many circumstances under which this simplification is not warranted and a more complex multi-period setting would then apply.

The choice of an economic maturity as opposed to a contractual maturity arises in three contexts: the pricing of the product, the evaluation of performance of a business unit and the selection of a hedge against interest rate risk.<sup>2</sup> As the relevant maturity might not be the same for the three applications, the choice of the FTP maturity has to be set in a specific context.

To illustrate the nature of the issue, consider the following scenario. A Lebanese bank raises deposits in U.S. dollars. Those in charge of pricing deposits argue that, since these deposits are fairly stable, they could be invested in a 5-year fixed-rate U.S.\$-denominated eurobond issued by the Lebanese government. As the return on such bonds is 7%, they propose to pay 5% on a very competitive market for U.S.\$-denominated savings deposits. At the time, the 3-month interest rate on a U.S.\$-denominated Lebanese government bond is 4%. Those in charge of asset and liability management (ALM) wonder whether the single effective maturity of five years chosen to identify the benchmark market rate is warranted, while those in charge of managing risks wonder about the hazard that investing these savings deposits in Lebanese five-year fixed rate bond represents. This illustrates the problematic nature of choosing an effective maturity for deposits with undefined maturity and the desire to simplify and convert this case into one with a single effective maturity.

The review of the foundation approach to fund transfer pricing in Section 2 is followed by a discussion of FTP for deposits with undefined maturity in Section 3. Numerical examples are used to illustrate the nature of the problem and solutions.

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<sup>2</sup> An additional issue not discussed in the paper is the measurement of liquidity risk on deposits with undefined maturity.

## THE FOUNDATION APPROACH: PRODUCTS WITH DEFINED MATURITY

The foundation approach to fund transfer pricing for products with fixed maturity is represented in Figure 1.

The horizontal line represents the market rate, i.e., the interest rate observed on the interbank market (LIBOR).<sup>3</sup> The line is horizontal as the interest rate is set on large international markets and is independent of the volume of transactions initiated by the bank. The two other lines represent the marginal income on loans and the marginal cost of deposits. As a bank wishes to increase its loan portfolio, the expected income from an additional dollar of loan – the marginal or incremental income – will go down because the bank needs to reduce the interest rate to attract the additional dollar of loan, or because the bank is willing to agree to a loan of lower quality. Similarly, the cost of collecting an additional dollar of deposits – the marginal or incremental cost of deposits – will go up because the bank either needs to raise the deposit rate to attract the additional dollar of deposits or to open additional branches in remote areas. In Figure 1, the optimal volume of deposits,  $D^{OPT}$ , is reached when the marginal cost of deposits is equal to the opportunity market rate. One would not want to go beyond  $D^{OPT}$  because the incremental

cost of deposits would be higher than the return earned on the money markets.<sup>4</sup> Similarly, the optimal volume of loans,  $L^{OPT}$ , is reached when the marginal revenue from loans is equal to the marginal investment return, the market rate. One would not want to increase the loan portfolio beyond  $L^{OPT}$  because the incremental income on the new loan would be lower than the return available on the money markets. The maturity of the market rate used for fund transfer pricing should correspond to the maturity of the fixed-term product. For shorter maturities (up to one-year) the interbank market rates are frequently used, while for longer fixed-rate maturities the swap rates are used.<sup>5</sup> Matching maturities not only has intuitive appeal for the search of a relevant opportunity cost, it also insulates the commercial units against the impact of interest rate (or currency) fluctuations. Interest rate (or currency) mismatches are transferred to the ALM department in charge of managing these sources of risk [Dermine (2015)], which is implicitly assumed to have the tools necessary to manage the maturity mismatch created by the loan and deposit commercial units. There is a separation between the profit earned from margins on loans and deposits (benchmarked against a matched-maturity market rate) and the profit realized by the ALM department in mismatching the book. This is justified by the respective types of expertise required to price loans and deposits, and in forecasting interest rates.<sup>6</sup>

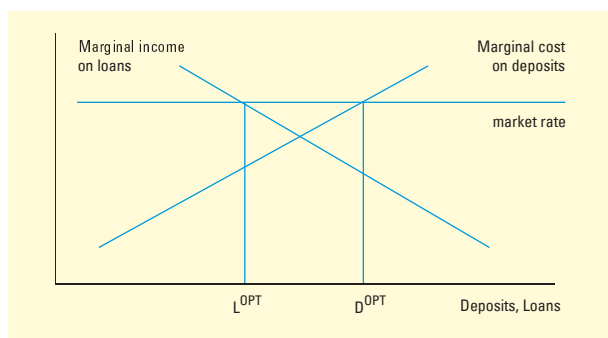


Figure 1 – The separation theorem

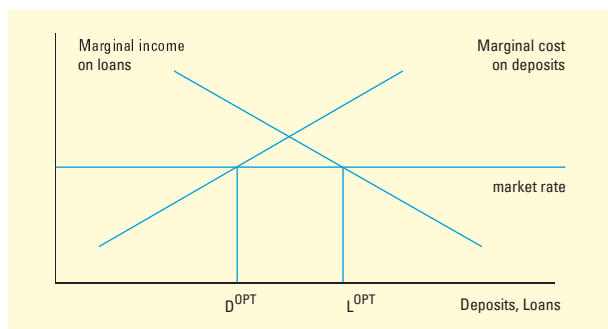


Figure 2 – The separation theorem

Note that there is a separation between the lending and funding decisions. Separation theorem states that loans and deposits must be priced with reference to the market rate and that these decisions are independent of one another. The difference between the optimal volumes of deposits and loans ( $D^{OPT} - L^{OPT}$ ) is the net position in treasury, bonds or interbank assets. In Figure 1 it is positive with deposits exceeding the volume of loans. The bank is a net lender in the money market. But it could be negative with the bank being a net borrower, as illustrated in Figure 2. In this case, the difference between the volume of loans and deposits ( $L^{OPT} - D^{OPT}$ ) must be funded in the money markets.

3 In countries with illiquid interbank markets, the relevant market rate is the interest rate on government bonds.

4 We ignore reserve requirements with the central bank, which reduce the revenue earned on deposits.

5 The swap rate gives the long-term cost of the roll-over of short-term interbank funding that is hedged with a swap. This is likely to differ from the actual cost of funding the long-term asset with a long-term debt that would include a liquidity or credit spread. The use of a swap rate is appropriate when the bank performs the traditional function of maturity transformation, funding long-term assets with short-term debt. The case of maturity matching – long-term assets funded with long-term debt – is analyzed in Dermine (2013).

6 When the ALM department does not have access to treasury products to manage the maturity mismatch at reasonable cost, the separation between deposits and loans breaks down because the interest rate risk is the result of the joint decision on loans and deposits. In this more complex case, one needs to find the appropriate joint mix of loans and deposits that maximizes the present value of future expected profits under reasonable interest rate risk constraints.

## FOUNDATION APPROACH, PRODUCTS WITH UNDEFINED MATURITY

In the foundation approach to fund transfer pricing, the relevant maturity for the marginal return is that of the deposit or loan. A two-year deposit should be priced against the two-year matched maturity market rate. However, there are several well-known cases, such as demand or savings deposits, for which the contractual maturity (very short as withdrawable on demand) is different from the effective economic maturity. Indeed, many deposits are fairly sticky with a longer effective maturity and the Basel Committee (2015) refers to non-maturity deposits (NMDs). Often bankers and regulators attempt to identify a behavioral maturity that would make it possible to convert a case with undefined maturity into one with a fixed effective maturity. Hence the question as to the conditions needed for the conversion.

Let us return to the example mentioned in introduction. In Lebanon, the deposit dollarization ratio reaches 66.1% in 2013 [Bank Audi (2013)]. Due to stiff competition, the bank proposes to pay 5% on U.S.\$ savings deposits withdrawable on demand as it can invest the money in a fixed-rate 7% U.S.\$ 5-years-to-maturity bond issued by the Lebanese government in international markets. The choice of a long-maturity is justified by the argument that savings deposits are fairly stable. The above discussion illustrates the search for a single relevant maturity that makes it possible to apply the framework of the foundation approach for products with defined maturity.

In some cases, one assumes that a fraction of the deposits ( $\alpha$ ) is volatile, equivalent to short-term “transient” deposits, while the complement ( $1 - \alpha$ ) behaves like long-term “core” deposits. The effective maturity becomes a weighted-average maturity of short- and long-term deposits. Again, the conditions needed to apply a weighted-average maturity for the choice of the benchmark rate merits further investigation.

With reference to pricing a product and performance evaluation, a related question arises as to whether the analysis can be conducted over a short period – say one year – and whether this process will lead to optimal decisions. Indeed, to reward bank executives, it is customary to evaluate their performance over a relatively short period, such as one quarter or one year, but it is important to ensure that this does not create sub-optimal short-term biases in decision making.

To illustrate the sources of complexity arising from deposits with undefined maturity, we consider the case of deposits over a 2-year horizon, although this could be extended to a more realistic multi-year setting.<sup>7</sup>

The 1-year-to-maturity bond rate in Year 1,  $b_1$ , is 4% and the 1-year-to-maturity bond rate expected at the start of year 2,  $b_2$ , is 6%. The fixed coupon  $c$  on a 2-year-to-maturity bond is 4.971%.

The coupon of 4.971% ensures that the fair value of the bond, the present value of future cash flows,<sup>8</sup> is equal to 100:  $100 = (4.971/1.04) + (104.971/1.04 \times 1.06)$

The coupon rate of 4.971% (see calculation in Appendix 1) also ensures that a 1-year investment strategy with roll-over yields the same return as a 2-year investment strategy with reinvestment of the annual coupon. Consider the case of an initial investment of 100:

1-year investment strategy with rollover:  $100 \times 1.04 \times 1.06 = 110.24$   
2-year investment strategy with reinvestment of interim coupon:  $(4.971 \times 1.06) + 104.971 = 110.24$

It is assumed that the ALM department of the bank can hedge the interest rate risk, so that the focus of the commercial units is entirely on the interest margins, taken as given by the current market rates and those implied in the yield curve.<sup>9</sup>

In this 2-year scenario, deposits with undefined maturity are collected. Undefined maturity has two dimensions. It refers first to the fact that some of the deposits collected in year 1 will still be deposited in the bank in year 2. This introduces the notion of temporal dependence – in our case over two years – between the volumes collected in year 1 and year 2. A second potential source of temporal dependence arises from price rigidity when the deposit rate set in year 2 is related to the interest rate set the previous year.

Several cases likely to be observed in the real world will be considered. The objective is to identify the conditions in which deposits with undefined maturity can be converted into a simpler one with a single effective maturity, taking into account three perspectives: pricing, evaluation of performance, and management of interest rate risk.

A parsimonious approach with five cases is chosen to focus and illustrate the sources of the time dependence. They are as follows:

7 A spreadsheet with solutions to examples is available from the author upon request.

8 We implicitly assume a risk neutral world and a liquid bond market. Interest rate and liquidity risk premia are assumed to be 0%.

9 Note that the ALM department might decide not to hedge the position. There is a separation between the commercial units focusing on interest margins based on current interest rates and the role of ALM in managing the maturity mismatch.

Case 1: Independence between deposits collected in year 1 and those collected in year 2, and flexible deposit rates in both years.

In the next four cases, we analyze the implications of temporal dependence originating from deposit volumes and/or deposit rates.

Case 2.1: Log-linear dependence between deposit volumes and flexible deposit rates.

Case 2.2: Complete rigidity of volumes and deposit rates over the two years.

Case 2.3: Linear-additive volume dependence and rigid deposit rate.

Case 2.4: Linear-additive volume dependence and discriminatory pricing.

### CASE 1: INDEPENDENCE BETWEEN YEAR 1 AND YEAR 2

Consider a first case in which the supply of deposits in year  $i$  ( $D_i$ ),  $i = 1, 2$ , is positively related to the deposit rate  $d_i$  offered in year  $i$  and negatively related to an investment opportunity competitive bond rate  $b_i$ . Let us assume that both the volume of deposits  $D_2$  and the deposit rate  $d_2$  chosen in Year 2 are independent of what happened in Year 1. That is, there is independence between Year 1 and Year 2. The case of complete independence is used as a benchmark to study the sources of temporal dependence.

The supply of deposits in years 1 and 2 are given by the following log-linear relations<sup>10</sup>:

$$D_1 = 100,000 \times b_1^{-1.5} \times d_1^2$$

$$D_2 = 100,000 \times b_2^{-1.5} \times d_1^2$$

The log-linear function is chosen because, as shown in Appendix 2, the price elasticity is the exponent of the deposit rate variable. Assuming that the deposits collected are invested in 1-year-to-maturity bonds,<sup>11</sup> the maximization of the present value of future profits, evaluated at the end of Year 1, is equal to:

$$\text{MaximizeValue} = (4\% - d_1) \times D_1 + \frac{(6\% - d_2) \times D_2}{1 + 6\%}$$

In case of independence between years 1 and 2 – the volume of deposits and the deposit rate in year 2 are unrelated to what happened in year 1 – one can maximize the profit of each year separately. In this case, the fund transfer price to be used in pricing and in evaluating the performance in year 1 is the matched-maturity 1-year maturity bond rate  $b_1 = 4\%$ , and the FTP for the second year is  $b_2 = 6\%$ .

As is shown in Appendix 2, the optimal deposit rate that maximizes profit in years 1 and 2 are given by the following relations,  $\epsilon$  denoting the deposit rate elasticity:

$$d_1^{\text{OPTIMAL}} = b_1 \times \frac{1}{(1 + \frac{1}{\epsilon})} = 4.0\% \times \frac{1}{(1 + \frac{1}{2})} = 4.0\% \times 0.6666 = 2.67\%$$

$$\text{Profit in year 1} = (4.0\% - 2.67\%) \times 100,000 \times 4.0^{-1.5} \times 2.67^2 = 1,185.19$$

$$d_2^{\text{OPTIMAL}} = b_2 \times \frac{1}{(1 + \frac{1}{\epsilon})} = 6.0\% \times \frac{1}{(1 + \frac{1}{2})} = 6.0\% \times 0.6666 = 4.0\%$$

$$\text{Profit in year 2} = (6.0\% - 4.0\%) \times 100,000 \times 6.0^{-1.5} \times 4.0^2 = 2,177.32$$

$$\text{PresentValue of Profits} = 1,185.19 + 2,177.32/1.06 = 3,239.26$$

The current 1-year interest rate – the marginal income on each dollar collected – has to be used for pricing and evaluating the performance of the manager over each year separately. Single-year pricing and performance evaluation is optimal as it will lead to the highest value over the 2-year horizon.

In addition to pricing and performance evaluation, a third question to consider is hedging interest rate risk. What maturity assets should the money collected at the start of year 1 be invested in? A matched-maturity of one year, shorter or longer?

To understand the nature of interest rate risk, we run the following simulation. The current upward rising yield curve being at 4% - 4.971%, deposits in year 1 are priced optimally at 2.67%. But what happens next year if the 1-year-rate falls from 6% to 5%? Let us calculate the profit in year 2 in a lower rate environment:

$$d_2^{\text{OPTIMAL}} = 5.0\% \times \frac{1}{(1 + \frac{1}{2})} = 5.0\% \times 0.6666 = 3.33\%$$

$$\text{Profit in year 2} = (5.0\% - 3.33\%) \times 100,000 \times 5.0^{-1.5} \times 3.33^2 = 1,656.35$$

When market interest rate in year 2 falls from 6% to 5%, profit in that year falls by 24% from 2,177.32 to 1,656.35. This is caused by the relative rigidity of the deposit rate. For a fall in market rates of 1% from 6% to 5%, the deposit rate fell by only 0.67% from 4% to 3.33%, which generated a fall in interest margin. To hedge against

<sup>10</sup> In the deposits supply relations, an interest rate of 4% is entered as "4.0."

<sup>11</sup> Deposits could be invested in 2-year maturity bond with coupon  $c$ . Arbitrage ensures that the two investment strategies yield the same return over two years.

<sup>12</sup> An alternative hedging tool would be to invest in a one-year maturity bonds and purchase an interest rate futures contract that creates a gain in case of a fall in interest rates.

the fall in profits in year 2, the bank can increase the maturity of the bond purchased at the start of year 1 beyond one year to create a capital gain when interest rate<sup>12</sup> falls by an amount equal to the fall in profitability in Year 2.

The above example illustrates that the relevant maturity used for hedging interest rate risk does not need to be the same as the maturity used for pricing and performance evaluation. For hedging, one would use a maturity of asset longer than one year, while for pricing and evaluation of performance, one would use the marginal rate, the 1-year-maturity rate.

With regards to evaluating the performances of the business units, a practical question arises as to which business units should receive the benefit of the hedge (capital gains if interest rate falls): the ALM department or the commercial deposit gathering unit? If the objective is to insulate the commercial unit against the negative impact of a lower interest rate environment on profit one could allocate the benefits of the hedge to the commercial unit. However, we take the position that this should not be done for the following reason. Allocation of capital gains to the profit in year 2 would risk distorting the evaluation of profitability in that year. In a low interest rate environment, profit margins are smaller and the correct lower marginal transfer price should be recognized to create managerial incentives to be more efficient and to reduce costs, possibly reducing the number of branches and exiting some locations. This does not imply that managers of commercial units should be penalized with lower bonuses when they operate in a low interest rate environment. To achieve this and reward outstanding performance, bonuses should be based not on the absolute revenue of a business unit but on the difference between realized revenue and a benchmark target. Obviously, in a lower rate environment, the benchmarked target would be reduced.<sup>13</sup>

In the context of independence between the two years, the analysis leads to the following observations. One would use a matched maturity current rate for both pricing and evaluation of performances in years 1 and 2. Maximization of 1-year revenue leads to optimal decisions that maximize total value. Hedging the interest rate risk requires consideration of profits over the two years. Capital gains on the asset is needed in the case of a fall in the interest rates. An additional performance measurement issue relates to the allocation of profits and losses of the hedging strategies among the commercial units. We argue against this in order to ensure that there is a recognition of the current lower interest rate environment. To avoid penalizing commercial units in a low interest rate environment, performances would be compared to a more flexible benchmark target.

## **CASE 2: DEPENDENCE BETWEEN YEAR 1 AND YEAR 2**

In Case 1, the decision taken in year 1 had no impact on the profit in year 2: there was complete separation between the two decisions. However, there are two potential reasons why independence might not be apply: (1) the volume of deposits in year 2 might be related to that collected in Year 1 and (2) the deposit rate applied in year 2 might be related to that set in year 1.

This situation is likely to be observed in retail banking markets with deposits with undefined maturity. Slow adjustments by depositors suggests that some customers will take their time in moving their deposits to another bank or financial product. This creates a time dependence between the volumes of deposits. Deposits in year 2 are partly related to what was collected in year 1. The second reason is that for marketing reasons (menu cost), one wants to avoid changing the interest rate on deposits too often. This creates a second type of dependence: the interest rate paid in year 2 is the rate chosen in year 1. Time dependence will force that bank to analyze the impact of the first year decision on the profit of the second year. Value maximization should be conducted on a multi-period basis.<sup>14</sup>

The stability of deposit volumes and the rigidity of the deposit rates suggest that the effective maturity of short-term deposits is longer than the contractual maturity. Hence we need to analyze how an effective maturity can be identified.

For banks, a standard practice when dealing with retail deposits with undefined maturity is to split them into two categories: volatile "transient" deposits and stable "core" deposits that are equivalent to long-term fixed rate maturity deposits. The fund transfer price is the weighted sum of the short-term and long-term interest rates, with the weights being the volume of volatile and stable deposits. Although one accepts the desire for a simple managerial rule, doubts may persist about the concept of an effective fixed-rate maturity. In the banking world the volume of sticky deposits is not completely fixed and the interest rate chosen in year 2 is not totally rigid. One objective of this paper is to specify the conditions in which the use of an effective long-term maturity can be justified, and to present a coherent value-maximization framework to deal with cases in which these conditions do not apply.

<sup>13</sup> And if we leave the capital gains to the ALM department, this should not lead automatically to a bonus for ALM managers. Performance of the ALM department's mismatch strategy should be relative to that of a fully hedged strategy (which in this case would include the capital gains).

<sup>14</sup> Although this issue is often discussed with reference to deposits, it applies as well to retail loans, such as consumer and credit card loans with relatively rigid interest rates.

Consider the case where the volume of deposits in Year 2,  $D_2$  (.), is a function not only of the deposit rate paid that period,  $d_2$ , but also of the volume of deposits collected in year 1,  $D_1$ . Below, a log-linear specification, Case 2.1, is first analyzed, which allows us to compare with Case 1 using a similar specification for the deposit supplies.

### Case 2.1: Log-linear dependence of the volume of deposits with flexible deposit rates

The supplies of deposits in years 1 and 2 are equal to:

$$D_1 = 100,000 \times b_1^{-1.5} \times d_1^{-2}$$

$$D_2 = 300 \times b_2^{-1.5} \times d_2^2 \times D_1^{0.5}$$

The specification of the volume of deposits in year 2 is log-linear, in which case the elasticity of the volume of deposits in year 2 to deposits collected in year 1 is the exponent 0.5. It is assumed that the deposit rate in year 2 is flexible.

The maximization of the present value of future profits, evaluated at end of year 1, is equal to:

$$\text{MaximizeValue} = (4\% - d_1) \times D_1 + \frac{(6\% - d_2) \times D_2}{1 + 6\%}$$

Intuitively, one should pay a bit more to attract deposits in year 1 because this will increase the supply of profitable deposits in year 2. In the case of dependence overtime, one needs to work with dynamic optimization [Intriligator (1971)], which has two stages. First, compute the optimal deposit rate in year 2, the last period, and then identify the deposit rate in the first year that will maximize the present value of profits earned in years 1 and 2.

The optimal pricing in the last year, year 2, is identical to that of the case of independence:

$$d_2^{\text{OPTIMAL}} = 6.0\% \times \frac{1}{(1 + \frac{1}{2})} = 6.0\% \times 0.6666 = 4.0\%$$

$$\text{Profit in year 2} = (6.0\% - 4.0\%) \times 300 \times 6.0^{-1.5} \times 4.0^2 \times D_1^{0.5}$$

Having computed the optimal deposit rate in year 2, one can then compute the optimal deposit rate in year 1 that maximizes value.

$$\text{MaximizeValue} = (4\% - d_1) \times D_1 + \frac{(6.0\% - 4.0\%) \times 300 \times 6.0^{-1.5} \times 4.0^2 \times D_1^{0.5}}{1 + 6\%}$$

A closed-form solution for optimal pricing in year 1 is given in Appendix 3. Alternatively, one can use the function “optimizer” or “solver” in a spreadsheet to identify the deposit rate in year 1 that maximizes value. The optimal deposit rate in year 1 is 3.235 %, a rate

significantly higher than that obtained under myopic optimization of 2.67%. Given the higher market rate of 6% accompanied by a higher margin in year 2, there is an incentive to attract more loyal “core” deposits in year 1. Relative to a myopic optimization, a dynamic optimization generates lower profits in year 1 (1,000.98 versus 1,185.19), but higher profits in year 2 (2,362.25 versus 1,947.46). The impact on the present value of profits over the two years is positive, 3,229.52 versus 3,022.41.

The above case shows that in a situation of intertemporal linkage – in our case deposits in year 2 are related but not identical to those of year 1 and the deposit rate in year 2 is not rigid – maximization over several periods is necessary and there is no simple concept for an effective maturity. One could artificially increase the FTP in year 1 to 4.852% to ensure that single-period optimization leads to the optimal deposit rate of 3.235% (= 4.852% x (1 + 1/ε)<sup>-1</sup>). This “blown-up” FTP rate is lower than the 2-years-to-maturity coupon rate of 4.973%. It is equal to a weighted average of the 1-year rate  $b_1$  and 2-year fixed coupon interest rate  $c$  with weights<sup>15</sup> of 12.25% and 87.75%, respectively. Hence, the market practice of applying weighted average rate to compute the FTP could be used but one must note that the weighting is sensitive to the market rates  $b_1$  and  $b_2$ , the price elasticity and the time-dependence factor. This differs from the ad hoc practice of applying shares of volatile deposits and stable deposits; the reason being that long-term deposits are not completely sticky but sensitive to the second year interest rate and that the deposit rate is not constant.

Three partly related issues have been identified in the context of products with undefined maturity: pricing, evaluation of performance and hedging. The analysis of the log-linear case of volume dependence over time with flexible deposit rates leads to the following conclusions:

- **Pricing:** to achieve optimal pricing, one needs to conduct multi-period optimization. If a bank intends to maximize profits over one period, it could artificially increase the FTP to ensure optimal pricing in year 1, but the blown-up transfer price will not be equal to a two-year maturity fixed interest rate or a weighted average of short- and long-term rate; with the weights being the volumes of volatile and stable deposits.
- **Evaluation of performance:** optimal pricing is shown to reduce profit in year 1. Once again, “superior” performance should be evaluated against a benchmark, not in absolute terms. An alternative is to use a blown-up FTP.

<sup>15</sup> The weights are obtained from the following relationship: effective FTP = 4.852% = ((1 - α) x b1) + (α x c).



- **Hedging:** as was done for case 1, one needs to assess the impact of a change in interest rates on the present value of future profits to determine the hedged maturity for the assets.

In the following case (2.2), we assume that the volume of deposits in year 2 is identical to that of year 1 and that the deposit rate in year 2 is identical to that applied in year 1.

### Case 2.2: Complete rigidity of the volume of deposits and of the deposit rate

The log-specification for the deposit volume in year 1 is identical to that of the first case but the deposits volume and the interest rate set in year 2 are identical to those of year 1. This is a case of complete rigidity of both volume and deposit rates.

$$D_1 = 100,000 \times b_1^{-1.5} \times d_1^2$$

$$D_2 = D_1$$

$$\text{And } d_2 = d_1$$

$$\text{Maximize Value} = (4\% - d_1) \times D_1 + \frac{(6\% - d_1) \times D_1}{1 + 6\%}$$

In the case of complete rigidity (Appendix 4), the multi-period maximization problem can be converted into a one-period maximization, with the deposit rate in year 1 being priced against the two-year fixed coupon rate  $c$  (calculated in Appendix 1).

$$d_1^{\text{OPT}} = c \times \frac{1}{(1 + \frac{1}{\epsilon})}$$

Within the parameters of the example, the optimal deposit rate in the case of constant volume and deposit rate is equal to:

$$d_1^{\text{Optimal}} = d_2 = 4.971\% \times \frac{1}{(1 + \frac{1}{2})} = 3.31\%$$

With reference to the example in Lebanon, using a 5-year 7% fixed rate to price short-term deposits is only warranted if the volume of deposits is constant and the deposit rate is fixed. This is an extreme situation since in a period of rising interest rates, the case of a positive yield curve, one can anticipate an increase in the deposit rate in the future driven by competition with a corresponding impact on volume. The case of extreme rigidity of both interest rates and volumes is very unlikely in reality, with the consequence that one cannot rely on maximization over one year with an effective maturity interest rate. A more complex multi-period maximization is needed.

One could argue that the above results are due to the log-linear specification in year 2. This specification does not allow for segmentation in year 2 between the “old” loyal deposits collected the previous year and the “new” deposits collected in year 2. Such a

segmentation would allow year 1-deposits to be treated as quasi longer-term 2-year-to-maturity deposits. A linear-additive specification is introduced to allow for such segmentation. Again, the purpose of the analysis is to identify conditions that allow the multi-period maximization to be simplified into an effective fixed maturity problem.

### Case 2.3: Linear additive volume dependence and fixed deposit rates

The volume of deposits in year 2 is made up of two components: a fraction of the deposits collected in year 1 (the loyal deposits) and new deposits ( $ND_2$ ).

We consider two settings for pricing. In the first one, case 2.3, the deposit rate chosen in year 1 applies in the second year. In the second case, case 2.4, we allow price discrimination. Deposits collected in year 1 keep receiving the same deposit rate while new deposits ( $ND_2$ ) received a rate set in year 2. Again, the objective is to understand the nature of the maximization over two years.

The bank maximizes the present value of future profits,

$$\begin{aligned} \text{Value} &= (b_1 - d_1) \times D_1 + \frac{\alpha D_1 \times (b_2 - d_1) + (b_2 - d_1) \times ND_2}{1 + b_2} \\ &= (1 - \alpha) \times D_1 \times (b_1 - d_1) + \alpha D_1 \times (b_1 - d_1) + \frac{\alpha D_1 (b_2 - d_1) + (b_2 - d_1) \times ND_2}{1 + b_2} \\ &= (1 - \alpha) \times D_1 \times (b_1 - d_1) + \alpha D_1 \times (c - d_1) + \frac{\alpha D_1 (c - d_1) + (b_2 - d_1) \times ND_2}{1 + b_2} \end{aligned}$$

The last relationship follows from the arbitrage that ensures that investing stable deposits in a 2-years-to-maturity bond with coupon  $c$  is equivalent to investing in a 1-year asset with roll-over at the forward rate  $b_2$ . Value is the sum of two terms: profit in year 1, which is a weighted sum of profits on volatile  $(1 - \alpha)$  and stable  $(\alpha)$  1-year deposits, and the value of profit on year 2-deposits.

The volume of deposits in years 1 and 2 are given by:

$$\text{Deposits}_1 = 100,000 \times b_1^{-1.5} \times d_1^2$$

$$\text{Deposits}_2 = \alpha \times D_1 + ND_2 = \alpha \times D_1 + 100,000 \times (1 - \alpha) \times b_2^{-1.5} \times d_1^2$$

The above is related to Case 2.2 with two differences: only a fraction ( $\alpha$ ) of deposit in Year 1 will transfer to Year 2 and the constant deposit rate set in Year 1 will affect the new deposits collected in Year 2 ( $ND_2$ ).

Using the function “optimizer” in a spreadsheet, one can identify the deposit rate in year 1 that maximizes value. Using as an example a retention rate,  $\alpha$ , of 90%, the optimal deposit rate in year 1 is



3.298%. Compared to the extreme case of fixed volume and fixed deposit rates discussed above, two forces are at work: the retention rate of year 1 deposits is less than 100% ( $\alpha < 1$ ) while there is a need to keep the deposit rate high enough to attract the new volatile deposits of year 2. Again, one could identify a fund transfer price that allows optimization over one year. The FTP equivalent is 4.948%, lower than the fixed coupon of 4.971% and higher than a weighted average (90% x 2-year coupon rate, 10% x 1-year rate) of 4.87%. Multi-period maximization is again warranted in this case.

In the final case, we allow discriminatory pricing, with the flexible deposit rate set in year 2 affecting only the new deposits.

#### Case 2.4: Linear-additive volume of deposits and discriminatory pricing

The deposit rate set in year 1 applies to the stable deposits that remain in year 2. New deposits collected in year 2 receive the rate  $d_2$ . The bank is said to apply discriminatory pricing between the “old” and “new” deposits  $ND_2$ . To circumvent laws that prohibit price discrimination, “revenue management” consulting companies advise the creation of new products targeted at a specific segment, the new depositors. This brings us closer to the case of a fixed interest rate and fixed volume of deposits. But there is a difference, as only a fraction ( $\alpha$ ) of the deposit collected in year 1 will transfer to year 2. The objective is to see whether the ad hoc rule of a weighted average of short- and long-term interest rate can be applied or not.

The bank maximizes the present value of future profits as follows:

$$\begin{aligned} \text{Value} &= (b_1 - d_1) \times D_1 + \frac{\alpha \times D_1 \times (b_2 - d_1) + (b_2 - d_2) \times ND_2}{1 + b_2} \\ &= (1 - \alpha) \times D_1 \times (b_1 - d_1) + \alpha \left[ D_1 \times (b_1 - d_1) + \frac{D_1 \times (b_2 - d_1)}{1 + b_2} \right] + \frac{(b_2 - d_2) \times ND_2}{1 + b_2} \\ &= (1 - \alpha) \times D_1 \times (b_1 - d_1) + \alpha \times D_1 \times (c - d_1) + \frac{\alpha \times D_1 \times (c - d_1) + (b_2 - d_2) \times ND_2}{1 + b_2} \end{aligned}$$

Value is the sum of two terms: profit in year 1, which is a weighted sum of profits on volatile  $(1 - \alpha)$  and stable  $(\alpha)$  1-year deposits invested in respectively the 1-year asset and 2-year-fixed coupon asset, and the value of profit on year 2-deposits. As is shown in Appendix 5, the optimal interest rate for year 1 is given by the following relation:

$$\begin{aligned} d_1^{\text{Optim}} &= \frac{\frac{\alpha \times c}{w + 1 + b_2}}{(1 - \varepsilon^{-1}) \times \left(1 + \frac{\alpha}{1 + b_2}\right)} \text{ with } w = (1 - \alpha)b_1 + \alpha c \\ &= \frac{b_1(1 + b_2) + \alpha b_2}{(1 + \varepsilon^{-1}) \times (1 + b_2 + \alpha)} \end{aligned}$$

At the optimum, the value of the marginal costs incurred over two years on one dollar of deposits collected in year 1 with a retention rate  $\alpha$  in year 2 must equal the value of the marginal revenue earned over two years. The marginal revenue includes the weighted average return  $w$  earned in investing the volatile deposits in a 1-year bond and the loyal stable deposits in a 2-year bond, and the revenue earned on deposits retained in the second year.

$$d \times (1 - \varepsilon^{-1}) \times \left(1 + \frac{\alpha}{1 + b_2}\right) = w + \frac{\alpha c}{1 + b_2}$$

In the general case, the maximization of value must be conducted over two years, and one cannot focus solely on the weighted average return  $w$  earned on volatile and stable deposits. The reason being that if the transient and loyal deposits can be invested in a weighted average of 1-year and 2-year assets, one cannot ignore the revenues and costs faced in year 2 in the value maximization.

Two special cases stand out. If  $\alpha = 1$ , the deposits collected in year 1 have effectively a 2-year fixed-rate maturity and the FTP becomes the two-year fixed coupon rate  $c$ . This situation is identical to that of case 2.2 with complete rigidity of both volumes of deposits and interest rates. If the yield curve is flat ( $b_1 = b_2$ ), the two-year maximization simplifies into a one-year myopic optimization with  $FTP = b_1 = b_2$ . In all other situations, the single effective maturity FTP given by the optimal pricing rule is different from the two-year fixed coupon  $c$  or from a weighted average  $w$  of the 1-year- and 2-years-to-maturity rates  $b_1$  and  $c$ .

The case is illustrated numerically with a specification for deposit supply function similar to that of case 2.3.

$$\begin{aligned} \text{Deposits}_1 &= 100,000 \times b_1^{-1.5} \times d_1^2 \\ \text{Deposits}_2 &= \alpha \times D_1 + ND_2 = \alpha \times D_1 + 100,000 \times (1 - \alpha) \times b_2^{-1.5} \times d_2^2 \end{aligned}$$

Dynamic optimization is applied. The optimal deposit rate in year 2 for the new deposits  $ND_2$  is equal to the myopic case of 4%. The optimal deposit rate in year 1 that maximizes total value over the two years is 3.28%, less than the 3.31% fixed deposit rate case (case 2.2). As the retention of year 1-deposits is imperfect ( $\alpha < 1$ ), there are fewer profitable deposits in year 2 and the deposit rate is reduced. The FTP equivalent to allow myopic one-period optimization is 4.92%, smaller than the 2-Year coupon rate  $c$  and different from the weighted average market rate of 4.87% with weights of 10% for the 1-year rate  $b_1$  and 90% for the two year coupon rate  $c$ .

In the case of linear-additive deposit supply function and price discrimination, one observes again that a multi-period maximization is needed. It cannot be readily converted into a single effective period maximization. Only in two cases would such a simplification be possible: extreme stickiness ( $\alpha = 1$ ) or the case of a flat yield curve.

## CONCLUSION

Demand and savings deposits are a significant source of funds for banks. These products with undefined maturity have generated heated debates on the effective maturity that should be applied to the fund transfer price used to evaluate their profitability. Furthermore, the 2016 Basel proposal for a capital regulation on interest rate risk on the banking book also raises the issue of the choice of a behavioral maturity for non-maturing deposits. This paper has presented a pedagogical yet rigorous value-based management approach to the management of deposit with undefined maturity. I have focused on three issues associated with the management of these deposits: pricing, performance evaluation, and hedging of interest rate risk. Such deposits raise the question of their effective behavioral maturity. I have identified the conditions in which the multi-period maximization problem can be converted into one with a single effective maturity, and I have evaluated the market use of a weighted average maturity obtained by breaking down the portfolio of deposits with undefined maturity into buckets with short-term volatile deposits and stable longer-term deposits.

Managing deposits with undefined maturity is a multi-period problem as there are two intertemporal issues: the volume of deposits in year 2 is related to the deposits collected in year 1 and the deposit rate can be relatively rigid. Under most assumptions analyzed here, the management issue cannot be simplified into a profit maximization over a single period. Multi-period maximization simplifies into maximization over one period in two extreme cases: constant deposit volume/deposit rate or flat yield curve with price discrimination. Since these conditions are unlikely to be met, one needs to work with a more complex multi-period optimization.

I have also shown that the maturity of assets needed to hedge the bank against interest rate risk can be different from the fund transfer price maturity used for pricing or for measuring performance. The reason for this is that the management of interest rate risk entails an analysis of the impact of interest rates on all future profits (the franchise value), while the maturity relevant for pricing is the period over which intertemporal dynamics apply.

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## APPENDICES

### Appendix 1. Computation of 2-year fixed coupon rate c consistent with 1-Year rates $b_1$ and $b_2$ .

In equilibrium, the value of a short term investment strategy with roll-over must equal the value of investing in a bond with fixed coupon c.

$$\frac{b_1}{1+b_1} + \frac{1+b_2}{(1+b_1)(1+b_2)} = \frac{c}{1+b_1} + \frac{1+c}{(1+b_1)x(1+b_2)}$$

$$(1+b_2)xb_1 + 1+b_2 = c(1+b_2) + c + 1$$

$$(1+b_2)x(1+b_1) = c(2+b_2) + 1$$

$$c = \frac{(1+b_2)(1+b_1) - 1}{2+b_2} = \frac{b_2 + b_1x(1+b_2)}{2+b_2}$$

### Appendix 2. Deposit pricing<sup>16</sup>

Given a supply of deposits  $D(d)$ , a positive function of the deposit rate  $d$ , and denoting by  $b$  and  $\epsilon$  the market rate and the deposit volume price-elasticity, one has: Revenue =  $(b - d) \times D(d)$

To maximize revenue, one has:

$$\begin{aligned} \frac{\partial \text{Revenue}}{\partial d} &= -D + (b-d)D' = 0 \\ -\frac{Dxd}{D} + (b-d) &= 0 \\ -d \times \epsilon^{-1} + b - d &= 0 \end{aligned}$$

$$dx(1+\epsilon^{-1}) = b$$

$$d^{Optimal} = bx \frac{1}{1+\frac{1}{\epsilon}}$$

In the case of a log-linear supply of deposits, the price elasticity ( $\epsilon$ ) is the exponent of the deposit rate variable:

$$\text{Deposits}_t = \alpha_0 x d_t^{\alpha_1} x b_t^{-\alpha_2}$$

$$\text{Elasticity}(\epsilon) = \frac{\partial \log D}{\partial \log d} = \alpha_1$$

### Appendix 3. Log-linear dependence with flexible deposit rates (Case 2.1)

$$D_1 = 100,000xb_1^{-1.5}xd_1^2$$

$$D_2 = 300xb_2^{-1.5}xd_2^2xD_1^{0.5}$$

$$\text{Value} = (b_1 - d_1)D_1 + \frac{(b_2 - d_2)D_2(d_2, D_1)}{(1+b_2)}$$

The dynamic optimization starts with maximization of profits in Year 2 and the choice of the deposit rate in Year 2

$$\begin{aligned} -D_2 + (b_2 - d_2)D_2' &= 0 \\ -\frac{D_2xd_2}{D_2'xd_2} + (b_2 - d_2) &= 0 \\ -d_2x\epsilon^{-1} + b_2 - d_2 &= 0 \\ d_2^{Optimal} &= b_2x \frac{1}{1+\frac{1}{\epsilon}} \end{aligned}$$

Having calculated the deposit rate in Year 2, one can then calculate the optimal deposit rate in Year 1

$$-D_1 + (b_1 - d_1)D_1' + \frac{(b_2 - d_2) \frac{\partial D_2}{\partial D_1} x \frac{\partial D_1}{\partial d_1}}{(1+b_2)} = 0$$

$$\text{define } D_2' = \frac{\partial D_2}{\partial D_1}$$

$$-\frac{D_1}{D_1'} x \frac{d_1}{d_1} + (b_1 - d_1) + \frac{(b_2 - d_2)D_2'}{(1+b_2)} = 0$$

$$d_1(1+\epsilon^{-1}) = b_1 + \frac{(b_2 - d_2)D_2'}{(1+b_2)}$$

$$d_1^{Optimal} = \left[ b_1 + \frac{(b_2 - d_2)D_2'}{(1+b_2)} \right] x (1+\epsilon^{-1})^{-1}$$

$$\text{with } D_2' = 0.5x300xb_2^{-1.5}xd_2^2xD_1^{0.5-1}$$

<sup>16</sup> Discussed in Chapter 11 of Dermine (2015).

**Appendix 4. Complete rigidity of the volume of deposits and of the deposit rate (case 2.2)**

The log-linear specification for the deposit volume in year 1 is given below. The deposits volume and the interest rate set in year 2 are identical to those of year 1. This is a case of complete rigidity of both volume and deposit rate.

$$D_1 = 100,000xb_1^{-1.5}xd_1^2$$

$$D_2 = D_1$$

$$\text{and } d_2 = d_1$$

$$Value = (b_1 - d_1)x D_1 + \frac{(b_2 - d_1)D_1}{1 + b_2}$$

$$\begin{aligned} \frac{\partial Value}{\partial d_1} &= -D_1 + (b_1 - d_1)D_1' + \frac{-D_1 + (b_2 - d_1)D_1'}{1 + b_2} = 0 \\ &= -\frac{D_1 d_1}{D_1 d_1} x(b_1 - d_1) + \frac{-D_1 d_1 + (b_2 - d_1)}{1 + b_2} = 0 \\ &= b_1 - d_1 x(1 + \varepsilon^{-1}) + \frac{b_2 - d_1 x(1 + \varepsilon^{-1})}{1 + b_2} \\ &= b_1 + \frac{b_2}{1 + b_2} - d_1 x(1 + \varepsilon^{-1}) x(1 + \frac{1}{1 + b_2}) = 0 \end{aligned}$$

$$\begin{aligned} d_1 x(1 + \varepsilon^{-1}) &= \left( \frac{b_1(1 + b_2) + b_2}{1 + b_2} \right) x \left( \frac{1 + b_2}{2 + b_2} \right) \\ &= \frac{b_1(1 + b_2) + b_2}{2 + b_2} = c \end{aligned}$$

**Appendix 5. Linear-Additive Volume of Deposits and Discriminatory Pricing (Case 2.4)**

The deposit rate set in year 1 applies to the deposits that remain in year 2. The new deposits collected in year 2 receive the rate  $d_2$ . The present value of future profits is as follows:

$$\begin{aligned} Value &= (b_1 - d_1)x D_1 + \frac{\alpha x D_1 x(b_2 - d_1) + (b_2 - d_2)x D_2}{(1 + b_2)} \\ &= (1 - \alpha)x D_1 x(b_1 - d_1) + \alpha \left[ D_1 x(b_1 - d_1) + \frac{D_1 x(b_2 - d_1)}{1 + b_2} \right] + \frac{(b_2 - d_2)x D_2}{1 + b_2} \\ (1 - \alpha)(b_1 - d_1(1 + \varepsilon^{-1})) + \alpha \left[ b_1 + \frac{b_2}{1 + b_2} - d_1(1 + \varepsilon^{-1})x(1 + \frac{1}{1 + b_2}) \right] &= 0 \\ -d_1 x(1 + \varepsilon^{-1})x(1 - \alpha + \alpha(1 + \frac{1}{1 + b_2})) + (1 - \alpha)b_1 + \alpha(b_1 + \frac{b_2}{1 + b_2}) &= 0 \\ -d_1 x(1 + \varepsilon^{-1})x(1 + \frac{\alpha}{1 + b_2}) + (b_1 + \frac{\alpha b_2}{1 + b_2}) &= 0 \end{aligned}$$

$$d_1^{opt} = \frac{b_1(1 + b_2) + \alpha b_2}{(1 + \varepsilon^{-1})x(1 + b_2 + \alpha)}$$

An alternative derivation of the last two relations is as follows:

$$\begin{aligned} -d_1 x(1 + \varepsilon^{-1})x(1 + \frac{\alpha}{1 + b_2}) + (b_1 + \frac{\alpha b_2}{1 + b_2}) &= 0 \\ d_1 x(1 + \varepsilon^{-1})x(1 + \frac{\alpha}{1 + b_2}) + (1 - \alpha)b_1 + \alpha b_1 + \frac{\alpha b_2}{1 + b_2} &= 0 \\ d_1 x(1 + \varepsilon^{-1})x(1 + \frac{\alpha}{1 + b_2}) + (1 - \alpha)b_1 + \alpha c + \frac{\alpha c}{1 + b_2} &= 0 \\ d_1 x(1 + \varepsilon^{-1})x(1 + \frac{\alpha}{1 + b_2}) + w + \frac{\alpha c}{1 + b_2} &= 0 \end{aligned}$$

$$d_1^{opt} = \frac{w + \frac{\alpha c}{1 + b_2}}{(1 + \varepsilon^{-1})x(1 + \frac{\alpha}{1 + b_2})} \quad \text{with } w = (1 - \alpha)xb_1 + \alpha xc$$