Funding Constraints and Informational Efficiency*

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Abstract

We develop a tractable rational expectations model that allows for general price-dependent portfolio constraints and study a setting where constraints arise because of margin requirements. We argue that constraints affect and are affected by informational efficiency, leading to a novel amplification mechanism. A decline in wealth tightens constraints and reduces investors' incentive to acquire information, lowering price informativeness. Lower informativeness, in turn, increases the risk borne by financiers who fund trades, leading them to further tighten constraints. This *information spiral* implies that risk premium, return volatility, and Sharpe ratio may rise significantly as investors' wealth declines.

JEL Classification:

Keywords: informational efficiency, information acquisition, funding constraints, margin requirements

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1 Introduction

One of the basic tenets of financial economics is that market prices aggregate information of investors. The core of the argument is that investors acquire information about future asset values and trade on it, thereby impounding that information into price. This argument presupposes that investors have incentives to acquire information and the capacity to trade on it, where each of these factors is crucially affected by investors' ability to fund their trades. Thus an important question arises: how do funding constraints faced by investors affect price informativeness? Conversely, since lower informativeness might have an effect on the financier's risk of funding a trade, another important question is: how price informativeness affects the tightness of funding constraints? Answering these questions requires a model in which price informativeness and funding constraints are jointly determined in equilibrium. Our paper develops such a model and examines its implications for asset pricing.

The main challenge in studying the interplay between funding constraints and informational efficiency is that most noisy rational expectation equilibrium (REE) models, which are instrumental in analyzing informational efficiency, cannot accommodate constraints in a tractable manner.[1] Our first contribution is to develop a tractable REE model with *general portfolio constraints* that can depend on prices; we then apply our methodology to study a model in which portfolio constraints arise because of margin requirements set by financiers. Our second contribution is to show that investors' funding both affects and is affected by informational efficiency, which leads to a novel amplification mechanism that we call the *information spiral*. This mechanism implies that the risk premium, conditional volatility of returns and Sharpe ratio may rise significantly as investors' wealth falls.

We consider a canonical CARA-Normal REE model in which investors first acquire information and then, as in Diamond and Verrecchia (1981), trade in order to profit from their private signals about the risky asset's fundamental value and also to hedge their endowment

[1] Two noteworthy exceptions are Yuan (2005) and Nezafat, Schroder, and Wang (2017); these authors analyze borrowing constraints and short-sale constraints, respectively.

shocks. The novelty is that we allow for general portfolio constraints: investors can trade up only to some maximal long and short positions of the risky asset, and these portfolio constraints can depend on price. This general, price-dependent specification of portfolio constraints subsumes many types of real-world trading constraints (e.g., short-sale constraints, borrowing constraints, margin requirements). Without constraints, the model is standard: (i) an investor's demand is linear in his private signal, the endowment shock, and the price; (ii) the equilibrium price itself is linear in the fundamental value and aggregate endowment shock; and (iii) investors' initial wealth is irrelevant for asset prices.

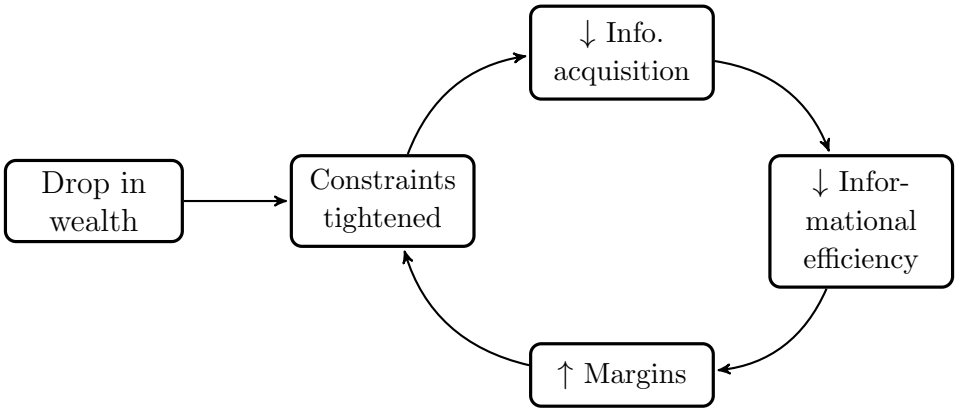
Under portfolio constraints, the financial market equilibrium is as follows. (i) Investors' *desired* demand (i.e., the amount they would like to trade) is still linear, but their *actual* demand is the desired demand truncated to the maximal long or short positions. (ii) Although the price function needs not be linear, it is informationally equivalent to a linear combination of the fundamental value and the aggregate endowment shock; hence inference remains tractable. (iii) Investors' initial wealth matters for asset prices provided that it affects constraints. With the methodology of solving equilibrium with constraints at hand, we turn to study the paper's primary concern: the equilibrium relationship between constraints and informational efficiency.

We commence with an analysis of how constraints affect informational efficiency. Without further specifying the source or form of constraints, we show that they hinder such efficiency. It is intuitive that, when constraints become tighter, investors must take smaller positions and so profit less on their private information. Anticipating the reduced scope for profit, they acquire less information ex ante. As investors acquire less information, the price becomes less informative about asset fundamentals in equilibrium. And to the extent that investors' wealth relaxes their constraints, a wealth effect emerges in our model despite investors' absolute risk aversion being constant: lower wealth impedes information acquisition and hence reduces informational efficiency.

Next we study the reverse channel of informational efficiency affecting constraints. Mo-

tivated by real-world margin requirements, we follow Brunnermeier and Pedersen (2009) in assuming that investors finance their positions through collateralized borrowing from financiers who require margins that control their value-at-risk (VaR).[2] We show that lower informational efficiency leads to tighter margins. Here it is intuitive that, when prices are less informative, financiers face more uncertainty about fundamentals; that uncertainty implies a greater risk of the trade they finance, leading them to set higher margins. When we combine these analyses, our model yields two key implications. First, tighter funding constraints reduce the information acquired by investors, which reduces informational efficiency; second, reduced informational efficiency leads to higher margins, which tightens investors' constraints. This interdependence gives rise to an information-based amplification mechanism, illustrated in Figure 1, that we call the *information spiral*.

Figure 1: Amplification mechanism



There are two key implications of this information spiral. First, small shocks to investors' wealth could be amplified and cause large fluctuations in asset prices. A drop in investors' wealth directly tightens their constraints, discouraging acquisition of information. The resulting lower informational efficiency in turn causes financiers to set higher margin requirements, further tightening investors' constraints. We show that, owing to this amplification mechanism, such a

[2]Our results are robust to alternative risk-based margins, such as tail value-at-risk (TVaR) and expected shortfall (ES).

shock can lead to large increase in the risk premium, return volatility, and Sharpe ratio. Each of these results match empirical observations made during crisis periods.[3] Although the literature has proposed other amplifying mechanisms for the effect of wealth shocks, ours is distinct in this sense: it works via the interaction between the informational efficiency of financial markets and the funding constraints of investors.

The information spiral's second key implication is that investors' decisions to acquire information could become *strategic complements*, i.e., an investor's incentive to acquire information decreases when other investors acquire less information. The reason is that a reduction in information acquired by others makes price less informative, which increases the margin requirements faced by the investor and induces him to acquire less information. Furthermore, this complementarity effect occurs only when investors' wealth is low, which we interpret as a crisis period. Therefore, our result provides a new rationale for why financial markets can be more fragile in crises.

This paper makes several methodological contributions. We present and solve a REE model with general portfolio constraints and compute the marginal value of information for an investor facing these constraints in closed-form.[4] In our main application we consider constraints arising from margin requirements, but one can also utilize our methodology to study other types of constraints.[5]

Related Literature

This paper lies at the intersection of various strands of literature. It shares the emphasis of seminal studies that address the role played by financial markets in aggregating and disseminating

[3]Financial crises, such as the hedge fund crisis of 1998 and the 2007–2008 subprime crisis, have several common characteristics: risk premia rise, the conditional volatility of asset prices rises, and the Sharpe ratio rises.

[4]By using stochastic calculus, we compute the marginal value of information for an investor facing general portfolio constraints directly, without first calculating the value of information and then differentiating it with respect to investor's choice of signal precision.

[5]In Appendix C we apply our methodology to study borrowing constraints as in Yuan (2005).

information, which include Grossman (1976), Grossman and Stiglitz (1980), Hellwig (1980) and Diamond and Verrecchia (1981). In these models, it is generally assumed that investors can borrow or lend freely at the riskless rate—in other words, there are no funding constraints. We contribute to this literature by developing an REE model that incorporates general portfolio constraints. Some particular types of portfolio constraints have been examined before: Yuan (2005) studies an REE model with borrowing constraints; Venter (2015) and Nezafat, Schroder, and Wang (2017) study REE models with short-sale constraints. Our methodology extends the work of Nezafat et al. to explore price-dependent constraints of a more general nature, allowing us to consider constraints resulting from risk-based margin requirements. Albagli, Hellwig, and Tsyvinski (2011) derive various asset pricing implications in a model with exogenous portfolio constraints and exogenous information. Our work differs from these papers in that we study investors' information acquisition problem and focus on the interplay between the tightness of constraints and the equilibrium informational efficiency.

Our work is related to the literature on information acquisition in REE models. Grossman and Stiglitz (1980); Verrecchia (1982); Peng and Xiong (2006); Van Nieuwerburgh and Veldkamp (2009) study financial investor's information acquisition problem in the case of no funding constraints. Peress (2004) and Breugem and Buss (2017) use approximation and numerical methods, respectively, to investigate the effect of investors' wealth on information acquisition in a setting with investors who exhibit constant relative risk aversion (CRRA). Our tractable model also features wealth effects, despite the investors having constant absolute risk aversion, because investors' wealth relaxes their funding constraints.

In addition, we contribute to the literature on strategic complementarities in information acquisition, for example, Veldkamp (2006); Hellwig and Veldkamp (2009); Garcia and Strobl (2011); Ganguli and Yang (2009); Goldstein and Yang (2015); Avdis (2016) and Dow, Goldstein, and Guembel (2017). The main distinguishing feature of our model is that complementarities arise only when wealth is low—that is, during times of crisis.

Our paper is also related to the literature on secondary financial markets as a source of information for decision makers; see Bond, Edmans, and Goldstein (2012) for a survey. We contribute to this literature by studying how financiers can use the information in prices to set their margins, and we find that lower informational efficiency leads to tighter margins.

Finally, our work contributes to the literature on the effect of investors' wealth and the associated amplification mechanisms. For example, Xiong (2001) and Kyle and Xiong (2001) study wealth constraints as amplification and spillover mechanism, respectively. Gromb and Vayanos (2002, 2017) develop an equilibrium model of arbitrage trading with margin constraints to explain contagion. Brunnermeier and Pedersen (2009) examine how funding liquidity and market liquidity reinforce each other. He and Krishnamurthy (2011, 2013) and Brunnermeier and Sannikov (2014) study how declines in an intermediary's capital reduce her risk-bearing capacity and lead to higher risk premia and conditional volatility; see also He and Krishnamurthy (2018) for a survey of the topic. None of these paper studies informational efficiency, which is the crux of our paper.

The rest of our paper is organized as follows. In Section 2, we solve for the financial market equilibrium and the value of information in an REE model with general portfolio constraints. Section 3 introduces margin requirements and shows how funding constraints affect—and are affected by—informational efficiency. In Section 4, we explore the implications of our information spiral for asset prices. After summarizing our predictions in Section 5, we conclude in Section 6. Appendices A and B contain all the proofs. Appendix C contains an alternative application of our methodology.

2 An REE model with general portfolio constraints

In this section we develop a model with general portfolio constraints. In Section 3, we will apply our model to study constraints that arise from margin requirements.

2.1 Setup

There are three dates (i.e., $t \in \{0, 1, 2\}$) and two assets. The risk-free asset has exogenous (net) return normalized to zero. The payoff (fundamental value) of the risky asset is $f = v + \theta$ (which is paid at date 2), where v is the learnable (i.e., information about which can be acquired) component of fundamentals, $v \sim N(0, \tau_v^{-1})$ and θ is the unlearnable component of fundamentals, $\theta \sim N\left(0, \tau_\theta^{-1}\right)$ and is independent of v. The aggregate supply of the asset is assumed to be constant 1 unit. The economy is populated by a unit continuum of investors, indexed by $i \in [0, 1]$, with identical CARA preferences over terminal wealth with CARA parameter γ. There is also a competitive market maker with CARA preferences over terminal wealth with CARA parameter γ_m.[6] Investors acquire information at $t = 0$ and trade the risky asset with the market maker at $t = 1$. All agents consume at $t = 2$.

Investors trade the risky asset for hedging and profit reasons. Specifically, at date 2, each investor receives a random, non-tradable, and non-pledgeable endowment b_i, which has a payoff that is correlated with the unlearnable component of the risky asset's payoff, θ. We assume that the endowment is given by $b_i = e_i\theta$. The coefficient e_i measures the sensitivity of the endowment shock to the payoff of the risky asset and is known to the investor at $t = 1$. Hereafter, we will refer to e_i as the endowment shock of investor i. Finally, the investor i's endowment shock e_i has systematic and idiosyncratic components: $e_i = z + u_i$. Both components are normally distributed and independent of v and θ, with $z \sim N\left(0, \tau_z^{-1}\right)$ and $u_i \sim N(0, \tau_u^{-1})$. Moreover, idiosyncratic shocks u_i are independent across investors and independent of z. This formulation implies that there is uncertainty about the aggregate endowment shock z, which will create noise in the price. Differences in exposures "e_i" across investors motivate trade in the risky asset.

At date 1, each investor i receives a signal $s_i = v + \epsilon_i$, where the ϵ_i are independent

[6] This specification of market maker nests two benchmarks. When $\gamma_m = 0$, our market maker is risk-neutral as in Vives (1995). When $\gamma_m = \infty$, market maker does not trade, hence our model is equivalent to a model without market maker, such as Diamond and Verrecchia (1981).

across investors with $\epsilon_i \sim N(0, \tau_{\varepsilon_i}^{-1})$. The precision of his private signal τ_{ϵ_i} is optimally chosen by investor i at date 0, subject to a cost function $C(\tau_{\epsilon_i})$. We assume that this cost function is identical for all investors. When forming their expectations about the fundamental, investors use all the information available to them. The information set of investor i at time 1 is $\mathcal{F}_i = \{p, s_i, e_i\}$, where p is the equilibrium price at time 1. A competitive market maker faces no endowment shocks and receives no signals about the asset payoff. Hence, the market maker's information set at time 1 is $\mathcal{F}_m = \{p\}$.

Constraints. The investors in our model—but not the market maker—are subject to the funding constraints described here.[7] Given the price p, the minimum and maximum positions that an investor can take are $a(p)$ and $b(p)$, respectively, with $a(p) < b(p)$. The functions $a(p)$ and $b(p)$ may depend on investors' initial wealth W_0 and other aggregate equilibrium variables, such as volatility of returns. We do not indicate this dependence explicitly whenever no confusion could arise. In short: at date 1, investors solve the problem

$$\begin{aligned} \max_{x_i(p,s_i,e_i)} \quad & E[-\exp(-\gamma W_i) \mid p, s_i, e_i], \\ \text{subject to} \quad & a(p) \leq x_i(p, s_i, e_i) \leq b(p), \\ \text{where} \quad & W_i = W_0 + x_i(v + \theta - p) + e_i\theta. \end{aligned} \tag{1}$$

The equation above states that the terminal wealth of investor i is the sum of his initial wealth, the profit or loss from trading the risky asset, and his endowment.

Similarly, the market maker solves

$$\begin{aligned} \max_{x_m(p)} \quad & E[-\exp(-\gamma_m W_m) \mid p], \\ \text{where} \quad & W_m = W_{0,m} + x_m(v + \theta - p). \end{aligned} \tag{2}$$

[7] We assume the market maker is unconstrained because our focus is on the interplay between investors' constraints and informational efficiency. Nevertheless, our model remains tractable if the market maker is also subject to constraints.

Finally, the equilibrium price is set to clear the market:

$$\int x_i(p, s_i, e_i)di + x_m(p) = 1. \tag{3}$$

We proceed to solve the model via backward induction. In Section 2.2 we characterize the financial market equilibrium at $t = 1$ for given investors information-acquisition decisions made at $t = 0$. Then, in Section 2.3, we solve for the investor's optimal information acquisition decision.

2.2 Financial market equilibrium at $t = 1$

We first solve for equilibrium in the unconstrained setting (i.e., when $a(p) = -\infty$ and $b(p) = \infty$), which was studied previously in Biais, Bossaerts, and Spatt (2010).[8] We review this setting here because it is an important step in characterizing the equilibrium with constraints.

2.2.1 Unconstrained setting

Our first proposition characterizes the unconstrained equilibrium and its key features. Unless stated otherwise, proofs of all propositions are given in Appendix A. From now on, we use superscript "u" for variables characterizing the unconstrained setting. The corresponding variables without superscript are used for the constrained setting.

Proposition 1. *(Financial market equilibrium without portfolio constraints) Suppose investors have identical signal precision τ_ϵ and $\tau_u^2\tau_\theta^2 < 3\gamma^2(\tau_u + \tau_z)\tau_v$. Then there exists a unique linear equilibrium in which the price is informationally equivalent to a statistic $\phi^u = v - \frac{z}{\beta^u} = g_0^u + g_1^u p$. The aggregate demand of investors and the market maker can be written as*

$$X^u(p, \phi) = c_0 + c_\phi \phi - c_p p \quad \textit{and} \quad x_m(p, \phi) = c_0^m + c_\phi^m \phi - c_p^m p,$$

[8] See also Ganguli and Yang (2009) and Manzano and Vives (2011) who analyzed related settings.

respectively. The individual demand of investor i can be written as follows:

$$x_i^u = X^u + \xi_i, \quad \textit{where } \xi_i \sim \mathcal{N}(0, \sigma_\xi^2) \textit{ are i.i.d. across investors,}$$

and β^u is the unique root (β) which solves

$$\beta^3 \gamma (\tau_u + \tau_z) - \beta^2 \tau_u \tau_\theta + \beta \gamma (\tau_\epsilon + \tau_v) - \tau_\theta \tau_\epsilon = 0. \tag{4}$$

Moreover, β^u increases with τ_ε, the precision of investors' information. All the coefficients are reported in Appendix A.

The analysis of unconstrained equilibrium highlights some important features of the model that will continue to hold in the constrained setting. We observe first of all that, in equilibrium, price is informationally equivalent to a linear combination of the (learnable) fundamental payoff v and the aggregate endowment shock z. Second, the extent of fundamental information revealed by price is captured by an endogenous signal-to-noise ratio (β^u). More precisely, the conditional variance of the learnable fundamental decreases as β^u increases:

$$Var(v|p) = Var(v|\phi^u) = (\tau_v + (\beta^u)^2 \tau_z)^{-1} \tag{5}$$

Hence we refer to β^u as the *informational efficiency* of the market when investors are unconstrained in their trading. It is important to bear in mind that investors' information acquisition (higher signal precision τ_ϵ) improves the informational efficiency β^u of the market.

Note that the condition $\tau_u^2 \tau_\theta^2 < 3\gamma^2 (\tau_u + \tau_z) \tau_v$ is sufficient to guarantee the uniqueness of a linear equilibrium without constraints. We shall proceed under the assumption that this condition continues to hold.

2.2.2 Constrained setting

We now impose the portfolio constraints $a(p)$ and $b(p)$ on the investor's problem. We posit and then verify that there exists a generalized linear equilibrium in the economy, which we define as follows.

Definition 1. *An equilibrium is generalized linear if there exists a function $g(p)$ and a scalar β, such that $\phi = v - \frac{z}{\beta}$ is informationally equivalent to price and is given by $\phi = g(p)$.*[9]

The ϕ and β defined here are the counterparts of ϕ^u and β^u in the economy without portfolio constraints. In a generalized linear equilibrium, the price function may be nonlinear but the statistic ϕ is still linear in (v, z) and so is normally distributed; therefore, the inference from price remains tractable. Since equation (5) holds in a generalized linear equilibrium, we continue using β to denote informational efficiency.

When there are constraints, the individual demand of investor i can be written as follows:

$$x_i(p, s_i, e_i) = \begin{cases} x_i^d(p, s_i, e_i), & \text{if } a(p) \leq x_i^d(p, e_i, s_i) \leq b(p), \\ b(p), & \text{if } x_i^d(p, s_i, e_i) > b(p), \\ a(p), & \text{if } x_i^d(p, s_i, e_i) < a(p), \end{cases}$$

where $x_i^d(p, s_i, e_i)$ denotes investor i's desired demand, or the amount he would like to trade, in the absence of constraints.

To solve for the equilibrium with constraints, one needs to pin down the informational efficiency β, the function $g(p)$, and investors' desired demand $x_i^d(p, s_i, e_i)$. We do that in the following proposition.

Proposition 2. *(Financial market equilibrium with portfolio constraints) Suppose that investors face portfolio constraints and have identical signal precision τ_ϵ. Then there exists a*

[9] We say that ϕ is informationally equivalent to price p if conditional distributions of $v|\phi$ and $v|p$ are the same. Our notion of a generalized linear equilibrium follows Breon-Drish (2015).

unique pair $\{g(p), \beta\}$ *that constitutes a generalized linear equilibrium in which informational efficiency* $\beta = \beta^u$. *Furthermore, investor i's desired demand* $x_i^d(p, s_i, e_i)$ *is equal to* $x_i^u(p, s_i, e_i)$, *where* $x_i^u(p, s_i, e_i)$ *is characterized in Proposition 1. The function* $g(p)$ *is determined as follows. For every* p, $g(p)$ *is the unique* ϕ *that solves*

$$X(p, \phi) + x_m(p, \phi) = 1;$$

here the demand $x_m(p, \phi)$ *of market makers is given in Proposition 1 and the closed-form expression for investors' aggregate demand* $X(p, \phi)$ *is given in Appendix A. If both* $a(p)$ *and* $b(p)$ *are continuously differentiable, then* $g(p)$ *can be determined by solving the ordinary differential equation (ODE)*

$$g'(p) = -\frac{\pi_1(p, g(p))a'(p) + \pi_3(p, g(p))b'(p) - \pi_2(p, g(p))c_p - c_p^m}{\pi_2(p, g(p))c_\phi + c_\phi^m} \tag{6}$$

subject to the boundary condition $g(0) = g_0$, *where the constant* g_0 *is the unique solution to* $X(0, g_0) + x_m(0, g_0) = 1$. *The term* $\pi_1(p, \phi) = \Phi\left(\frac{a(p) - X^u(p,\phi)}{\sigma_\xi}\right)$ *is for the fraction of investors whose lower constraint binds,* $\pi_3(p, \phi) = 1 - \Phi\left(\frac{b(p) - X^u(p,\phi)}{\sigma_\xi}\right)$ *denotes the fraction of investors whose upper constraint binds,* $\pi_2(p, \phi) = 1 - \pi_1(p, \phi) - \pi_3(p, \phi)$ *is the fraction of unconstrained investors, and* $\Phi(\cdot)$ *stands for the cumulative distribution function (CDF) of a standard normal distribution.*

Proposition 2 is our first main result establishing the existence of a tractable, generalized linear equilibrium in an REE model with portfolio constraints, even when price may be nonlinear. It also states that, for an *exogenously given signal precision* τ_ϵ, portfolio constraints are irrelevant for the informational efficiency ($\beta = \beta^u$). This result is the key to our model's tractability. Instead of solving for β in the complex model with constraints, we can solve the simpler unconstrained model. Nonetheless, it would be premature to conclude that constraints do not matter for informational efficiency: in Section 2.3, we show that constraints affect the

amount of information acquired by investors at $t = 0$. It is when the signal precision τ_ϵ becomes *endogenous* that constraints affect informational efficiency.

Our irrelevance result is not only instrumental for the model's tractability, but also sheds light on the way price aggregates information in an economy with portfolio constraints. In essence, this result underscores that, even with constraints, the aggregate demand of investors (and hence the market-clearing price) still varies with and reflects fundamentals via changes in the *fractions* of constrained investors. Consider an improvement in the asset fundamental v (while fixing the endowment shock z), which leads investors to increase their demand for the risky asset. Although some investors cannot increase their demand owing to the upper portfolio constraint, in aggregate more (resp. fewer) investors become constrained by a maximal long (resp. short) position. According to the exact law of large numbers, aggregate demand will increase almost surely and thereby reveal the improved asset fundamentals via a higher market-clearing price.[10]

Besides the informational efficiency β, the other important equilibrium object $g'(p)$, which is given in equation (6), captures how much the statistic ϕ changes when the price p changes by a single unit. The numerator in (6) represents aggregate demand's price sensitivity, which derives from four sources. First is the fraction π_1 of investors constrained by the lower constraint, whose demand has price sensitivity $a'(p)$. Second, a similar effect applies for the fraction π_3 of investors for whom the upper constraint $b(p)$ binds. Third, there is a fraction π_2 of unconstrained investors whose demand has price sensitivity $\frac{\partial X^u}{\partial p} = -c_p$. The numerator's last term is the market maker's demand sensitivity to price, $\frac{\partial x_m}{\partial p} = -c_p^m$. The denominator of (6), which represents the sensitivity of aggregate demand to ϕ, can be interpreted similarly. Equation (6) clearly demonstrates that constraints affect the shape of the function $g(p)$. In

[10]The irrelevance result we describe is related to—yet differs from—the one in Dávila and Parlatore (2017). Instead of portfolio constraints, these authors study the impact of various forms (quadratic, linear, or fixed) of trading cost on informational efficiency. They find that, when investors are ex ante homogeneous, trading cost reduces *each* investor's trading incentives symmetrically with respect to information and hedging. In equilibrium, then, the signal-to-noise ratio of price is unaffected. However, it is important to note that portfolio constraints differ from trading costs in that the former affect some investors' trading ex post but not the trading of others. Thus the logic underlying our irrelevance result differs from that of Dávila and Parlatore.

general—and in contrast to standard CARA-Normal models—this function is nonlinear.[11]

2.3 Information acquisition at $t = 0$

Having solved for the financial market equilibrium at $t = 1$, we now study how portfolio constraints affect the incentives of investors to acquire information at $t = 0$. We maintain the assumption $a(p) \leq 0 \leq b(p)$ and say that constraints are *tightened* when $a(p)$ increases and/or $b(p)$ decreases. We start by deriving an expression for the marginal value of information under general portfolio constraints, after which we show that an investor's marginal value of information declines if his constraints are tightened.

At date 0, investor i decides on the optimal amount of information to acquire by solving this problem:

$$\max_{\tau_{\epsilon_i}} \quad E\left[u_0\left(E\left[-e^{-\gamma\left(W_i - C\left(\tau_{\epsilon_i}\right)\right)}|\mathcal{F}_i\right]\right)\right].$$

The investors' preference at $t = 0$ depends on the specification of the function u_0, which governs their preference for the timing of resolution of uncertainty. If u_0 is linear, the investor is an expected utility maximizer and is indifferent about timing of the resolution of uncertainty. If u_0 is convex, the investor has preferences for early resolution of uncertainty. See Van Nieuwerburgh and Veldkamp (2010) for further discussion. We consider two specifications commonly used in the literature: $u_0(x) = x$, under which investors are expected utility maximizers; and $u_0(x) = -\frac{1}{\gamma}\log(-x)$, under which investors are mean-variance maximizers who prefer an early resolution of uncertainty. We find that our results remain qualitatively the same under both specifications. We will write down the investors' preferences under both cases, with the help of the investors' date-1 certainty equivalent $CE_{1,i} \equiv E[W_i|\mathcal{F}_i] - \frac{\gamma}{2}Var[W_i|\mathcal{F}_i]$, characterized in

[11] The fact that constraints affect the sensitivity $g'(p)$ distinguishes our setting from those in Dávila and Parlatore (2017) and Nezafat et al. (2017), where trading costs and short-sale constraints, respectively, are irrelevant not only for informational efficiency β but also for sensitivity $g'(p)$.

the following Lemma.

Lemma 1. *The date-1 certainty equivalent is given by*

$$CE_{1,i} = \underbrace{W_0 + \frac{\gamma}{2\tau_i}\left(x_i^u\right)^2 - \frac{\gamma}{2\tau_\theta}e_i^2}_{\textit{CE absent constraints}} \underbrace{-\frac{\gamma}{2\tau_i}\left(x_i^u - x_i\right)^2}_{\textit{Effect of constraints, <0}},$$

where $\tau_i = Var(f|\mathcal{F}_i)$.

The certainty equivalent at date 1 includes a new term because of the constraints. This term captures the "distance" between an investor's desired demand and his actual demand. It is immediate to see that, ceteris paribus, the certainty equivalent decreases as constraints become tighter.

We now write the investor's preferences at date 0 and solve his information acquisition problem

Case 1: $u_0(x) = x$. The investor's problem at $t = 0$ becomes

$$\max_{\tau_{\epsilon_i}} \quad E\left[-e^{-\gamma\left(CE_{1,i} - C\left(\tau_{\epsilon_i}\right)\right)}\right],$$

which is equivalent to

$$\max_{\tau_{\epsilon_i}} \quad CE_0\left(\tau_{\epsilon_i}\right) - C\left(\tau_{\epsilon_i}\right),$$

where the date-0 certainty equivalent CE_0 is the solution to $e^{-\gamma CE_0} = E[e^{-\gamma CE_{1,i}}]$.

Case 2: $u_0(x) = -\frac{1}{\gamma}\log(-x)$. Now the investor's problem at $t = 0$ becomes

$$\max_{\tau_{\epsilon_i}} \quad E[CE_{1,i}] - C(\tau_{\epsilon_i}),$$

which is equivalent to

$$\max_{\tau_{\epsilon_i}} \quad CE_0\left(\tau_{\epsilon_i}\right) - C\left(\tau_{\epsilon_i}\right),$$

where the date-0 certainty equivalent is given by $CE_0 = E[CE_{1,i}]$.

In both cases, we define the *marginal value of information* as $\text{MVI} \equiv \text{CE}_0'(\tau_{\varepsilon_i})$. In the next proposition, we characterize this marginal value of information under general portfolio constraints and show that it declines when an investor's constraints tighten.

Proposition 3. *(Marginal value of information) The marginal value of information for an investor i choosing signal precision τ_{ϵ_i}, while others' signal precisions are τ_ϵ, is given by:*

$$\text{MVI}(\tau_{\epsilon_i}, \tau_\epsilon) = \underbrace{\frac{\tau_i}{2\tau_{v,i}^2\gamma}}_{\text{MVI } \textit{absent constraints}} + \underbrace{\frac{\tau_i}{2\tau_{v,i}^2\gamma}\left(\frac{U_0^u}{U_0} - 1\right)}_{\textit{Effect of constraints, } <0}, \tag{7}$$

in Case 1, and

$$\text{MVI}(\tau_{\epsilon_i}, \tau_\epsilon) = \underbrace{\frac{\gamma}{2\tau_{v,i}^2}E[(x_i^u)^2] + \frac{\tau_i}{2\gamma\tau_{v,i}^2}}_{\text{MVI } \textit{absent constraints}} + \underbrace{\frac{\gamma}{2\tau_{v,i}^2}E[(x_i)^2 - (x_i^u)^2] + \frac{\tau_i}{2\gamma\tau_{v,i}^2}(E[\mathbb{I}_{x_i^u = x_i}] - 1)}_{\textit{Effect of constraints, } < 0}, \tag{8}$$

in Case 2. In these expressions, $\tau_{v,i} = Var(v|\mathcal{F}_i)$ is the total precision of investor i's information about the learnable component; $U_0^u(\tau_{\epsilon_i}, \tau_\epsilon) = E[-e^{-\gamma CE_{1,i}}\mathbb{I}_{x_i^u = x_i}]$ is the expectation of utility in the states when constraints do not bind; $U_0(\tau_{\epsilon_i}, \tau_\epsilon) = E[-e^{-\gamma CE_{1,i}}]$ is date-0 expected utility; $E[\mathbb{I}_{x_i^u = x_i}]$ is the ex ante probability of being unconstrained.

In both cases, the marginal value of information decreases when individual investor's constraints become tighter, ceteris paribus.

Proposition 3 shows how portfolio constraints affect an investor's incentive to acquire information. The introduction of constraints reduces that incentive, as the terms capturing the effects of constraints—in equations (7) and (8)—are negative. It makes sense that an investor considers information valuable to the extent he can profit from it.

Next, we study how the equilibrium information acquisition changes when the portfolio constraints of all investors become tighter. Tightening constraints for all investors is more complicated because the equilibrium price distribution will change due to the market maker's

risk aversion, which in turn affects price-dependent constraints. If the market maker is risk neutral, then we can prove that tightening the constraints for all investors reduces each investor's marginal value of information. This result is stated formally in our next proposition.

Proposition 4. *Suppose that all investors face portfolio constraints $a(p)$ and $b(p)$ and that the market maker is risk neutral. If constraints become tighter for all investors, i.e. $a(p)$ increases and $b(p)$ decreases $\forall p$, the marginal value of information decreases for all investors for both specifications of the function $u_0(\cdot)$.*

Proposition 4 illustrates one of the key forces of our mechanism: tighter constraints reduce investors' incentive to acquire information and hence the informational efficiency of prices. To close the model with the characterization of the effect of informational efficiency on the tightness of constraints, we need to further specify the nature of the portfolio constraints. In the rest of the paper, we focus on margin requirements and study their interactions with informational efficiency.

3 Portfolio constraints arising from margin requirements

So far we have studied general, price-dependent portfolio constraints. In this section, we apply our model to study the constraints arising from margin requirements. Toward the end of demonstrating analytically our paper's main mechanism—namely, the interaction between funding constraints and informational efficiency—we shall assume throughout that the market maker is risk neutral (i.e., $\gamma_m = 0$). Our analytical results hold for both specifications of the function $u_0(\cdot)$, but we simplify the exposition by reporting numerical results only for $u_0(x) = -\frac{1}{\gamma}\log(-x)$.[12] In Section 4 we relax the assumption of a risk-neutral market maker and illustrate numerically our model's implications for asset prices.

[12] Our numerical results are qualitatively similar under the expected utility preference $u_0(x) = x$.

3.1 Wealth effect with margin requirements

Our notion of margin requirements is standard and closely follows Brunnermeier and Pedersen (2009). To build a long position in the risky asset, an investor can borrow from a financier at the risk-free rate but he has to pledge cash margin of $m^+(p) \geq 0$ per unit of asset to the financier as collateral. The investor can similarly establish a short position by providing, as collateral, a cash margin of $m^-(p)$ per unit of asset. Thus investors face a funding constraint that the total margin on their positions cannot exceed their initial wealth:

$$m^-(p)[x_i]^- + m^+(p)[x_i]^+ \leq W_0,$$

where $[x_i]^-$ and $[x_i]^+$ are respectively the positive and negative parts of x_i.[13] We can rewrite the margin requirements in the form of portfolio constraints as

$$a(p) = -\frac{W_0}{m^-(p)}, \quad b(p) = \frac{W_0}{m^+(p)} \tag{9}$$

Equations (9) show that an investor faces tighter constraints when his initial wealth is lower and/or if the financier's margin requirements are higher. We shall delay until Section 3.2 a discussion of how the financier sets margins. For now, we simply assume that margins are independent of the price; we later prove that, in equilibrium, this is indeed the case with a risk-neutral market maker.

We proceed with solving the model under margin requirements backwards. The financial market equilibrium at $t = 1$ is just a special case of Proposition 2, so the next result is a straightforward extension.

Corollary 1. *Suppose that investors have identical signal precisions τ_ϵ and face margin requirements that do not depend on prices, then there exists a unique generalized linear equilibrium in which informational efficiency $\beta = \beta^u$ and the function $g(p) = g_0 + g_1 \cdot p$, where the constants*

[13] Since the endowment b_i is not pledgeable, it cannot be used as a collateral to satisfy the margin requirements.

g_0 and g_1 are given in Appendix A.

Working backwards, we next characterize the equilibrium at $t = 0$, the information acquisition stage. In any symmetric equilibrium, investors acquire information until the marginal cost of doing so equals the marginal value of information. The result below builds upon Propositions 3 and 4.

Proposition 5. *Equilibrium information acquisition (τ_ϵ^*) at $t = 0$ satisfies, for Case 1 (i.e., $u_0(x) = x$) and Case 2 (i.e., $u_0 = -\frac{1}{\gamma}\log(-x)$), respectively:*

$$C'(\tau_\epsilon^*) = \frac{\tau_i}{2\tau_{v,i}^2\gamma}\frac{U_0^u(\tau_\epsilon^*, \tau_\epsilon^*)}{U_0(\tau_\epsilon^*, \tau_\epsilon^*)},$$
$$C'(\tau_\epsilon^*) = \frac{\gamma}{2\tau_{v,i}^2}E\left[(x_i)^2\right] + \frac{\tau_i}{2\gamma\tau_{v,i}^2}E\left[\mathbb{I}_{x_i^u = x_i}\right].$$

In addition, for both cases, the equilibrium precision τ_ϵ^* and equilibrium informational efficiency β in a stable equilibrium decrease when initial wealth W_0 drops and/or margins m^+ and m^- increase for all investors.[14]

Proposition 5 implies that wealth plays an important role in our model with constraints—in contrast to typical CARA-Normal models. As investors' initial wealth decreases, they become more constrained and hence acquire less information, reducing price informativeness in equilibrium. Similarly, an increase in the margins m^+ and m^- reduces price informativeness. The effects of wealth and margins on informational efficiency are the key results in this subsection and contribute towards the information spiral, which we discuss in Section 3.3.

[14]Our notion of stability is as in Manzano and Vives (2011) and Cespa and Foucault (2014) and is standard in game theory (see Fudenberg and Tirole (1991), Chapter 1, Section 1.2.5). We call an equilibrium stable if the fixed point determining equilibrium precision of investors' signals is stable. More specifically, we call an equilibrium stable if $|\tau'_{\epsilon_i}(\tau_\epsilon^*)| < 1$, where $\tau_{\epsilon_i}(\tau_\epsilon^*)$ is investor i's optimal choice of precision given that all other investors' precisions are equal to τ_ϵ^*. Numerically, we find that the equilibrium is stable for fixed margins.

3.2 Value-at-risk based margin requirements

Until now we have assumed that margins are fixed—in other words, they are not determined as part of the equilibrium. Here we assume that each financier sets her margin in order to control her value-at-risk, as in Brunnermeier and Pedersen (2009):

$$m^+(p) = \inf\{m^+(p) \geq 0 : \Pr(p - v > m^+(p) \mid p) \leq 1 - \alpha\},$$
$$m^-(p) = \inf\{m^-(p) \geq 0 : \Pr(v - p > m^-(p) \mid p) \leq 1 - \alpha\}; \tag{10}$$

here "Pr" signifies "probability". It says that the financier require the investors to set aside a minimum amount of cash (i.e., margin) large enough to cover, with probability α, the potential loss from trading. We assume that the financier is uninformed but can condition her margins on prices. As detailed in Brunnermeier and Pedersen (2009, Appendix. A), this margin specification is motivated by the real-world margin constraints faced by hedge funds and the capital requirements imposed on commercial banks. Observe that we allow margins to depend on prices; yet we later show that, in equilibrium, they do not depend on prices if the market maker is risk neutral.

We describe our financial market equilibrium with VaR-based margin constraints as follows. (i) Financiers set their margin requirements according to (10), given a conjectured price function. (ii) Investors and the market maker choose their optimal demand given the margin requirements and the conjectured price function. (iii) In equilibrium, the conjectured price function is consistent with market clearing. As before, we take the precisions of investors' signals as given.

Proposition 6. *(Financial market equilibrium under VaR-based margin requirements) If portfolio constraints are of the form of margin requirements, as in equation (9) and if margins are determined by value-at-risk, as in (10), then there exists a unique generalized linear equilibrium in which the function $g(p)$ is as characterized by Corollary 1 and the equilibrium margins are*

given by

$$m^+ = m^- = \Phi^{-1}(\alpha)\sqrt{Var[f-p|p]} = \Phi^{-1}(\alpha)\sqrt{(\tau_v + \beta^2\tau_z)^{-1} + \tau_\theta^{-1}}.$$

Consequently, for a given investors' wealth W_0, if informational efficiency (β) decreases then the margins (m^+ and m^- both) increase. This implies that the lower constraint (i.e., a) increases and the upper constraint (i.e., b) decreases. In other words: as informational efficiency declines, constraints become tighter.

The proposition above establishes the uniqueness of the equilibrium with VaR-based margin requirements. Moreover, in this unique equilibrium, a decrease in informational efficiency leads to higher margins and tighter constraints, which is the key result in this subsection. The intuition is as follows. Financiers use information embedded in prices to assess the risk of their losses from financing exceeding the margin. If prices are less informative then they face more uncertainty about fundamentals; hence they perceive greater risk of financing the trade and so require higher margins. In turn, higher margins imply tighter constraints.

Remark 1: *Informational efficiency affects constraints even if financiers do not learn from prices.* We emphasize that this section's results do not rely on financiers learning from prices. Indeed, one can compute the *unconditional* variance of returns as

$$Var[f-p] = E[Var[f-p|p]] + Var[E[f-p|p]] = E[Var[f-p|p]].$$

It follows from this expression that, for a given equilibrium signal precision τ_ϵ^*, the conditional variance $Var[f-p|p]$ is constant, and therefore equal to the unconditional variance $Var[f-p]$. This result the financier will set the same margins irrespective of whether (or not) she learns from prices.

Remark 2: *Alternative risk-based margins.* Our result that margins increase when informational efficiency falls holds also for alternative risk-based margins, such as tail value-at-risk (TVaR) and expected shortfall (ES). This is because all these risk measures depend on the

conditional distribution of the loss $p - f$ (resp. $f - p$) for a long (resp. short) position given p. If the market maker is risk-neutral, then $p = E[f|p]$ and this distribution is normal with mean zero and variance $Var[f|p]$. Hence, the distribution is parameterized by a single parameter, $Var[f|p]$. Since VaR, ES and TVaR are all monotone in $Var[f|p]$, it follows that results in this section are robust to using these alternative risk-based margins.

3.3 Information spiral

In Section 3.1, we undertook a partial equilibrium analysis and argued that given margins, tighter funding constraints (e.g., reductions in wealth) lead to lower informational inefficiency because investors acquire less information (Proposition 5). In Section 3.2, we argued that, for a given level of wealth, lower informational efficiency leads to higher margins (Proposition 6). Putting these two results together yields the amplification loop that we call the information spiral (see Figure 1, in Section 1, for an illustration). The main implication of this spiral is that small changes in the underlying funding constraints can lead to sharp reductions in information acquisition and hence in informational efficiency.

3.3.1 Amplification

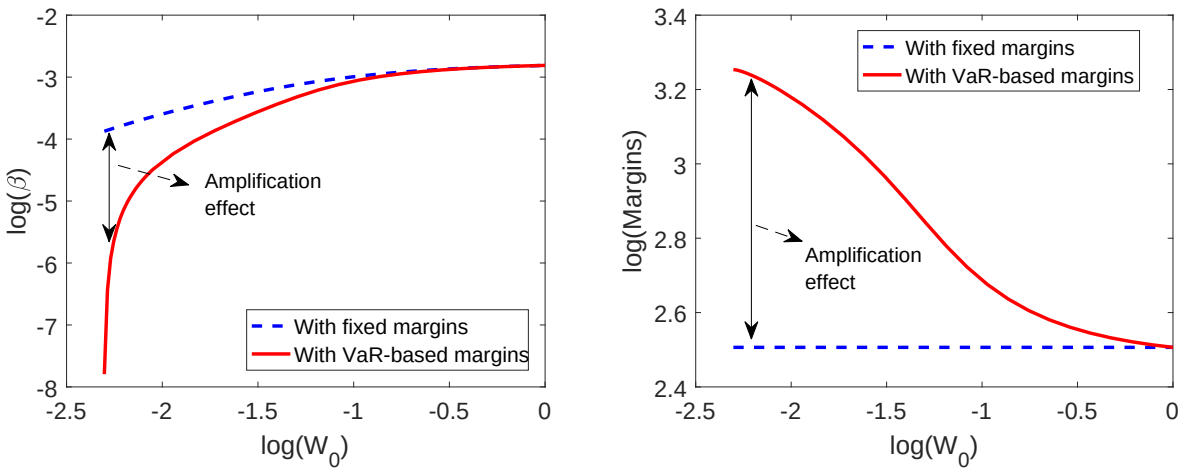
As illustrated in Figure 1, the information spiral could amplify a small shock to investor wealth into large decreases in informational efficiency (β) and large increases in margin requirements (m^+ and m^-). We illustrate these amplification effects numerically in Figure 2. Figure 2 illustrates the effect of reduced investor wealth on the informational efficiency of prices, in Panel (a), and on equilibrium margins, in Panel (b), under fixed and under VaR-based margins. In the case of fixed margins (the dashed lines), as investors have lower wealth, they face tighter funding constraints and acquire less information. This results in lower informational efficiency, whereas by assumption margins are fixed.

The amplification effects of VaR-based margins are shown by the solid lines in this figure.

The effect of a decline in wealth is more pronounced than in the case with fixed margins because the decrease in informational efficiency leads to greater volatility of returns and thus to higher margins, further tightening the funding constraints. Note that the amplification effect is much stronger in bad times—that is, when the investor's wealth is low. It follows from the observation that this amplification arises due to binding funding constraints and the constraints bind more often when the investors have less wealth.

Figure 2: The effect of investors' wealth on informational efficiency and margins

The figure plots the informational efficiency β and margins as a function of investors' wealth. The dashed line corresponds to the case of fixed margins and the solid line to value-at-risk based margins. The fixed margins are chosen to be equal to the VaR-based margins at log wealth level of 0. We assume that the cost function is the standard entropy cost function: $C(\tau_{\epsilon_i}) = k_0 \log(1 + \frac{\tau_{\epsilon_i}}{\tau_v})$. Other parameter values are set to: $\tau_u = \tau_\theta = 1$, $\tau_v = 0.01$,$\tau_z = 10$,$\gamma = 10$,$\gamma_m = 0$.



(a) Informational efficiency

(b) Margins

3.3.2 Complementarity in information acquisition

Our feedback mechanism suggests that investors' decisions to acquire information could be strategic complements. An investor's incentive to acquire information decreases when other investors acquire less information because of the VaR-based margins. As less information is acquired by other investors, the price becomes less informative about the asset fundamentals; hence financiers set higher VaR-based margins and so, with a tightened funding constraint, the

investor values information less.

As standard in REE models (e.g., Grossman and Stiglitz (1980)), there is also a substitutability effect in information acquisition: when other investors acquire more information, price is more informative about fundamentals and hence there is less incentive for an investor to acquire private information. The question is then when the complementarity effect dominates the substitutability effect. Since the complementarity effect arises due to constraints, which are more likely to bind when investors have lower wealth, we expect that the information acquisition by investors could be strategic complements when investor wealth is low.

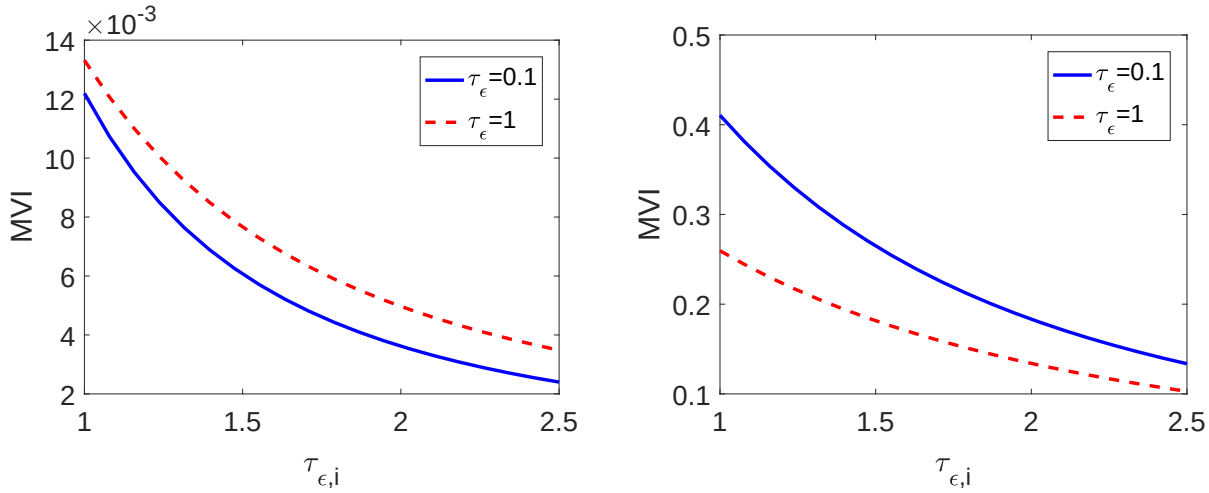
This conjecture is supported by numerical simulations. Figure 3 plots an investor's marginal value of information (MVI) as a function of his signal's precision (τ_{ϵ_i}) at various levels of the precision of others' signals (τ_ϵ). When the wealth of investors is low, as in Panel (a) of the figure, the marginal value of information is higher when others acquire more information (i.e., the dashed line is above the solid line); thus information acquisition choices are strategic complements. Yet if investor wealth is high, as in Panel (b), then those choices are strategic substitutes.

4 Asset pricing implications

In this section we derive the implications of a decline in investor wealth on the risky asset's equilibrium risk premium and return volatility. For that purpose, we need to relax the assumption of a risk-neutral market maker. In Appendix B, we describe how financiers determine margins and also characterize the financial market equilibrium with a risk-averse market maker. Our main result in this section is that a small drop in wealth, ceteris paribus, could lead to large rise in the risk premium, return volatility, and Sharpe ratio.

Figure 3: Complementarity vs. substitutability in information acquisition

The figure shows the marginal value of information as a function of the precision of an investor's signal for two different levels of other investors' precision. The solid line corresponds to the case where other investors have less precise signals ($\tau_\epsilon = 0.1$) and the dashed line to more precise signals ($\tau_\epsilon = 1$). Panel a represents the case when investors' wealth is low ($W_0 = 0.5$) and Panel b when it is high ($W_0 = 10$). Other parameter values are set to: $\tau_u = \tau_\theta = 1$,$\tau_v = 0.01$,$\tau_z = 10$, $\gamma = 3$, $\gamma_m = 0$.



(a) Low wealth (b) High wealth

4.1 Risk premium

We start by analyzing how the initial wealth of investors affects the risk premium. The conditional risk premium is formally defined as

$$rp(p) = E[f - p|p] = \frac{\gamma_m}{\tau_m}(c_0^m + c_\phi^m g(p) - c_p^m p). \tag{11}$$

Since an econometrician measures the unconditional risk premium, we will focus on it. It is given by:

$$\bar{rp}(W, \tau_\epsilon) \equiv E[f - p] = E[rp(p)] = \frac{\gamma_m}{\tau_m}(c_0^m - c_p^m E[p]).$$

The change in risk premium in response to a change in the investors' wealth can be decomposed as follows:

$$\frac{d\bar{rp}(W,\tau_\epsilon)}{dW} = \underbrace{\frac{\partial \bar{rp}}{\partial W}}_{\text{Direct Effect}} + \underbrace{\frac{\partial \bar{rp}}{\partial \tau_\epsilon}.\frac{\partial \tau_\epsilon}{\partial W}}_{\text{Indirect Effect}} \tag{12}$$

The first term in the right-hand side of equation (12) captures the direct effect that a change in investors' wealth has on the risk premium; the second term captures the indirect effect resulting from investors' endogenous information acquisition decisions. Note that since here price is a nonlinear function of fundamentals, we proceed with numerical analysis.

Figure 4: Risk premium

The figure plots risk premium as a function of precision of investors signal for different levels of wealth: $W = 0.5$ (dotted line) and $W = 1.5$ (solid line). Other parameter values are set to: $\tau_u = \tau_z = 1$, $\tau_v = 0.01$, $\tau_\theta = 0.5$, $\gamma = \gamma_m = 3$ and $\alpha = 0.99$.

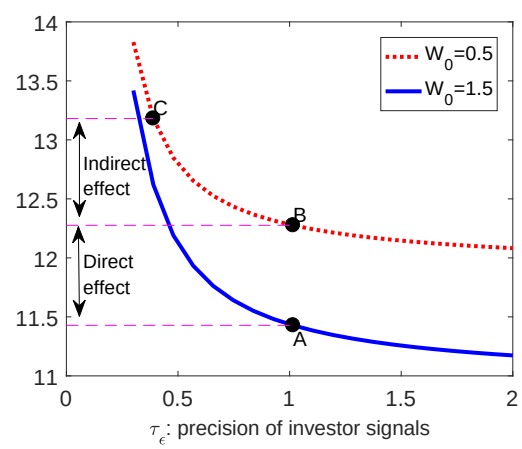


Figure 4 plots the unconditional risk premium in our model against τ_ϵ, the precision of investors' signals for two different levels of wealth. Let point A corresponds to the equilibrium with high wealth level, and consider a negative shock to investors' wealth. With a decreased wealth, constraints become tighter and investors' capacity to go long or short the asset is diminished, which is similar to the effect of lowering their risk-bearing capacity (i.e., increasing their risk aversion). Therefore, the risk premium rises. This argument implies that absent the information acquisition channel (i.e., holding τ_ϵ fixed), the wealth drop would cause an increase in risk premium that corresponds to the move from the graph's solid line (corresponding to

high wealth level) to its dotted line (corresponding to low wealth level)—that is, from point A to point B. This reflects the direct effect in equation (12). Moreover, because of the information spiral, investors in equilibrium acquire less information (lower τ_ϵ), which leads to an additional increase in risk premium; this increase corresponds to the move from point B to point C along the dotted line, which is the indirect effect given in (12). Thus the effect of a decline in wealth on risk premium is amplified by the information acquisition channel, so the equilibrium moves from point A in the graph all the way to point C .

4.2 Return volatility

We next examine the risky asset's return volatility. The variance of returns can be written as

$$\mathcal{V}(W, \tau_\epsilon) \equiv Var[f - p] \tag{13}$$

$$= Var[E(f - p|p)] + E[Var[f - p|p]] \tag{14}$$

$$= Var[rp(p)] + E[Var[v|p]] + \tau_\theta^{-1} \tag{15}$$

$$= \underbrace{\left(\frac{\gamma_m}{\tau_m}\right)^2 Var[x_m(p)]}_{\text{Variance of conditional risk premium}} + \underbrace{(\tau_v + \beta^2\tau_z)^{-1} + \tau_\theta^{-1}}_{\text{Conditional variance of payoff } f} . \tag{16}$$

As before, the change in return variance in response to a change in the investors' wealth can be decomposed as follows:

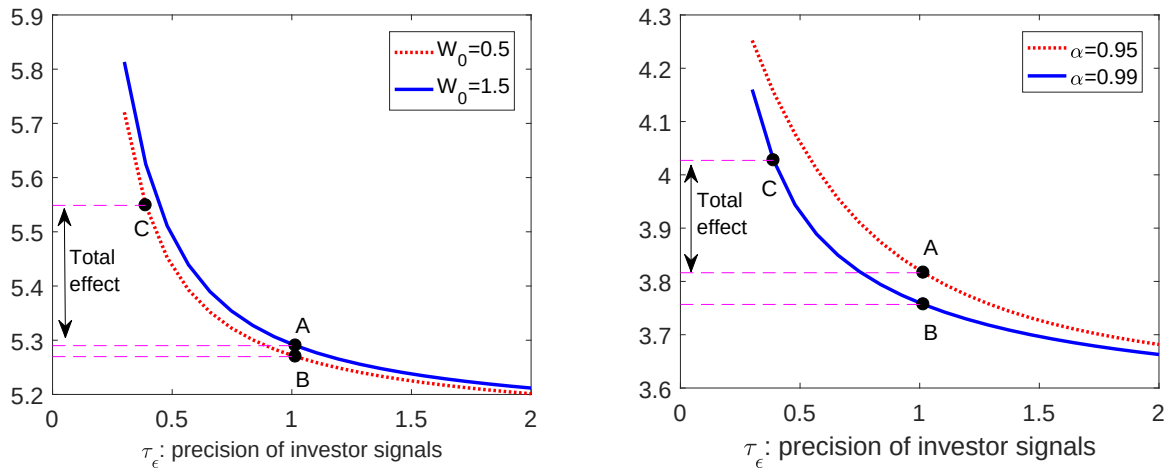
$$\frac{d\mathcal{V}(W, \tau_\epsilon)}{dW} = \underbrace{\frac{\partial \mathcal{V}}{\partial W}}_{\text{Direct Effect}} + \underbrace{\frac{\partial \mathcal{V}}{\partial \tau_\epsilon} \cdot \frac{\partial \tau_\epsilon}{\partial W}}_{\text{Indirect Effect}} . \tag{17}$$

Since the price is nonlinear, the preceding expression for the variance of returns cannot be further simplified. Hence we proceed numerically.

Panel (a) of Figure 5 plots return volatility against the precision of investors' signals (τ_ϵ), for two levels of wealth. Once again, we suppose that point A is an equilibrium with

Figure 5: Return volatility

The figure plots return volatility as a function of precision of investors signal for different levels of wealth, $W = 0.5$ (dotted line) and $W = 1.5$ (solid line) (panel (a)) and different levels of value-at-risk confidence level α, $\alpha = 0.95$ (dotted line) and $\alpha = 0.99$ (solid line) (panel (b)). Other parameter values are set to: $\tau_u = \tau_z = 1$, $\tau_v = 0.01$, $\tau_\theta = 0.5$, $\gamma = \gamma_m = 3$.



(a) (b)

high wealth level and then imagine reducing wealth. If τ_ϵ is held fixed then we can see that, as wealth declines, there is less volatility (corresponding to the move from point A to point B). The intuition follows from equation (16). With decreasing wealth, the second and third terms do not change when τ_ϵ is fixed but the first term decreases because investor demand is then less volatile; this is the direct effect. Yet, investors acquire less information when they are constrained, so there is an increase in volatility corresponding to the move from point B to point C. Thus, the indirect effect (which operates through the information acquisition channel) may end up dominating, which means that volatility increases overall as wealth declines.

We also examine the effects of margin requirements (as measured by VaR confidence level α) on volatility. It has long been argued that tighter margin requirements stabilize prices. The reasoning is that tighter margin requirements curb investors' positions, thereby limiting the price impact of their information and liquidity shocks. Panel (b) of Figure 5 illustrates the effect of margin requirements on volatility. As margin constraints tighten (α increases), volatility indeed drops—when information acquisition choices are held fixed—as we move from

point A to point B; this outcome confirms the conventional wisdom. With tighter constraints, however, investors acquire less information and so return volatility may increase when funding requirements are stricter (and thus we move from point B to point C on the graph). In this way, our model complements the results in Wang (2015) by giving an alternative information-based explanation for the positive association between tightening margin requirements and increased return volatility.

4.3 Sharpe ratio

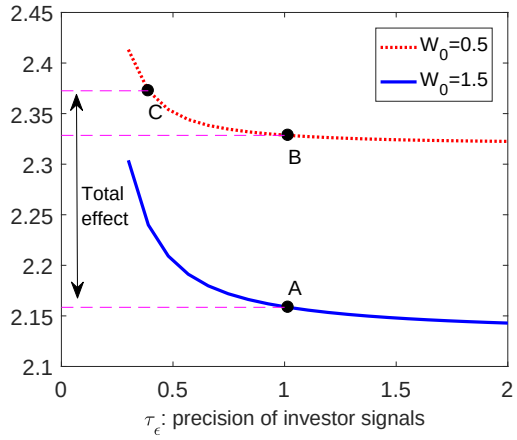
Our final asset pricing implication is about the risky asset's Sharpe ratio, defined as $SR = \frac{E[f-p]}{\sqrt{Var[f-p]}}$. We argued previously that, when τ_ϵ is fixed, the risk premium rises and volatility falls (from point A to point B in both plots) as the wealth of investors declines. These movements implies that falling wealth will cause the Sharpe ratio to rise (if τ_ϵ is held constant). We also argued that, with VaR-based margins and endogenous information acquisition, both the risk premium and return volatility rise; as a consequence, the indirect effect cannot be signed. Figure 6 depicts a case in which the direct effect (from A to B) and the indirect effect (from B to C) are in the same direction, thus amplifying the wealth shock's effect.

5 Empirical Predictions

Our paper offers two main testable predictions. First, information acquisition by investors—and hence price informativeness—will decrease when investors' constraints tighten. Second, shocks to price informativeness lead to higher margin requirements and thus to tighter constraints. Because of the interaction between the constraints and informational efficiency, a correlation between the two needs not indicate causality. It follows that testing the first (resp., second) prediction requires an exogenous shock to investors' funding constraints (resp., to price informativeness).

Figure 6: Sharpe ratio

The figure plots Sharpe ratio as a function of precision of investors signal for different levels of wealth, $W = 0.5$ (dotted line) and $W = 1.5$ (solid line). Other parameter values are set to: $\tau_u = \tau_z = 1$, $\tau_v = 0.01$, $\tau_\theta = 0.5$,$\gamma = \gamma_m = 3$ and $\alpha = 0.99$.



There are a number of different proxies developed for intermediaries' constraints. He, Kelly, and Manela (2017) argue that equity capital ratio of the New York Fed's primary dealers is a good proxy for soundness of financial intermediaries. In that case, shocks to this variable can proxy for shocks to the funding constraints of investors. Another way of identifying exogenous shocks to investors' funding constraints is to explore the changes in margin regulations established by the Board of Governors of the Federal Reserve System (see Jylha (2018)). Jylha (2015) argues that the New York Stock Exchange's portfolio Margining Pilot Program of 2005–2007 was an exogenous shock to the margin constraints of index options that had no effect on the margins of equity options. Using the difference-in-difference approach, Jylha (2015) finds evidence consistent with our first prediction: the loosening of funding constraints leads to an improvement in informational efficiency, represented by a reduction in the dispersion of changes in options' implied volatilities.

While there are many proxies for price informativeness in empirical literature, the one suggested by Bai, Philippon, and Savov (2016) is perhaps the closest to our notion; it captures the extent to which asset prices in a given year are able to predict future cash flows. The second

main implication of our model is that shocks to price informativeness affect funding constraints. Testing this prediction empirically would require exogenous shocks to price informativeness. The shutting down of a broker (Kelly and Ljungqvist, 2012) or the merger of brokers (Hong and Kacperczyk, 2010) could be shocks to the activity of analysts and hence to price informativeness. Our model implies that such a shock would lead to higher margin requirements.

6 Conclusion

In this paper we developed a tractable REE model with general portfolio constraints, and we applied our methodology to study a canonical REE model with margin constraints. We argued that funding constraints affect and are affected by informational efficiency, leading to a novel amplification mechanism that we call the information spiral. This spiral implies that the risk premium, conditional return volatility and Sharpe ratio each rise disproportionately as investor wealth declines. The information spiral also generates complementaries in the investors' acquisition of information during crises (i.e., when investor wealth is low). These results imply a new, information-based rationale for why the wealth of investors is important. Our analysis also yields novel testable predictions.

While many papers describe amplification mechanisms for amplification over the business cycle, ours is different because it involves changes in market informativeness. Given the important role capital markets play in aggregating and disseminating information, as argued by Bond et al. (2012), our mechanism should have significant real implications.

There are several potential extensions of our model. The first is to accommodate multiple risky assets. Such an extension can provide implications about spillovers of fundamental shocks of one asset to informational efficiency and margin requirements of other assets. In addition, our model also allows us to do welfare analysis. In some preliminary analysis, we find that tightening of constraints can lead to improvement of investors' welfare like in Wang (2015).

7 Appendix A: Proofs

Proof. (Proposition 1) At time 1, the first order condition for investor i solving problem (1) is given by

$$x_i = \frac{\tau}{\gamma}\left(E\left[v|\mathcal{F}_i\right] - p - \gamma e_i \tau_\theta^{-1}\right), \quad \text{where} \quad \tau^{-1} = Var\left[v + \theta|\mathcal{F}_i\right].$$

Similarly, the first order condition for the market maker solving problem (2) is

$$x_m = \tau_m \frac{E\left[v|\phi\right] - p}{\gamma_m}, \quad \text{where} \quad \tau_m^{-1} = Var\left[v + \theta|\phi\right].$$

Using Bayes's rule for jointly normal random variables, we can write

$$E\left[v|\mathcal{F}_i\right] = \frac{\tau_\epsilon s_i + \beta^2\left(\tau_u + \tau_z\right)\phi + \beta\tau_u e_i}{\tau_\epsilon + \beta^2\left(\tau_u + \tau_z\right) + \tau_v} \quad \text{and} \quad \frac{1}{\tau} = \frac{1}{\tau_\epsilon + \beta^2\left(\tau_u + \tau_z\right) + \tau_v} + \frac{1}{\tau_\theta},$$

$$E\left[v|\phi\right] = \frac{\beta^2\tau_z\phi}{\beta^2\tau_z + \tau_v} \quad \text{and} \quad \frac{1}{\tau_m} = \frac{1}{\beta^2\tau_z + \tau_v} + \frac{1}{\tau_\theta}.$$

Substituting these into the market clearing condition (3), we get

$$\frac{\tau}{\gamma}\left(\frac{\tau_\epsilon v + \beta^2\left(\tau_u + \tau_z\right)\phi + \beta\tau_u z}{\tau_\epsilon + \beta^2\left(\tau_u + \tau_z\right) + \tau_v}\right) - \frac{\tau}{\tau_\theta}z + \frac{\tau_m}{\gamma_m}\frac{\beta^2\tau_z\phi}{\beta^2\tau_z + \tau_v} = p\left(\frac{\tau}{\gamma} + \frac{\tau_m}{\gamma_m}\right).$$

One can express equilibrium price $p = p(v, z)$ from the above equation. Since it can only depend on v and z through $\phi = v - \frac{1}{\beta}z$, it must be true that $\frac{\partial p}{\partial v}/\frac{\partial p}{\partial z} = -\beta$. This implies that β satisfies:

$$\beta^3\gamma\left(\tau_u + \tau_z\right) - \beta^2\tau_u\tau_\theta + \beta\gamma\left(\tau_\epsilon + \tau_v\right) - \tau_\theta\tau_\epsilon = 0. \tag{18}$$

It can be seen from the above equation that the solution to it is always positive and there exists at least one solution. The solution is unique if the first derivative of the above polynomial does not change sign. The first derivative of the above equation is given by:

$$3\beta^2\gamma\left(\tau_u + \tau_z\right) - 2\beta\tau_u\tau_\theta + \gamma\left(\tau_\epsilon + \tau_v\right).$$

At $\beta = 0$, the slope is positive and the slope is always positive if the above equation has no roots. This is true if and only if

$$\tau_u^2\tau_\theta^2 < 3\gamma^2\left(\tau_u + \tau_z\right)\left(\tau_\epsilon + \tau_v\right).$$

Using implicit differentiation of (18), β increases in τ_ϵ if and only if $\tau_\theta - \beta\gamma > 0$, which always

holds for β solving equation (18).

Finally, we provide expressions for the coefficients mentioned in the proposition. Since the aggregate demand of investors and market makers can depend on v only through ϕ, we find

$$c_\phi = \frac{\tau}{\gamma}\frac{\partial E[v|\mathcal{F}_i]}{\partial v} = \frac{\tau}{\gamma}\left(\frac{\tau_\epsilon + \beta^2(\tau_u+\tau_z)}{\tau_\epsilon + \beta^2(\tau_u+\tau_z)+\tau_v}\right), \tag{19}$$

$$c_\phi^m = \frac{\tau_m}{\gamma_m}\frac{\partial E[v|\phi]}{\partial v} = \frac{\tau_m}{\gamma_m}\frac{\beta^2\tau_z}{\beta^2\tau_z+\tau_v}. \tag{20}$$

Similarly,

$$c_p = \frac{\tau}{\gamma}, \quad c_p^m = \frac{\tau_m}{\gamma_m}.$$

Finally,

$$\xi_i = \frac{\tau}{\gamma}\left(\frac{\tau_\epsilon\epsilon_i + \beta\tau_u u_i}{\tau_\epsilon + \beta^2(\tau_u+\tau_z)+\tau_v} - \gamma u_i \tau_\theta^{-1}\right),$$

$$\sigma_\xi^2 = \left(\frac{\tau}{\gamma}\right)^2\left(\frac{\tau_\epsilon\epsilon_i + \beta\tau_u u_i}{\tau_\epsilon + \beta^2(\tau_u+\tau_z)+\tau_v} - \gamma u_i \tau_\theta^{-1}\right).$$

The coefficients g_0^u and g_1^u can be expressed throgh the above coefficients as follows:

$$g_0^u = \frac{1-c_0-c_0^m}{c_\phi + c_\phi^m}, \quad g_1^u = \frac{c_p + c_p^m}{c_\phi + c_\phi^m}.$$

■

Proof. (Proposition 2) We first define a function $T(x;a,b)$ that truncates its argument x to the interval $[a,b]$:

$$T(x;a,b) = \begin{cases} x, & \text{if } a \le x \le b, \\ b, & \text{if } x > b, \\ a, & \text{if } x < a. \end{cases} \tag{21}$$

Conjecture that there exists a generalized linear equilibrium with informational efficiency β. Investor i's demand can then be written as

$$x_i = T\left(x_i^d; a(p), b(p)\right).$$

Moreover, as in the proof of Proposition 1, one can find investor i's *desired demand* x_i^d as

$$x_i^d = X^d + \xi_i^d,$$

where *aggregate desired demand* X^d is

$$X^d = \frac{\tau}{\gamma}\left(\frac{\tau_\epsilon v + \beta^2\left(\tau_u + \tau_z\right)\phi + \beta\tau_u z}{\tau_\epsilon + \beta^2\left(\tau_u + \tau_z\right) + \tau_v}\right) - \frac{\tau}{\tau_\theta}z - p\frac{\tau}{\gamma}$$

and the idiosyncratic part of the desired demand is

$$\xi_i^d = \frac{\tau}{\gamma}\left(\frac{\tau_\epsilon \epsilon_i + \beta\tau_u u_i}{\tau_\epsilon + \beta^2\left(\tau_u + \tau_z\right) + \tau_v} - \gamma u_i \tau_\theta^{-1}\right).$$

By the exact law of large numbers, one can write the aggregate demand of investors as

$$X = \int x_i di = E_{\xi_i}\left[T\left(X^d + \xi_i^d; a(p), b(p)\right)\right].$$

For a given price p, the aggregate demand X is an increasing (and thus invertible) function of the aggregate desired demand X^d. Therefore, given p, one can compute X^d, from which one can express $\eta(\phi, v, z) \equiv \frac{\tau}{\gamma}\left(\frac{\tau_\epsilon v + \beta^2(\tau_u+\tau_z)\phi + \beta\tau_u z}{\tau_\epsilon + \beta^2(\tau_u+\tau_z)+\tau_v}\right) - \frac{\tau}{\tau_\theta}z$. Thus, the price in the constrained economy is informationally equivalent to η. However, in the generalized linear equilibrium the price must be informationally equivalent to ϕ. For this to hold we need $-\frac{\partial\eta(\phi,v,z)}{\partial v}/\frac{\partial\eta(\phi,v,z)}{\partial z} = \beta$, which is equivalent to equation (18) that characterizes the informational efficiency in the unconstrained economy. Thus, $\beta = \beta^u$ and $x_i^d = x_i^u$. Moreover, for the aggregate demand of investors we can write

$$X = X(\phi, p) = E_{\xi_i}\left[T\left(X^u(\phi, p) + \xi_i; a(p), b(p)\right)\right],$$

where $X^u(\phi, p)$ and ξ_i are characterized in Proposition 1.

We now prove that for every p there exists unique $\phi = g(p)$ such that market clears. Indeed, the market clearing can be written as

$$X(\phi, p) + c_0^m - c_p^m p + c_\phi^m \phi = 1.$$

For a given p, aggregate investors' demand $X(\phi, p)$ is increasing in ϕ. Thus, there is at most one solution. At least one solution exists by the Intermediate Value Theorem. The aggregate demand at $+\infty(-\infty)$ is equal to $+\infty(-\infty)$, thus at some intermediate point aggregate demand has to be equal to 1.

We now compute a closed-form expression for the aggregate demand of investors $X(\phi, p)$. It can be split into three parts. For a fraction π_1 of investors the lower constraint $a(p)$ will bind. They contribute $\pi_1(\phi, p)a(p)$ to the aggregate demand. Similarly, a fraction π_3 of investors for whom the upper constraint $b(p)$ binds. They contribute $\pi_3(\phi, p)b(p)$. Finally a fraction π_2 will

be unconstrained. They contribute $\pi_2 \cdot (X^u + E[\xi_i|(\xi_i + X^u) \in [a(p), b(p)]])$. Using the standard results for the mean of truncated normal distribution, the last term can be further simplified to

$$\pi_2 E[\xi_i|(\xi_i + X^u) \in [a(p), b(p)]] = \sigma_\xi \left(\Phi' \left(\frac{a(p) - X^u}{\sigma_\xi} \right) - \Phi' \left(\frac{b(p) - X^u}{\sigma_\xi} \right) \right)$$

where $\Phi(\cdot)$ and $\Phi'(\cdot)$ stands for the cumulative distribution function (CDF) and probability density function (PDF) of a standard normal distribution. Combining all of the terms we get

$$X(\phi, p) = \pi_1 a(p) + \pi_3 b(p) + \pi_2 X^u + \sigma_\xi \left(\Phi' \left(\frac{a(p) - X^u}{\sigma_\xi} \right) - \Phi' \left(\frac{b(p) - X^u}{\sigma_\xi} \right) \right).$$

Now we determine the fractions π_1, π_2 and π_3. The fraction of investors constrained by the lower constraint, π_1, is given by

$$\pi_1(p, \phi) = \mathrm{P}\left(x_i < a(p)\right) = \mathrm{P}\left(X^u(p, \phi) + \xi_i < a(p)\right) = \Phi \left(\frac{a(p) - X^u(p, \phi)}{\sigma_\xi} \right)$$

The expressions for π_2 and π_3 can be derived analogously:

$$\pi_3(\phi, p) = 1 - \Phi \left(\frac{b(p) - X^u(p, \phi)}{\sigma_\xi} \right),$$

$$\pi_2(\phi, p) = 1 - \pi_1 - \pi_3.$$

Finally, we find the expression for the function $g'(p)$. Differentiating the market-clearing condition implicitly, we have

$$g'(p) = -\frac{\frac{\partial}{\partial p}(X(p, \phi) + x_m(p))}{\frac{\partial}{\partial \phi}(X(p, \phi) + x_m(p))}. \tag{22}$$

For the numerator, we have

$$\frac{\partial}{\partial p}(X(p, \phi) + x_m(p)) = \pi_1 a'(p) + \pi_3 b'(p) - \pi_2 c_p - c_p^m.$$

For the denominator, we have

$$\frac{\partial}{\partial \phi}(X(p, \phi) + x_m(p)) = c_\phi^m + \pi_2 c_\phi.$$

Substituting these expressions into 22 gives us the desired result. ■

Proof. (Proposition 3) For compactness of notation, we denote investor i's precision of signal

by t,

$$t \equiv \tau_{\epsilon_i}.$$

The key to computing MVI is to understand how the time-1 certainty equivalent $CE_{1,i}$ depends on the realizations of random variables $\{s_i = v+\epsilon_i, e_i, p\}$ and how investor i's choice of precision t affects the distribution of these variables. Clearly, only the distribution of ϵ_i is affected by the choice of t. To emphasize the latter fact we will use the notation $\epsilon_i(t)$. The key step in the proof is to substitute $\epsilon_i(t) = \frac{1}{t}B_t$, where B_t is a Brownian motion that is independent of all other random variables in the model. Indeed, such a substitution is valid, as ex-ante both $\epsilon_i(t)$ and $\frac{1}{t}B_t$ have the same distribution, $N(0, 1/t)$. Hence, computing $E[-e^{-\gamma CE_{1,i}}]$, with or without substitution of $\epsilon_i(t) = \frac{1}{t}B_t$, will produce the same result. With the substution at hand, to emphasize the fact that $CE_{1,i}$ depends on t only through the dependence of the distribution of B_t on t we will write $CE_{1,i} = CE_{1,i}(B_t)$. The advantage of substitution we've made is that now we can utilize Ito's lemma to compute $dCE_{1,i}(B_t)$.[15]

We start with Case 1 for function u_0 and note that marginal value of information (MVI) is given by

$$\text{MVI} = -\frac{\frac{d}{dt}E[e^{-\gamma CE_{1,i}}]}{\gamma e^{-\gamma CE_0}}.$$

We proceed as follows:

$$\frac{d}{dt}E[-e^{-\gamma CE_{1,i}}] = -E\left[\frac{de^{-\gamma CE_{1,i}(B_t)}}{dt}\right].$$

We use Ito's lemma to compute

$$de^{-\gamma CE_{1,i}(B_t)} = -\gamma e^{-\gamma CE_{1,i}(B_t)} dCE_{1,i} + \frac{\gamma^2}{2} e^{-\gamma CE_{1,i}(B_t)} dCE_{1,i}^2.$$

The expression for $dCE_{1,i}$ and $dCE_{1,i}^2$ depends on whether constraints bind for the investor i. The latter depends on realizations of random variables in his information set $\mathcal{F}_i = \{v + \frac{1}{t}B_t, e_i, \phi\}$.

We first consider the situation where agent i is unconstrained ex-post i.e., where $\{v + \frac{1}{t}B_t, e_i, \phi\}$ are such that $x_i^u \in (a(p), b(p))$. Then,

$$CE_{1,i} = \frac{\tau_i}{2\gamma}\left(v_i - p - \gamma e_i \tau_\theta^{-1}\right)^2 + \text{terms that do not depend on } t,$$

[15]There is a technicality here. Ito's lemma is applicable to $CE_{1,i}(B_t)$ that is C^2 in B_t. However, our function is only C^1 in B_t. One can also show that it is convex, which makes the Ito-Tanaka-Meyer rule applicable (see, e.g. Cohen and Elliott (2015), p. 352, and also Björk (2015) p. 18 for a more light reading). Since $CE_{1,i}$ is C^1, the local time terms in the Ito-Tanaka-Meyer rule disappear and we can write the Ito rule in the usual way.

where we denoted

$$v_i = E[v|\mathcal{F}_i] = \frac{t\left(v + \frac{1}{t}B_t\right) + \beta^2\left(\tau_u + \tau_z\right)\phi + \beta\tau_u e_i}{\tau_{v,i}},$$

$$\tau_{v,i} = Var[v|\mathcal{F}_i] = t + \beta^2\left(\tau_u + \tau_z\right) + \tau_v.$$

Differentiating $CE_{1,i}$ we get

$$dCE_{1,i} = \frac{d\tau_i}{2\gamma}\left(v_i - p - \gamma e_i \tau_\theta^{-1}\right)^2 + \frac{\tau_i}{\gamma}\left(v_i - p - \gamma e_i \tau_\theta^{-1}\right) dv_i + \frac{\tau_i}{2\gamma}\left(dv_i\right)^2,$$

$$\left(dCE_{1,i}\right)^2 = \left(\frac{\tau_i}{\gamma}\left(v_i - p - \gamma e_i \tau_\theta^{-1}\right)\right)^2 \left(dv_i\right)^2. \tag{23}$$

We now differentiate v_i and $\tau_i = \left(\frac{1}{\tau_{v,i}} + \frac{1}{\tau_\theta}\right)^{-1}$ to get

$$d\tau_i = \left(\frac{\tau_i}{\tau_{v,i}}\right)^2 dt, \;\; dv_i = v\frac{dt}{\tau_{v,i}} + \frac{dB_t}{\tau_{v,i}} - \frac{dt}{\tau_{v,i}} v_i, \;\; \left(dv_i\right)^2 = \left(\frac{dB_t}{\tau_{v,i}}\right)^2 = \frac{dt}{\tau_{v,i}^2}.$$

We now compute $E\left[de^{-\gamma CE_{1,i}(B_t)}\right]$. We use the law of iterated expectations and write $E\left[de^{-\gamma CE_{1,i}(B_t)}\right] = E\left[E_t\left[de^{-\gamma CE_{1,i}(B_t)}\right]\right]$, where we introduced notation

$$E_t[\cdot] = E\left[\cdot|v + 1/t \cdot B_t, e_i, \phi\right].$$

Moreover, we can write

$$E_t\left[de^{-\gamma CE_{1,i}(B_t)}\right] = -\gamma e^{-\gamma CE_{1,i}(B_t)} E_t[dCE_{1,i}] + \frac{\gamma^2}{2} e^{-\gamma CE_{1,i}(B_t)} E_t[dCE_{1,i}^2]. \tag{24}$$

Note that since $E_t[v] = v_i$ and $E_t[dB_t] = 0$, we have $E_t[dv_i] = 0$. Hence,

$$E_t[dCE_{1,i}] = \frac{d\tau_i}{2\gamma}\left(v_i - p - \gamma e_i \tau_\theta^{-1}\right)^2 + \frac{\tau_i}{2\gamma}\frac{dt}{\tau_{v,i}^2}. \tag{25}$$

Substituting (23) and (25) into (24), we get

$$E_t\left[de^{-\gamma CE_{1,i}(B_t)}\right] = -e^{-\gamma CE_1}\frac{\tau_i}{2}\frac{dt}{\tau_{v,i}^2}.$$

We now consider the situation where agent i is constrained by lower bound i.e., where $\{v +$

$\frac{1}{t}B_t, e_i, \phi\}$ are such that $x_i^u < a(p)$. Then,

$$CE_{1,i} = a(p)\left(v_i - p - \gamma e_i \tau_\theta^{-1}\right) - \frac{\gamma}{2\tau_i} a(p)^2.$$

Differentiating this expression, we get

$$dCE_{1,i} = a(p)dv_i + \frac{\gamma}{2\tau_i^2} a(p)^2 d\tau_i.$$

We therefore get

$$E_t\left[dCE_{1,i}\right] = \frac{\gamma}{2} a(p)^2 \left(\frac{1}{\tau_{v,i}}\right)^2 dt.$$

$$E_t\left[\left(dCE_{1,i}\right)^2\right] = a(p)^2 \left(dv_i\right)^2 = a(p)^2 \frac{dt}{\tau_{v,i}^2}.$$

Substituting the above two equations into (24), we get:

$$E_t\left[de^{-\gamma CE_{1,i}(B_t)}\right] = 0.$$

Proceeding analogously for the case $\{v + \frac{1}{t}B_t, e_i, \phi\}$ are such that $x_i^u > b(p)$ and combining the results, we obtain

$$\begin{aligned} E_t\left[de^{-\gamma CE_{1,i}(B_t)}\right] &= \begin{cases} -e^{-\gamma CE_{1,i}} \frac{\tau_i}{2} \frac{dt}{\tau_{v,i}^2}, & \text{if } x_i^u \in \left(a(p), b(p)\right), \\ 0, & \text{otherwise,} \end{cases} \\ &= -e^{-\gamma CE_{1,i}} \frac{\tau_i}{2} \frac{dt}{\tau_{v,i}^2} \mathbb{I}(x_i^u = x_i). \end{aligned}$$

For marginal value of information (MVI), we finally get

$$\text{MVI} = -\frac{\frac{dU_0}{dt}}{\gamma U_0} = \frac{\tau_i}{2\tau_{v,i}^2} \frac{E\left[e^{-\gamma CE_{1,i}} \mathbb{I}(x_i^u = x_i)\right]}{E\left[e^{-\gamma CE_{1,i}}\right]}.$$

We now consider Case 2 for the function u_0. In this case, the MVI is defined as

$$\text{MVI} = \frac{d}{dt} E[CE_{1,i}] = \frac{d}{dt} E[E_t[CE_{1,i}]].$$

From the calculations above, we know

$$E_t\left[CE_{1,i}\right] = \begin{cases} \left(\frac{1}{\tau_{v,i}}\right)^2 dt \frac{\gamma}{2} E_t\left[(x_i^u)^2\right] + \frac{\tau_i}{2\gamma}\frac{dt}{\tau_{v,i}^2} & \text{if } x_i^u \in (a(p), b(p)) \\ \frac{\gamma}{2} a(p)^2 \left(\frac{1}{\tau_{v,i}}\right)^2 dt & \text{if } x_i^u < a(p) \\ \frac{\gamma}{2} b(p)^2 \left(\frac{1}{\tau_{v,i}}\right)^2 dt & \text{if } x_i^u > b(p) \end{cases}$$

$$= \frac{\gamma}{2} E_t[x_i^2] \left(\frac{1}{\tau_{v,i}}\right)^2 dt + \frac{\tau_i}{2\gamma}\frac{dt}{\tau_{v,i}^2} \mathbb{I}(x_i^u = x_i)$$

Thus,

$$\text{MVI} = \frac{\gamma}{2\tau_{v,i}^2} E\left[x_i^2\right] + \frac{\tau_i}{2\gamma\tau_{v,i}^2} E[\mathbb{I}(x_i^u = x_i)].$$

The result that the marginal value of information decreases when individual investors constraints become tighter, holding everything else fixed, can be proved exactly as the Proposition 4, the proof of which we present below. ■

Proof. (Proposition 4) In Case 1, we write the expression for the marginal value of information as

$$\text{MVI} = \frac{\tau_i}{2\tau_{v,i}^2 \gamma} \frac{U_0^u}{U_0}. \tag{26}$$

In the case of risk-neutral market maker, constraints do not alter prices i.e., prices are independent of portfolio constraints. The only term affected by constraints is $\frac{U_0^u}{U_0}$. Consider first the nominator: $U_0^u = E[-e^{-\gamma CE_1}\mathbb{I}(x_i^u = x_i)] = E[-e^{-\left(W_0 + \frac{\gamma}{2\tau_i}\left(x_i^u\right)^2 - \frac{\gamma}{2\tau_\theta} e_i^2\right)}\mathbb{I}(x_i^u = x_i)]$. It increases (becomes less negative) as constraints become tighter: recall that investors get negative utility; as constraints become tighter, they get it in fewer states of the world. The denominator U_0 decreases (becomes more negative) as with constraints the certainty equivalent $CE_{1,i}$ in all states weakly decreases. Thus, the ratio decreases as constraints become tighter.

In Case 2, the marginal value of information is given by

$$\text{MVI} = \frac{\gamma}{2\tau_{v,i}^2} E[x_i^2] + \frac{\tau_i}{2\gamma\tau_{v,i}^2} E[\mathbb{I}(x_i^u = x_i)]. \tag{27}$$

Again, prices are independent of portfolio constraints. As constraints become tighter, the first term $E[x_i^2]$ decreases and the second term $E[\mathbb{I}(x_i^u = x_i)]$ also decreases. This implies that the marginal value of information decreases. ■

Proof. (Corollary 1) With risk-neutral market maker, $p = E[v|\phi] = \frac{\beta^2\tau_z\phi}{\beta^2\tau_z+\tau_v}$. Thus,

$$g_1 = \frac{\beta^2\tau_z + \tau_v}{\beta^2\tau_z}, \quad g_0 = 0.$$

The rest follows directly from Proposition 2. ■

Proof. (Proposition 6) We prove that in a stable equilibrium $\frac{d\tau_\epsilon^*}{dm^+} < 0$ and $\frac{d\beta}{dm^+} < 0$. The rest of the results either can be proved analogously or follow directly from Propositions 3 ans 4.

Given that other investors choose precision τ_ϵ^* it is optimal for an investor i to choose τ_{ϵ_i} such that:

$$C'(\tau_{\epsilon_i}) = \text{MVI}(\tau_{\epsilon_i}, \tau_\epsilon^*),$$

$$C''(\tau_{\epsilon_i}) - \text{MVI}_1(\tau_{\epsilon_i}, \tau_\epsilon^*) > 0.$$

The first (second) equation above corresponds to the first (second) order condition in investor i's optimization problem and $\text{MVI}_k(\cdot,\cdot)$ denotes the derivative of $\text{MVI}(\cdot,\cdot) > 0$ with respect to its' k-th argument. Differentiating the first equation above implicitly one can get

$$\tau'_{\epsilon_i}(\tau_\epsilon^*) = \frac{\text{MVI}_2(\tau_{\epsilon_i}, \tau_\epsilon^*)}{C''(\tau_{\epsilon_i}) - \text{MVI}_1(\tau_{\epsilon_i}, \tau_\epsilon^*)}. \tag{28}$$

In a symmetric equilibrium $\tau_{\epsilon_i} = \tau_\epsilon^*$, therefore:

$$C'(\tau_\epsilon^*) = \text{MVI}(\tau_\epsilon^*, \tau_\epsilon^*),$$

$$C''(\tau_\epsilon^*) - \text{MVI}_1(\tau_\epsilon^*, \tau_\epsilon^*) > 0.$$

Moreover, since in a stable equilibrium $|\tau'_{\epsilon_i}(\tau_\epsilon^*)| < 1$, from (28) we also have

$$C''(\tau_\epsilon^*) - \text{MVI}_1(\tau_\epsilon^*, \tau_\epsilon^*) - \text{MVI}_2(\tau_\epsilon^*, \tau_\epsilon^*) > 0. \tag{29}$$

To calculate $\frac{d\tau_\epsilon^*}{dm^+}$ we differentiate $C'(\tau_\epsilon^*(m^+)) = \text{MVI}(\tau_\epsilon^*(m^+), \tau_\epsilon^*(m^+); m^+)$ with respect to m^+ to get

$$\frac{d\tau_\epsilon^*}{dm^+} = \frac{\text{MVI}_3(\tau_\epsilon^*, \tau_\epsilon^*)}{C''(\tau_\epsilon^*) - \text{MVI}_1(\tau_\epsilon^*, \tau_\epsilon^*) - \text{MVI}_2(\tau_\epsilon^*, \tau_\epsilon^*)}.$$

It follows from Proposition 4 that $\text{MVI}_3(\tau_\epsilon^*, \tau_\epsilon^*) < 0$. Combining it with (29) we get $\frac{d\tau_\epsilon^*}{dm^+} < 0$.

To see that $\frac{d\beta}{dm^+} < 0$ note that β still satisfies equation (4) and β decreases as investors acquire less information. ■

Proof. (Proposition 6) Note that with risk-neutral market maker, the margins are given by

$$m^+(p) = m^-(p) = \frac{\Phi^{-1}(\alpha)}{\sqrt{\tau_m}}$$

where

$$\frac{1}{\tau_m} = \frac{1}{\tau_v + \beta^2\tau_z} + \frac{1}{\tau_\theta}.$$

As informational efficiency (β) decreases, margins increase. This implies that the constraint $a(p) = \frac{W_0}{m^+(p)}$ decreases and the constraint $b(p) = -\frac{W_0}{m^-(p)}$ increases. This implies that constraints tighten as informational efficiency decreases. ■

Appendix B: Equilibrium characterization with risk-averse market maker

With risk-averse market maker, the price can be written as $p = E[v|p] - rp(p)$. We assume the financiers use information from prices to set margin in order to control VaR:

$$m^+(p) = \inf\{m^+(p) \geq 0 : Pr(p - v > m^+(p)|p) \leq 1 - \alpha\}.$$

$$m^-(p) = \inf\{m^-(p) \geq 0 : Pr(v - p > m^-(p)|p) \leq 1 - \alpha\}.$$

$m^+(p)$ and $m^-(p)$ are the margins on long and short positions (per unit of asset) respectively. We now derive the expressions for margins. To compute $m^+(p)$, we first determine the function $m_n^+(p)$ that satisfies

$$\begin{aligned} 1 - \alpha &= Pr(E[v|p] - rp(p) - v > m_n^+(p)|p) \\ &= Pr\left(\sqrt{\tau_m}(E[v|p] - v) > \sqrt{\tau_m}(m_n^+(p) + rp(p))\big|p\right) \\ &= 1 - \Phi\left(\sqrt{\tau_m}(m_n^+(p) + rp(p))\right). \end{aligned}$$

Thus, we find

$$m^+(p) = [m_n^+(p)]^+ = \left[\frac{\Phi^{-1}(\alpha)}{\sqrt{\tau_m}} - rp(p)\right]^+ \tag{30}$$

Similarly, one can define $m_n^-(p)$ which satisfies $Pr(v - p > m_n^-(p)|p) = 1 - \alpha$ and get

$$m^-(p) = [m_n^-(p)]^+ = \left[\frac{\Phi^{-1}(\alpha)}{\sqrt{\tau_m}} + rp(p)\right]^+ \tag{31}$$

The VaR-based margins are determined by three variables. Both margins on long and short positions increase in the exogenous level of confidence α and decrease in the endogenous informational efficiency of price β (through $\frac{1}{\tau_m} = \frac{1}{\tau_v+\beta^2\tau_z} + \frac{1}{\tau_\theta}$). In addition, the margin on long (short) position decreases (increases) in the endogenous risk premium $rp(p)$. We would like to emphasize the fact that informational efficiency of price affects the tightness of margin constraint.

Formally, our financial market equilibrium with VaR-based margin constraints is defined as follows: (1) financiers and investors determine demands and margins anticipating a particular price function (2) in equilibrium demands and margins are consistent with anticipated price function. We hold the precisions of investors' signals fixed.

Proposition 7. *(Equilibrium with VaR-based margin requirements) When the portfolio constraints are of the form of margin as in equation (9) and margins are determined by value-at-risk, there exists a unique generalized linear equilibrium. Moreover, in this unique equilibrium the function $g(p)$, i.e. the sufficient statistic ϕ, is increasing in price.*

Proof. (Proposition 7) To prove that $g(p)$ is invertible, we plug expression for our VaR-based margins into ODE (6) assuming that both m_n^+ and m_n^- are positive. We get

$$g'(p) = \frac{c_p^m + \pi_2 c_p - \left(\frac{\pi_1 W_0}{m^-(p)^2} + \frac{\pi_3 W_0}{m^+(p)^2}\right) rp(p)'}{\pi_2 c_\phi + c_\phi^m}.$$

from which, accounting for (11) we find

$$g'(p) = \frac{c_p^m + \pi_2 c_p + \frac{\tau_m}{\gamma_m}\left(\frac{\pi_1 W_0}{m^-(p)^2} + \frac{\pi_3 W_0}{m^+(p)^2}\right) c_p^m}{\pi_2 c_\phi + c_\phi^m + \frac{\tau_m}{\gamma_m}\left(\frac{\pi_1 W_0}{m^-(p)^2} + \frac{\pi_3 W_0}{m^+(p)^2}\right) c_\phi^m}.$$

Both m_n^+ and m_n^- are positive, when $g(p) \in \left[-\Phi^{-1}(\alpha)\frac{\tau_m^{-1/2}}{c_\phi^m}\frac{\gamma_m}{\tau_m} - \frac{c_0^m}{c_\phi^m} + \frac{c_p}{c_\phi^m}p; \Phi^{-1}(\alpha)\frac{\tau_m^{-1/2}}{c_\phi^m}\frac{\gamma_m}{\tau_m} - \frac{c_0^m}{c_\phi^m} + \frac{c_p}{c_\phi^m}p\right] = [g^-(p); g^+(p)]$ Proceeding similarly, one can get

$$g'(p) = \begin{cases} \frac{c_p^m+\pi_2 c_p+\frac{\gamma_m}{\tau_m}\frac{\pi_3 W_0}{m^+(p)^2}c_p^m}{\pi_2 c_\phi+c_\phi^m+\frac{\gamma_m}{\tau_m}\frac{\pi_1 W_0}{m^-(p)^2}c_\phi^m}, & \text{if } g < g^-(p), \\ \frac{c_p^m+\pi_2 c_p+\frac{\gamma_m}{\tau_m}\left(\frac{\pi_1 W_0}{m^-(p)^2}+\frac{\pi_3 W_0}{m^+(p)^2}\right)c_p^m}{\pi_2 c_\phi+c_\phi^m+\frac{\gamma_m}{\tau_m}\left(\frac{\pi_1 W_0}{m^-(p)^2}+\frac{\pi_3 W_0}{m^+(p)^2}\right)c_\phi^m}, & \text{if } g^-(p) < g < g^+(p), . \\ \frac{c_p^m+\pi_2 c_p+\frac{\gamma_m}{\tau_m}\frac{\pi_1 W_0}{m^-(p)^2}c_p^m}{\pi_2 c_\phi+c_\phi^m+\frac{\gamma_m}{\tau_m}\frac{\pi_1 W_0}{m^-(p)^2}c_\phi^m}, & \text{if } g > g^+(p). \end{cases}$$

Clearly, the derivative above is always positive, which means that the equilibrium function $g(p)$ is invertible. Thus, for each fundamental ϕ there exists a unique p clearing the market. The

initial condition for the ODE above can be found by clearing the market for a particular price, e.g., price $p = 0$. ■

Appendix C: Application to Yuan (2005)

In this appendix, we apply our methodology developed in section 2 to study borrowing constraints introduced in Yuan (2005). In this case, borrowing-constrained informed investors' demand is bounded above by $b(p) = \delta_0 + \delta_1 p$ where $\delta_1 > 0$ and there is no lower bound on investor demand i.e., $a(p) = -\infty$.

The borrowing constraint is a function of the price. The lower the asset price, the harder it is for informed investors to raise outside financing to invest in the risky asset. In this case, proposition 2 implies that

$$g'(p) = \frac{c_p^m + (1-\pi_3)c_p - \pi_3\delta_1}{(1-\pi_3)c_\phi + c_\phi^m}$$

where all the coefficients are positive and π_3 denotes the mass of investors for which the constraint binds. The following theorem gives conditions under which there will be multiple equilibria.

Proposition 8. *When the constraint is of the form $b(p) = \delta_0 + \delta_1 p$ where $\delta_1 > 0$, equilibrium is unique when $\delta_1 < \frac{\tau_m}{\gamma_m}$ and there will be multiple equilibria otherwise.*

Proof. (Proposition 8) In this case,

$$g'(p) = \frac{c_p^m + (1-\pi_3)c_p - \pi_3\delta_1}{(1-\pi_3)c_\phi + c_\phi^m} \tag{32}$$

where π_3 denotes the mass of investors for which the constraint binds. As p decreases, π_3 increases and numerator of equation (32) increases. In the extreme case, as p tends to low number, constraints bind for most of the informed investors and numerator tends to $c_p^m - \delta_1$. If this term is positive, we will always have unique equilibrium because $g'(p) > 0 \quad \forall p$. If this term becomes negative, there will be multiple equilibria. Finally, recall that $c_p^m = \frac{\tau_m}{\gamma_m}$ and τ_m is a constant parameter for a given τ_ϵ chosen at $t = 0$. ■

What we mean by multiple equilibria is that some realization of fundamentals can be supported by two prices. This results from the interaction of substitution and information effects in the model. In typical REE model (for example Hellwig (1980)), the substitution effect always dominates the information effect leading to unique equilibrium. In those models, the information effect is fixed as prices reveal the same amount of information regardless of

level. In our setting, due to the borrowing constraint imposed on informed investors, unit change in price does not reflect the same information. This implies that information effect can dominate substitution effect for some realization of prices and there can be multiple equilibria.

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