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When Consumers Regret

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Advance Selling When Consumers Regret

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We characterize the effect of anticipated regret on consumer decisions, firm profits and policies, in an advance selling context where buyers have uncertain valuations. Advance purchases trigger action regret if valuations turn out to be lower than the price paid, whereas delaying purchase may cause inaction regret from missing a discount or facing a stock-out. Emotionally rational consumers act strategically in response to the firm's policies and in anticipation of regrets. In this context, regret explains two types of behavioral patterns: inertia (delayed purchase) and frenzies (buying early at negative surplus). We further show how firms should optimally respond to consumer regrets, and characterize a normative regret threshold above which they should not advance sell. Action regret reduces profits, as well as the value of advance selling and booking limit policies for price-setting firms; inaction regret has the opposite effects. These effects are diminished by capacity constraints, and reversed for firms facing price pressure in the advance period, e.g. due to competition or market heterogeneity. Regret heterogeneity explains premium advance selling for capacitated firm, who may benefit from larger shares of regretful buyers. Finally, we show how the negative effects of regret on profits can be mitigated by regret-priming marketing campaigns, as well as offering refunds, options, or allowing resales. Our results emphasize the importance of assessing the relative strength of regrets within and across market segments, and accounting for these factors in pricing and marketing policies.

Key words: Advance Selling; Behavioral Pricing; Consumer Regret; Refunds

1. Introduction

Until September 27, INFORMS members can preregister for the November 2011 INFORMS Annual Meeting in Austin and pay \$360, instead of the full \$410 price after that date. Early September, Professor Regrette is still uncertain about her future preferences and possible conflicts early November. As these uncertainties materialize, closer to the conference date, she might regret having committed to attend the conference. Anticipation of this regret *ex-ante* may lead her to forego the early registration. On the other hand, as she stands in line to pay for the registration on-site in

Austin, she might regret having foregone the \$50 early bird discount. What is the effect of anticipated regret on Professor Regrette's decisions? On INFORMS' profits? How should INFORMS account for regret in its pricing policy and marketing campaign? This paper proposes to answer such questions in a general advance selling context.

Consumers often make purchase decisions while uninformed about their true valuations for a product or service. Such decisions have emotional consequences once uncertainties (regarding valuation and product availability) are resolved, and consumers learn if they have made, in hindsight, the wrong choice. As consumers reflect on forgone alternatives, wrong decisions trigger emotions of *action* or *inaction regret* ("I should have waited" or "I should have bought"), and the anticipation of these emotions affects purchase decisions. A consequence of decision making under uncertainty, regret is a negative, cognitive emotion experienced upon realizing *ex-post* that we would have been better off had we made a different decision, even if the decision was *ex-ante* right (Zeelenberg 1999). There is ample empirical validation for regret and the effect of its anticipation on individual behavior in diverse contexts (Zeelenberg 1999), and in particular for purchase timing decisions (Cooke et al. 2001, Simonson 1992).

Our goal in this paper is to understand, in an analytical framework, the effects of anticipated consumer regrets on purchase decisions, firm policies and profits, and provide prescriptions for firms to better respond to regret in an advance selling context when consumers face valuation uncertainty.

In such a context, we are first interested to understand: (1) *what is the effect of regrets on advance purchase behavior, in particular what departures from rationality in buy-or-wait decisions are explained by regret?* Consumers often delay purchase, a pattern known as buyer inertia; for example a significant fraction of regular participants at INFORMS conferences register on site, at a surcharge, or end up having to stay at other hotels. On the other hand, consumers rush to buy lottery tickets, limited edition Nintendo games they never tried or tickets to a sports game (before knowing the qualifying teams, or whether they can attend). Our results show that action and inaction regrets provide alternative explanations for such behavior.

Consumer regret is a potential concern for companies selling to uninformed consumers, e.g. for internet retailers — as consumers may be wary of purchasing products they can not try, or for providers of opaque services, such as Expedia’s undisclosed-name hotel bookings or Last-minute.com’s short-lived “surprise destination” holiday campaign. The benefits of advance selling are then in question. On the other hand, as consumers may fear a stock out (e.g. not finding tickets for a Broadway show or World Cup match), companies can potentially leverage their inaction regret by selling at high prices in advance. This raises the second question addressed in this research: *(2) what is the effect of consumer regret on profits, and how should firms optimally respond to regret?*

Organizations are not oblivious of the effect of regret on consumer behavior, and often leverage it in their marketing strategies. Advertising campaigns prime the anticipation of regret with slogans such as “Don’t miss this...” or “Buy now or regret it later!”¹ On the other hand, retailers try to mitigate consumer regret by offering price protection mechanisms, “a sales tactic that can give a buyer peace of mind and entice shoppers to buy immediately instead of looking elsewhere or delaying a purchase. It’s a kind of regret insurance” (Chicago Tribune, January 2008). For example, General Motors’ recent “May The Best Car Win” campaign aimed to revive sales by guaranteeing buyers that, if they don’t like their new car, they have 60 days to bring it back for a full refund (The New York Times, September 2009) — essentially a regret mitigating mechanism.² Yet not all companies offer refunds. Open box video-games or DVDs cannot be returned. Tickets for the Bregenz (outdoor) opera festival are non-refundable, even if a performance is canceled due to weather conditions. This motivates the third, and last question addressed in this research: *(3) when should firms prime or mitigate regret, and what mechanisms should they use to do so?*

To answer these three broad questions, we propose a model where consumers are strategic and “emotionally rational”, in that they time their purchase decisions in response to the firm’s pricing policies, and the anticipation of regret. Zeelenberg (1999) provides multiple arguments for the

¹ e.g. J.T.Hughes car dealership or Topshop women clothing.

² Similar returns policies are offered for Rosetta Stone language learning software, or by Amazon for the Kindle.

rationality of anticipated regret.³ Our model relies on a formal, axiomatic theory of regret in consumer choice (Bell 1982, Loomes and Sugden 1982), in order to understand and characterize: (1) consumer buy-or-wait decisions under anticipated regret in an advance selling context (Section 3); (2) the effect of regret and its heterogeneity on firm profits and advance selling policies (Sections 4 and 6), and (3) mechanisms for firms to mitigate the negative consequences of consumer regret (Section 5). Our main findings, summarized in the last section, suggest when it is important for a firm to measure consumer regret in an advance selling context, and how marketing policies should respond to it. We next present our model and assumptions, and relate these to the literature.

2. The Model and Relation to the Literature

A profit seeking firm with capacity C sets static prices p_1, p_2 for advance and spot period sales, respectively, assuming consumer's best response. Given the firm's policy, strategic consumers with unit demand and uncertain valuation \mathbf{v} decide whether to advance purchase, or wait until their valuation is realized. Their emotionally rational choice maximizes expected surplus *net of anticipated regret*, as detailed in Section 3. Consumers' valuation \mathbf{v} has common knowledge cumulative distribution $F(\cdot)$ on $[0, v_{max}]$, finite mean $E[\mathbf{v}] = \mu$, and survival function $\bar{F} = 1 - F$. For technical convenience, we occasionally assume that the revenue rate $p\bar{F}(p)$ is unimodal, or \mathbf{v} follows a two-point distribution, but most of our results hold for general distributions.

We focus on relatively large markets, which motivate a fluid model with infinitesimal (atomic) consumers, i.e. the decision of one consumer does not affect how other consumers behave, and spot demand is proportional to $\bar{F}(p)$.⁴ Without loss of generality, we normalize market size to 1, so $C \in (0, 1]$ is the fraction of the market that can be served. In case of excess demand, $C < 1$, consumers face the same rationing probability k (proportional rationing), and form rational expectations

³ Consistent with the argument that rationality applies to “what we do with our regrets, and not to the experience itself” (Zeelenberg 1999, p. 325), we focus on anticipated regret and do not consider here the (observed, yet irrational) effect of experienced regret on decisions. For example, the regret experienced upon missing a discount may prevent consumers from purchasing later at full price, even if that is below their valuation. Such behavior, consistent with loss aversion and modeled in Diecidue, Rudi and Tang (2010), has been evidenced, but is not (emotionally) rational.

⁴ The fluid model has been shown to yield insights that hold asymptotically in stochastic models (e.g. Gallego and Sahin 2009).

about it. The firm commits to prices, e.g. through mechanisms like Ticketmaster, and optimizes profits assuming consumers' best response. Lack of credibility (e.g. DeGraba 1995, Courty 2005) emerges as a special case of spot price constraints in our model. These assumptions, mainly relevant when capacity is tight ($C < 1$), are consistent with a large literature on strategic customers and advance selling, e.g. Xie and Shugan (2001), Gallego and Şahin (2009), Yu, Kapuscinski and Ahn (2010), Liu and van Ryzin (2008); the latter provides an excellent review of modeling assumptions.

The literature in marketing, economics and operations provides abundant reasons why firms should advance sell. In the operations literature, advance selling has been shown to reduce inventory risk (Cachon 2004), improve demand information (Boyaci and Özer 2010) and information sharing in supply chains (Tang and Girotra 2010). Unlike e.g. Prasad, Zhao and Stecke (2010) in a newsvendor setting, work in this area focuses largely on advance selling in supply chains (B2B).

By contrast, our focus is on behavioral aspects emerging from selling directly to consumers, when their valuation is uncertain. In this case, advance selling allows firms to extract additional surplus, because consumers are (more) homogeneous before their valuation is realized (Xie and Shugan 2001, Möller and Watanabe 2009). We explore how these insights are moderated by consumer regrets, as triggered by valuation uncertainty. For simplicity, we do not consider additional features that support advance selling, such as aggregate demand uncertainty (Nocke and Peitz 2008), late arrivals and correlated valuations (Yu et al. 2010), or competition (Dana 1998).

Despite abundant arguments provided by the literature, advance selling is not a universal practice. Capacity constraints can diminish the benefits of advance selling (Xie and Shugan 2001, Yu et al. 2010), as does consumer risk aversion (Che 1996, Prasad et al. 2010). We show that consumer action regret provides an additional reason for firms not to sell in advance. To counter this negative effect, we further investigate regret mitigating mechanisms, providing an alternative, psychological argument in favor of returns (Su 2009b), refunds (Che 1996, Liu and Xiao 2010), options (Gallego and Şahin 2009) and resale markets (Golzolari and Pavan 2006).

Capacity constraints make advance sales more appealing to consumers who want to avoid stock-outs. This can justify strategically limiting supply (deGraba 1995, Liu and van Ryzin 2008) or

advance selling at a premium (Möller and Watanabe 2009, Nocke and Peitz 2007). We provide two new explanations why firms can sell at high prices in advance: dominant inaction regret (commission bias, Kahneman and Tversky 1982), and, if supply is tight, regret heterogeneity.

Our work contributes to a growing literature on behavioral operations (see e.g. Loch and Wu 2007 for a review), which studies how firms should optimally set prices in response to “predictably irrational” consumers. Closest to our work, Su (2009a) and Liu and Shum (2009) model forward-looking consumers prone to inertia, respectively disappointment.

Liu and Shum (2009) study a model where rationing first period sales causes disappointment to consumers with certain valuations; they find that disappointment affects the firm only if it correlates with valuation. Disappointment and regret are both results of counterfactual thinking (Zeelenberg et al. 2000), but differ in the nature of the counterfactual comparison. Intuitively, we regret wrong choices (buy or wait), but are disappointed by poor outcomes of a given decision (low valuation, or stock-out).⁵ This leads to structurally different models and insights.

Su (2009a) provides a stylized model of buyer inertia, a tendency to postpone purchase decisions, and shows that its strength adversely affects profits, but a larger share of inertial consumers can be beneficial. Inertia, modeled holistically, as a positive constant threshold on consumer surplus, can be explained by various behavioral regularities, including anticipated action regret, but also hyperbolic discounting, probability weighting and loss aversion. By contrast, we focus on modeling consumers’ regret, based on its theoretical foundations; regret translates into a non-constant, possibly negative surplus threshold, leading to a richer set of implications for consumers and firms.

Our paper is among few in the literature to model consumer regret in an operational context, and provide prescriptive insights for a firm’s decisions. Regret has been used to explain market behavior, including why too much choice decreases demand (Irons and Hepburn 2007), preferences for standardized vs. customized products (Syam et al. 2008), demand for insurance (Braun and

⁵ For example, in absence of spot sales, consumers with uncertain valuation feel no disappointment upon waiting (because there is no surplus), but may regret not having bought early if they would have afforded to.

Muermann 2004), excess volatility of stock returns (Dodonova and Khoroshilov 2005), and over-bidding in auctions (Engelbrecht-Wiggans and Katok 2006). In a similar spirit, we show that, in an advance selling context, regret explains buyer inertia, i.e. delayed purchase (consistent with Su 2009a, Diecidue et al. 2010), but also frenzies, i.e. buying at negative surplus. Unlike these authors, we further use this model to derive the optimal policies of a firm in response to a regretful market.

3. Consumer Purchase Behavior under Regret

This section presents our model of how regret averse consumers behave in an advance selling setting, given prices for advance and spot sales, p_1, p_2 , and the probability $k \in [0, 1]$ of finding the product available in the spot period. Emotionally rational consumers do not discount utility, and act to maximize expected surplus, net of anticipated regret. An advance purchase triggers action regret from paying above valuation, or from missing a subsequent mark-down. In contrast, a consumer anticipates inaction regret from foregoing an affordable advance purchase discount, or facing a stock-out. Following Bell (1982) and Looms and Sugden (1982), we posit that consumer surplus features a separable regret component, proportional to the foregone surplus, as described next.

3.1. Consumer Surplus with Action and Inaction Regret

We consider two cases, depending if the product is available on spot, which occurs with probability k ; consumers are assumed to form rational expectations about availability, which affect their anticipated regrets.⁶ If the product is unavailable on spot (either due to a stock-out or if the firm only advance sells), the consumer regrets buying if $v < p_1$, otherwise she regrets waiting. So the foregone surplus from buying, respectively waiting correspond to the negative, respectively positive part of $v - p_1$, denoted $(v - p_1)^-$ and $(v - p_1)^+$. Anticipated regret is proportional to their expected values.

If, on the other hand, the product is available on spot, that is with probability k , then the spot price p_2 introduces an additional anchor for regret. A consumer who buys early regrets not only if her valuation falls short of the price paid, i.e. if $v < p_1$, but also missing a markdown, if $p_2 < p_1$. The foregone surplus from an advance purchase is then the larger of the two losses, i.e.

⁶ Our insights remain valid if, in absence of spot availability feedback or information, consumers do not regret a foregone markdown.

$(\min(v, p_2) - p_1)^-$. On the other hand, a consumer who decides to wait, either buys on spot if $v \geq p_2$, and regrets missing an advance purchase discount $(p_2 - p_1)^+$, or she cannot afford to purchase, if $v < p_2$, so she leaves the market empty-handed, but regrets not having bought in the first period, if $v \leq p_1$. In sum, the foregone surplus from waiting, in this case, is $(\min(v, p_2) - p_1)^+$. To summarize, the total expected surplus from buying, respectively waiting, are as follows:

$$S_1 = S_1(\rho) = \mu - p_1 + \rho((1 - k)E[\mathbf{v} - p_1]^- + kE[\min(\mathbf{v}, p_2) - p_1]^-), \quad (1)$$

$$S_2 = S_2(\delta) = kE[\mathbf{v} - p_2]^+ - \delta((1 - k)E[\mathbf{v} - p_1]^+ + kE[\min(\mathbf{v}, p_2) - p_1]^+), \quad (2)$$

where $\rho, \delta \geq 0$ measures the strength of action, respectively inaction regrets; in particular $\rho = \delta = 0$ for unemotional buyers. The first terms, $S_1(0) = \mu - p_1, S_2(0) = kE[\mathbf{v} - p_2]^+$, reflect the expected economic surplus from buying, respectively waiting (in absence of regret), whereas the second terms capture the corresponding emotional surplus. Intuitively, the symmetry in emotional surplus is explained by viewing inaction regret as the positive counterpart of action regret; a consumer regrets waiting, whenever she would have rejoiced over buying, and vice versa. Our model thus implicitly captures rejoice — the positive emotional surplus from making the right choice ex-post.

3.2. A Sufficient Regret Statistic for Consumer Choice

Consumers' differential expected surplus from an advance purchase can be written as

$$\Delta S = S_1 - S_2 = (1 + \delta)\Delta S(0) + (\rho - \delta)((1 - k)E[\mathbf{v} - p_1]^- + kE[\min(\mathbf{v}, p_2) - p_1]^-), \quad (3)$$

where $\Delta S(0) = S_1(0) - S_2(0) = \mu - p_1 - kE[\mathbf{v} - p_2]^+$ is the differential expected surplus without regret.

A consumer buys early whenever the expected surplus from doing so, S_1 , exceeds the expected surplus from waiting S_2 , i.e. if $\Delta S \geq 0$. We follow the standard assumption that a consumer indifferent between buying and waiting chooses to buy. Denoting $R(x) = E[\mathbf{v} - x]^-$, we obtain:⁷

LEMMA 1. *Consumers advance purchase if and only if*

$$\gamma = \frac{\rho - \delta}{1 + \delta} \leq \bar{\gamma}(p_1, p_2, k) = \frac{\mu - p_1 - k(\mu - p_2 - R(p_2))}{-R(p_1) + k(p_1 + R(p_1) - p_2 - R(p_2))^+}, \quad (4)$$

⁷ We use the terms increasing/decreasing, positive/negative in the weak sense throughout.

and $\bar{\gamma}(p_1, p_2, k)$ is decreasing in k and p_1 , and increasing in p_2 . All else equal, consumers are less likely to buy early the more (less) they regret actions, ρ (inactions, δ).

The second term in the denominator vanishes for markup policies $p_1 \leq p_2$, because $x + R(x) = E[\min(\mathbf{v}, x)]$ is increasing in x . The result confirms that, all else equal, lower advance prices p_1 , higher spot prices p_2 and higher rationing risk (lower k) increase propensity to buy early.

Interestingly, Lemma 1 shows that $\gamma = \frac{\rho - \delta}{1 + \delta}$ is a sufficient regret statistic for characterizing regret-averse consumer choice in an advance selling setting. In other words, (ρ, δ) consumers can be segmented into equivalence classes, according to the unique regret parameter $\gamma = \frac{1 + \rho}{1 + \delta} - 1 \geq -1$. Any (ρ, δ) consumer makes the same buy-or-wait choices as a $(\gamma, 0)$ consumer if $\gamma \geq 0$, or a $(0, -\gamma)$ consumer if $\gamma < 0$, so, for simplicity, we refer to her as a γ -consumer.

In particular, if action and inaction regret have the same strength, $\rho = \delta$, consumers behave as if they do not anticipate regret. Emotional buyers derive additional value from optimally managing their regrets, which magnify differential surplus (3) by a factor of $1 + \delta$, but this does not change the outcome of their advance purchase decision (4).

3.3. On The Relative Strength of Regrets

Lemma 1 implies that regret averse consumers are less (more) likely to advance purchase than unemotional buyers whenever they regret actions more than inactions, i.e. $\gamma > 0$ ($\gamma < 0$). It is therefore important to understand which type of regret is dominant.

Experimental research suggests that the relative strength of regrets is context dependent. On the short term, actions are typically regretted more than inactions, i.e. $\gamma > 0$, consistent with the omission bias (Kahneman and Tversky 1982), and labeled as “the clearest and most frequently replicated finding in the entire literature on counterfactual thinking” (Gilovich and Medvech 1995).

A reversal of the omission bias ($\gamma < 0$) has been evidenced, however, in purchase timing decisions (Simonson 1992), in particular for long-term regrets (Keinan and Kivetz 2008) and limited purchase opportunities (Abendroth and Diehl 2006), including auctions (Filiz-Ozbay and Ozbay 2007, Engelbrecht-Wiggans and Katok 2008). For example, consumers are presumably more likely to

regret foregoing a limited time offer (50% off Curves Gym membership for signing up on the day of trial), not purchasing a special or limited edition (e.g. Omega moon watch, 2010 Chevrolet Camaro Transformers, Disney DVD collections), a travel souvenir, or a ticket to a unique event (graduation ball, U2 concert) — and more so when putting these decisions in a long term perspective.

3.4. Inertia and Buying Frenzies

Our consumer behavior model (4) explains two types of “predictably irrational” purchase behaviors: inertia and frenzies. As consumers regret buying more than waiting, they are more likely to delay a rational purchase, an observed behavioral pattern known as buyers’ inertia (Su 2009a, Zeelenberg and Pieters 2004). By contrast, the more consumers regret foregoing a purchase opportunity, the more they act myopically and accelerate purchase. In particular, we may observe buying frenzies.

Indeed, unlike traditional economic models, *emotionally rational consumers may advance purchase at a negative economic surplus* ($S_1 \leq \mu - p_1 < 0$), in order to avoid inaction regret. This is because (unlike expected utility models), the expected surplus from not buying early (waiting), S_2 , can be negative due to inaction regret. For example, in absence of spot sales ($k = 0$), at any price p_1 such that $0 > S_1(0) = \mu - p_1 > -\delta E[\mathbf{v} - p_1]^-$, a $\rho = 0$ consumer prefers to advance purchase, because doing so causes less (emotional and economic) pain than not buying at all, $-\delta E[\mathbf{v} - p_1]^+ = S_2 < S_1 < 0$. By contrast, in the same situation, unemotional consumers wouldn’t buy at all.

The fact that consumers may buy at negative expected surplus is a distinctive feature of our consumer model, driven by counterfactual thinking and the assumption that consumers cannot avoid regret (e.g. through self-control) even when they decide not to buy. This assumption has strong empirical support. Indeed, even in absence of counterfactual information consumers have been evidenced to anticipate regret (Simonson 1992), and often search for (costly, yet economically irrelevant) negative counterfactual information, which triggers regret (Shani et al. 2008). Our results are consistent with Gee (2010), who shows how the design of a lottery can lead regret averse buyers to choose options with a negative expected value, rather than abstain.

National Lotteries are a classical example where consumers purchase at negative expected surplus. The unusual popularity of the Dutch Postcode Lottery, which splits the jackpot among ticket

holders with the winning postcode, has been attributed to the anticipation of inaction regret (Zeelenberg and Pieters 2004).⁸ In an experimental setting, Nasiry and Popescu (2010) find that sport fans are willing to pay above their expected valuation for tickets to a game, when these are sold only in advance, before the qualifying teams are known.

4. The Effect of Regret on Profits and Policies

In this section, we investigate the impact of anticipated regret on the firm's profits and decisions when consumers are ex-ante homogeneous. Section 6 explores how our insights extend when consumers are heterogeneous in terms of regret.

4.1. The Uncapacitated Firm

In absence of capacity constraints, we set $C = k = 1$. Because consumers are ex-ante homogeneous, given a pricing policy (p_1, p_2) , either all consumers wait, and a fraction $\bar{F}(p_2)$ purchase on spot, or they all advance purchase, if $\Delta S \geq 0$. The latter occurs whenever $\mu - p_1 - E[\mathbf{v} - p_2]^+ \geq \gamma E[\min(\mathbf{v}, p_2) - p_1]^-$, by (3). For simplicity, we ignore marginal costs, which magnify the effects of regret, so the firm's profit is:

$$\pi(\gamma; p_1, p_2) = \begin{cases} p_1, & \text{if } \gamma \leq \bar{\gamma}(p_1, p_2, k = 1), \text{ see (4);} \\ p_2 \bar{F}(p_2), & \text{otherwise.} \end{cases} \quad (5)$$

The higher γ , the more consumers are likely to delay purchase, by Lemma 1. Therefore, higher action regret or lower inaction regret benefit a price taking firm whenever spot sales are more profitable than advance sales, i.e. $p_2 \bar{F}(p_2) \geq p_1$. These effects change when the firm is able to optimize the spot or advance price in (5) in response to regret. Our next result summarizes the effect of regrets on the profits of a firm who is constrained in either one or both periods.

PROPOSITION 1. (a) *The expected profit of a price taking firm is increasing in γ if $\frac{p_1}{p_2} \leq \bar{F}(p_2)$, otherwise it is decreasing in γ .* (b) *The optimal expected profit of a firm which is a price taker in the advance (spot) period, is increasing (decreasing) in γ .*

⁸ Unlike regular lotteries, feedback here is almost unavoidable: you don't have to buy a ticket to know your combination (zipcode) and would find out from the neighbors if you could have won. Given this feedback, regret aversion motivates purchase as a protection from the large regret experienced if you didn't buy and your postcode were drawn.

Action regret has negative profit consequences for firms facing spot price constraints, in particular those practicing markdown policies, or lacking credible commitment devices. On the other hand, action regret can benefit firms who offer relatively steep advance purchase discounts, or face price pressure in the advance period. Incumbent airlines, for example, offer steep advance purchase discounts due to competitive pressure from low cost carriers. Sport teams and rock bands keep advance prices low for image, fairness or social considerations. Section 6.2 illustrates a setting where market heterogeneity creates pressure on advance prices, making action regret beneficial.

A firm with full pricing flexibility optimally responds to regret by solving $\max_{p_1, p_2} \pi(\gamma; p_1, p_2)$, as given by (5). Consumers are ex-ante homogeneous, so the firm will sell only in one period. On spot, it obtains at most $\bar{\pi} = \max p\bar{F}(p)$, by charging $p_0 = \operatorname{argmax} p\bar{F}(p)$, unique by the unimodality assumption (Section 2). In advance, the firm extracts at best consumers' maximum wtp in absence of a spot market, $p_1 = \bar{w}(\gamma)$, which by (3), solves:

$$\mu - p_1 + \gamma R(p_1) = 0. \quad (6)$$

The left hand side is decreasing (because $\gamma \geq -1$), so $\bar{w}(\gamma)$ is decreasing in γ . In particular, $\bar{w}(\gamma) \geq \bar{w}(0) = \mu$ whenever consumers regret inactions more than actions ($\gamma \leq 0$); in this case, because $\bar{\pi} \leq \mu$, the firm always advance sells. Our next result shows that advance selling may not be profitable, however, if consumer regret exceeds a positive threshold $\bar{\gamma}$.

PROPOSITION 2. *A price-setting firm sells only in advance at $p_1^* = \bar{w}(\gamma)$, if $\gamma \leq \bar{\gamma} = -\frac{\mu - \bar{\pi}}{R(\bar{\pi})}$, otherwise it sells only on spot at $p_2^* = p_0$. The optimal profit, $\pi^*(\gamma) = \max\{\bar{w}(\gamma), \bar{\pi}\}$, is decreasing in regret, γ . In particular, the firm benefits from consumer regrets if and only if inactions are regretted more than actions, i.e. $\gamma < 0$.*

In contrast with the bulk of the advance selling literature (Dana 1998, Nocke and Peitz 2008, Xie and Shugan 2001), this result provides an argument for why firms might *not* benefit from advance selling, even when capacity is ample and buyers are homogeneous. This is justified if consumers regret purchases, relative to non-purchases, beyond a threshold $\bar{\gamma} > 0$, determined by

their valuation uncertainty. In other words, whether or not firms can profitably sell to uninformed consumers, depends on the relative intensity of their regrets and the uncertainty they face.

On the other hand, in contexts where not buying is associated with greater regret (such as limited purchase opportunities or unique events, see Section 3.3), advance selling allows firm to achieve a buying frenzy, whereby consumers advance purchase at a net loss in order to avoid inaction regret, as discussed in Section 3.4. Indeed, when $\gamma < 0$, Proposition 2 shows that the optimal pricing strategy is to advance sell at $\bar{w}(\gamma) > \mu$, in particular $S_2 \leq S_1 < 0$, so consumers buy at negative expected economic (and emotional) surplus.⁹ Our results provide an emotionally rational explanation for how firms can create frenzies by selling only in advance, even in absence of capacity constraints. This may explain why tickets to rare or unique events (e.g. graduation balls, Bregenz festival) are priced high and not sold at the door, while sports events fetch high prices by selling tickets only in advance before the teams or players are known.

In summary, the effect of regrets on the profits of a price-setting firm depend on the type of regrets — action regret hurts profits, whereas inaction regret is beneficial — and their relative magnitude, γ . This suggests that firms may benefit from mechanisms that mitigate consumers' action regret, such as refunds or resales (investigated in Section 5), but less so from price protection or other guarantees mitigating inaction regrets. On the other hand, price-setting firms benefit from priming inaction regret, for example through marketing campaigns that trigger appropriate counterfactual thinking (e.g. buy-now-or-regret-later advertising, or framing the offer as a special or limited opportunity, see Section 3.3).

4.2. Capacity Constraints

This section extends our analysis to the case when the firm faces capacity constraints, $C < 1$. Rationing risk triggers additional regrets, affecting both economic and emotional surplus. This risk is endogenously determined by consumer response to the firm's policy. For a given capacity C and spot price p_2 , the rationing probability when consumers wait is $k = k(p_2) = \min(1, C/\bar{F}(p_2))$,

⁹ For low enough γ , the firm can even advance sell at prices above the optimal spot-only price, i.e. $\bar{w}(\gamma) \geq p_0$.

and their wtp in advance, $w(\gamma; p_2, C)$ solves $\Delta S(\gamma; w, p_2, k(p_2)) = 0$, see (3). The following remarks highlight several effects of capacity on customer behavior which make the problem different from the uncapacitated case studied in Section 4.1; formal statements and proofs are in the Appendix.

Remark 1. *Customers are willing to advance purchase at a premium, i.e., for any γ , $w(\gamma; p_2, C) > p_2$, for a range of spot prices p_2 , which are low enough to induce excess spot demand, $\bar{F}(p_2) < C$.*

Remark 2. *Capacity constraints increase customers' wtp in advance, i.e. $w(\gamma; p_2, C)$ is decreasing in C for any given p_2 . This suggests that firms might benefit from artificially restricting supply.*

Remark 3. *Customers' wtp in advance is not necessarily increasing in the spot price, i.e. $w(\gamma; p_2, C)$ may be non-monotone in p_2 . Lower spot prices may indeed reduce expected surplus from waiting as the rationing probability increases. Firms can thus extract more in advance by offering lower prices on spot, suggesting that selling in both periods may be optimal.*

These remarks show that, for given prices (p_1, p_2) , capacity constraints bring important structural changes in customer behavior, further magnified by inaction regret (but atoned by action regret). This raises the question of whether the interaction of capacity constraints and inaction regret can induce buying frenzies, such as premium advance selling or artificial supply limits? For example, can this setup explain why tickets for La Scala or Broadway performances are sold in advance at a premium, or whether it is profitable for Disney and Nintendo to artificially limit supply of DVDs and game cartridges, respectively? Remarks 1 and 2 reveal a positive answer to these questions, if the firm faces sufficient price pressure in the spot market.¹⁰ We next show that these effects disappear in homogeneous markets if firms optimally respond to regret; we revisit this issue under regret heterogeneity in Section 6.1.

The insights in Section 4.1, in particular Propositions 1 and 2 extend under capacity constraints. Instead of revenue per customer $\bar{\pi}$, the relevant unit of analysis becomes revenue per capacity unit:

$$\bar{\pi}_C = \begin{cases} \bar{\pi}/C, & \text{if } C \geq \bar{F}(p_0); \\ \bar{F}^{-1}(C) = \inf\{x; \bar{F}(x) \geq C\}, & \text{otherwise.} \end{cases}$$

¹⁰ This replicates, under regret, the insights in deGraba (1995) and Courty (2005). Retailers face price pressure on spot due to end of season sales, as do sellers of perishable goods. A hotel in Moshi (Tanzania, at the Kilimanjaro base camp) is committed to offer low prices to customers who just show up; this is to support local business travel.

PROPOSITION 3. *A price-setting firm sells in advance at $\bar{w}(\gamma)$, solving (6), if $\gamma \leq \bar{\gamma}(C) = -\frac{\mu - \bar{\pi}_C}{R(\bar{\pi}_C)}$, otherwise it spot sells at $p_0(C) = \max(p_0, \bar{F}^{-1}(C))$. The optimal profit $\pi^*(C) = C \max\{\bar{w}(\gamma), \bar{\pi}_C\}$ is decreasing in γ and increasing in C , and $\bar{\gamma}(C)$ is increasing in C .*

Alternatively, the result shows that advance selling is optimal above a capacity threshold $\bar{C}(\gamma)$, given by $\bar{\gamma}(\bar{C}(\gamma)) = \gamma$, and this threshold increases with regret γ . Although premium advance selling can be beneficial for fixed prices, this is not the case at optimality if the firm has full price flexibility and optimally responds to regret. This is because, unlike Remark 3 might suggest, the firm cannot leverage consumers' unavailability regret in order to extract more than $\bar{w}(\gamma)$ in advance.

Capacity constraints limit the benefit of advance selling for a price setting firm, in the sense that they lower the threshold on regret $\bar{\gamma}(C)$, below which advance selling is optimal. From an estimation perspective, the results shows that it is only relevant to measure regret (γ) exactly, if it is likely to fall below the threshold $\bar{\gamma}(C) = -\frac{\mu - \bar{\pi}_C}{R(\bar{\pi}_C)} \geq -1$, determined by capacity and valuation; above this threshold, the firm should spot sell, and spot prices and profits are unaffected by γ . Unlike the uncapacitated case, this threshold is not necessarily positive: $\bar{\gamma}(C) < 0$ for $C < \bar{F}(\mu)$; in this case, the firm might not advance sell to customers who regret inactions more than actions. In particular, capacity constraints reduce the prevalence of buying frenzies observed in Section 3. Overall, we conclude that, at optimality, the effects of both action and inaction regrets on firm profits and policies are in fact diminished by capacity constraints.

4.3. Limited Advance Sales

We next investigate how limiting advance sales by setting a booking limit $B \in (0, C)$ interferes with consumer regrets in affecting the firms optimal policy and profits. The benefits of booking limits are theoretically well-established (Xie and Shugan 2001, Yu et al. 2010). A common practice in airlines, hotels, ticketing and other capacitated service industries, booking limits allow firms to reserve availability for late coming high-paying customers (we do not require second period arrivals or aggregate demand uncertainty).

Given a pricing and booking limit policy (p_1, p_2, B) , customers willing to advance purchase may get rationed because of the booking limit B . Consumers who get rationed remain in the market,

and will try to buy on spot if their valuation exceeds the spot price. Because consumers are ex-ante homogeneous they will all try to advance purchase, or they will all wait. If they all try to advance purchase, then $k = k(p_2, B) = \min(1, (C - B)/((1 - B)\bar{F}(p_2)))$. An optimal policy makes an atomic consumer indifferent between buying and waiting, so it satisfies $\Delta S(p_1, p_2, k(p_2, B)) = 0$, see (3).

PROPOSITION 4. *Suppose that the firm can limit advance sales, and \mathbf{v} has log-concave density or a two-point distribution. The optimal policy is to advance sell at a discount with a positive booking limit if $\gamma \leq \bar{\gamma}_B(C)$, where $\bar{\gamma}_B(C) \geq \bar{\gamma}(C)$; otherwise spot selling is optimal. Moreover, the optimal prices, booking limit and profits decrease with γ .*

Papers that use incentive compatible fluid pricing models (such as ours) to characterize booking limits without regret make similar distributional assumptions (e.g. two-point support in Möller and Watanabe 2009, continuous in Yu et al. 2010). A wide range of parametric families have log-concave density, e.g. uniform, exponential, normal and logistic (Bagnoli and Bergstrom 2005).

Booking limits are suboptimal in uncapacitated settings, and increase the prevalence of discount advance selling (premium advance selling remains sub-optimal). Intuitively, this is because booking limits enable the firm to sell to some high valuation consumers in the spot period, while clearing the remaining capacity at a discount in advance. Booking limits also increase the regret threshold above which spot selling is optimal. The profitability and magnitude of booking limits is diminished by action regret. On the other hand, the more consumers regret foregoing the discount or potentially being stocked out, the more relevant booking limits are.

A robust result of our analysis so far is that firms who set prices optimally are hurt by action regret, but benefit from inaction regret; this motivates us to study mechanisms to mitigate action regrets in the next section. On the other hand, limited purchase opportunities and perception of scarcity have been shown to increase inaction regret, triggering a reversal of the omission bias, i.e. $\gamma < 0$ (Simonson 1992, Abendroth and Diehl 2006, see Section 3.3). In such cases, our result suggest that firms with limited availability benefit from consumer regrets. Psychological effects of capacity on the relative scale of regrets, such as γ increasing in C or B , are not captured by our model, but offer an interesting query for empirical and analytical investigation.

5. Regret Mitigating Mechanisms

Action regret has been shown to adversely affect profits for price setting firms. We therefore propose in this section, three mechanisms that help mitigate action regret — refunds, options and resales, and study their impact on consumer behavior, firm decisions and profits. Because we are interested in mitigating action regret, we assume $\gamma \geq 0$ in this section.

5.1. Refunds

Refund policies, such as GM’s 60-day money-back offer (see Introduction), provide a vehicle to stimulate demand and profits by insuring consumers against the downside of their decisions. Similarly, by allowing returns, internet retailers are able to draw consumers to buy an item without trying it, as Amazon.com did for the Kindle. Not all companies offer full refunds. Domino’s Pizza, for example, went from a ‘30 minutes or it’s free’ guarantee to only \$3 off. We investigate the effectiveness and design of optimal refund policies to mitigate consumer regrets.

Consumer behavior. Upon an advance purchase, a customer returns the product for a refund r whenever her valuation turns out lower than the refund, i.e. $v < r$. Effectively, a refund policy shifts the valuation of consumers who advance purchase from \mathbf{v} to $\max(\mathbf{v}, r)$, insuring them against downside valuation risk. Consumers gain $-R(r)$ in both economic and emotional surplus, so expected surplus from an advance purchase (1) increases by $-(1 + \rho)R(r)$. Refunds do not affect expected surplus from waiting (2), in particular inaction regret is triggered only when $v > p_1$, in which case refunds are irrelevant. By (3), the differential expected surplus from buying early can be written as: $\frac{\Delta S^r}{1+\delta} = \frac{\Delta S}{1+\delta} - (1 + \gamma)R(r)$, so γ is again a sufficient regret statistic for customer choice.

Firm profits. GM resells returned cars as used, at a lower price. Assume for simplicity that the firm salvages items that are returned or unsold at $s \geq 0$ (e.g. Davis et al. 1995, Su 2009b).¹¹ The firm maximizes incremental profit on top of the salvage value, which can be written as:

$$\pi_s(\gamma; p_1, p_2, r) = \begin{cases} \min(C, 1)(p_1 - s - (r - s)F(r)), & \text{if all consumers buy early;} \\ \min(C, \bar{F}(p_2))(p_2 - s), & \text{if all consumers buy on spot.} \end{cases} \quad (7)$$

¹¹ An alternative setup where returned items are put back into circulation at full price is considered in the next section. All results extend for a marginal cost of production $c = s$.

From the customer's perspective, buying on spot at p_2 is equivalent to buying in advance with a full refund, $r = p_2 = p_1$. This policy also yields the same profits for a firm with excess supply, $C = 1$, by (7). Such firms can thus focus on advance selling with refunds.¹²

We first analyze the uncapacitated case, then argue that the main insights extend when capacity is limited. For a given refund r , consumers' maximum wtp, $\bar{w}(\gamma; r)$, solves: $\mu - p + \gamma R(p) = (1 + \gamma)R(r)$. In particular, $\bar{w}(\gamma; r)$ is increasing in r and decreasing in γ (see Appendix).

PROPOSITION 5. *The optimal policy is to offer partial refunds, $0 < r^* < p^* = \bar{w}(\gamma; r^*)$, solving*

$$1 + \gamma F(p^*) = (1 + \gamma)F(r^*) / (F(r^*) + (r^* - s)f(r^*)). \quad (8)$$

The optimal refund $r^ = r^*(\gamma)$ increases in γ , $r^*(0) = s$; the optimal profit $\pi_s^*(\gamma)$ is decreasing in γ .*

Refunds increase consumers' wtp in advance by insuring them against wrong purchase decisions. Such guarantees command a premium, i.e. the firm charges a higher price when it offers refunds than when it does not: $p^* = \bar{w}(\gamma; r^*) \geq \bar{w}(\gamma; 0) = \bar{w}(\gamma)$, as defined in (6). For unemotional buyers, the firm refunds the salvage value $r^*(0) = s$; so $r^*(\gamma) - s$ reflects the positive refund for regret.

We find it interesting that, *with refunds, the firm may actually charge higher prices to consumers who regret purchases more*, i.e. the optimal advance price $p^*(\gamma) = \bar{w}(\gamma; r^*)$ is increasing in γ , above a certain threshold, as illustrated in Figure 1a. This is in sharp contrast with our results in previous sections, where the optimal advance selling price, $\bar{w}(\gamma)$, was always decreasing in γ . Intuitively, as regret becomes a bigger issue for consumers, the positive effect of a higher refund on willingness to pay outweighs the negative action regret effect.

A full refund policy, such as GM's, eliminates action regret by providing full insurance against wrong advance purchase decisions. Yet, Proposition 5 shows that full refund policies are suboptimal, $r^* < p^*$ (supporting Domino Pizza's policy change). In contrast with Proposition 2, this shows that spot selling is suboptimal regardless of regrets if uncapacitated firms can offer refunds. While

¹² We ignore for simplicity both firm and consumer transaction costs associated with refunds. This is consistent with the literature (Su 2009b, Liu and Xiao 2010). We also ignore the fact that only a fraction of consumers who value the refund may redeem it; this would only magnify the observed effects.

dominated by partial refunds, *full refunds are better than no refunds* for sufficiently high salvage value s (regardless of regret), or if customers are sufficiently regret averse (see Appendix).¹³

A common finding in the literature is that the profitability of refunds in homogenous markets is determined by supply-side variables, e.g. marginal cost of production or salvage value (Liu and Xiao 2010, Su 2009b, Xie and Shugan 2001). In absence of such variables, refunds have been shown to be suboptimal in homogenous markets, even if capacity is constrained (Liu and Xiao 2010). Our results add a new dimension to this literature by showing that demand-side effects, such as anticipated regret, can trigger the profitability of refunds. As illustrated in Figure 1a, refunds can be profitable in our model even if $s = 0$, provided that consumers are sufficiently regret-averse.

Figure 1b indicates that the relative incremental profit gains from offering refunds can be significant. Interestingly, the value of offering refunds depends non-monotonically on regret. Refunds are increasingly beneficial up to a regret threshold, after which their relative benefits diminish (this is because, without refunds, the firms spot sells to sufficiently regretful buyers, and spot profits do not depend on regret). Before this threshold, both profits with and without refunds decrease with regret, but refunds are able to recapture a larger share of the revenues lost to regret.

Capacity Constraints. With capacity constraints, $C < 1$, the firm either advance sells with the partial refund policy given in Proposition 5, or sells only on spot if capacity is sufficiently tight; spot prices and profits are independent of regret. All other insights from this section remain valid under capacity constraints. These diminish the negative effects of regret on profits, and hence the prevalence of advance selling with refunds. We omit the analysis for conciseness.

5.2. Options

Suppose that, at a price x consumers can purchase the right to buy the product on spot, at an exercise price of r . A consumer exercises the option whenever $v \geq r$, so for her, this is technically equivalent to a partial refund policy ($p = x + r, r$). Options are different from refunds, however, and firms can potentially offer both. For example, car dealers sell options, framed as a non-refundable

¹³ Similarly, Che (1999) shows that full refunds are more profitable than no refunds if the firm faces sufficiently high marginal cost or if (CARA) consumers are sufficiently risk averse; he does not study partial refunds.

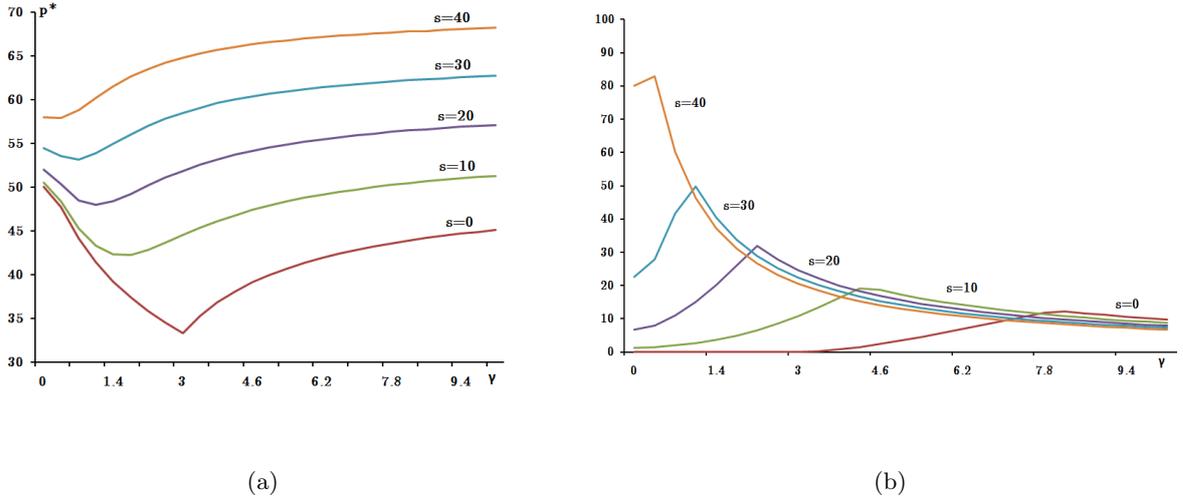


Figure 1 (a) The optimal price $p^*(\gamma)$ as a function of regret γ . (b) The relative increase in profits due to refunds, $\frac{\pi_s^* - \pi^*(r=0)}{\pi^*(r=0)}$. In both figures $v \sim U[0, 100]$.

deposit, on a Chevy Volt before it is produced; once the car is in the lot, the consumer can buy it upon paying in full, i.e. exercising the option. Unlike GM's refund policy, where a used car can be returned after two months, a capacitated firm can sell more options than capacity, on account that only a fraction will be exercised. So options allow better capacity utilization than refunds. Options are equivalent to a (full recirculation) refund model where returned products are put back in the market at no loss of margin; this is appropriate for services with delayed consumption, e.g. travel.

Our setup in this section follows that in Gallego and Şahin (2009), who show that options considerably improve profits for capacitated firms, and that the fluid model, used here, is asymptotically optimal. The optimal policy is derived in the Appendix.

PROPOSITION 6. *The optimal profit and option price $x^*(\gamma) > 0$ are decreasing in γ , and the optimal exercise price $r^*(\gamma)$ is increasing in γ . Moreover, $r^*(\gamma) > 0$ for $C < 1$ or for γ above a positive threshold.*

To mitigate regret, firms offer lower option prices to regretful buyers, but then charge them higher exercise prices. When capacity is limited, or consumers are sufficiently regret-averse, options dominate pure advance selling, i.e. $r^* > 0$. Unlike with refunds, spot selling, i.e. offering free options $x = 0$, is sub-optimal even if capacity is tight, supporting the practice of non-refundable deposits

practiced by hotels and car dealers. In an experimental setting, Sainam et al. (2009) show that options for sporting ticket events increase customers' wtp and seller profits. Our results suggest that an alternative explanation for these effect is that options mitigate regret.

5.3. Resales

We briefly argue that secondary markets can increase firm profits by mitigating regret. This may explain why entertainment venues such as theaters, concerts, and sporting events allow brokers and scalpers to resell primary tickets. Estimates suggest that roughly 10% of tickets for shows and sporting events are resold, reaching 20-30% for top-tiered seats (Happel and Jennings 2002).

Suppose that customers have the opportunity to resell in the spot period to a third party broker, for a price \mathbf{s} , which is a priori uncertain with known distribution $F_{\mathbf{s}}$. For simplicity, we assume that the broker sells to a different customer pool than the firm (Golzolari and Pavan 2006); our results extend as long as cannibalization between the two markets is limited.¹⁴ The model with resales can be shown to be equivalent to the basic model in Section 2 with a shifted valuation distribution, $\mathbf{w} = \max(\mathbf{v}, \mathbf{s})$. This implies that, for a given pricing policy, customers are more likely to advance purchase, and willing to pay a higher advance price when resale is allowed, because resales provide a protection against action regret. Resales thus provide the firm the opportunity to charge higher prices and obtain higher profit in the advance period. In addition, resales also allow the firm to extract higher spot profits. These results, formalized in the Appendix, together imply that allowing resales can be profitable as a means to mitigate action regret.

6. Regret Heterogeneity

In this section, we illustrate how regret heterogeneity affects our insights. Not everybody regrets, at least not to the same extent. Unlike Professor Regrette, her colleague Ilia Piaf prides himself to make rational decisions; he has no regrets for being wrong ex-post, as long as he can rationalize his choices ex-ante, so emotions do not influence his decisions. In this case, we show that a capacitated

¹⁴ For example, until recently, tickets to the Roland Garros tennis open were either sold in advance by mail to French residents, or at the door, if available; meanwhile secondary markets, such as eBay, targeted international sales. Cars, books and appliances are other typical examples where primary and resale (secondary) markets are largely disjoint.

firm may sell to Ilia in advance at a premium, and benefit from a larger share of consumers like Prof. Regrette. On the other hand, if she has higher valuation than he does, then the firm may be able to benefit from her regrets, by selling to her at a premium, on spot.

To illustrate these insights, we assume for simplicity that consumers have two-point valuation distributions, and a fraction α of the market anticipates regret ($\gamma > 0$) whereas the rest does not ($\gamma = 0$); our insights extend as long as one segment regrets more than the other, as measured by γ .

6.1. Premium Advance Selling

Premium advance selling, or mark-downs, are common practice for music or sporting events (Broadway shows, La Scala in Milan, Duke's basketball games), but also seasonal goods (skis, fashion) or travel. The next result provides an alternative explanation for premium advance selling, when consumers are heterogeneous in their regrets, and face the same uncertainty $\mathbf{v} = (H, q; L, 1 - q)$:

PROPOSITION 7. The firm's optimal pricing policy is as depicted in Figure 2. In particular, premium advance selling can be optimal in mixed markets (intermediate values of α), and more prevalent as γ increases. The optimal profit $\pi^(\gamma, \alpha, C)$ is increasing in C , decreasing in γ , and generally non-monotone in α .*

There are five possible policies at optimality, depending on capacity and market mix: (1) spot sell at H (for very tight capacity); (2) advance sell at μ to non-regretful buyers only (if they can clear capacity); (3) a mark-up policy that sells to non-regretful buyers in advance at a discount μ , and to regretful buyers on spot at H (when capacity is not cleared by the non-regretful buyers in advance); (4) a mark-down policy which offers the product on spot at L , and charges a premium in advance to non-regretful buyers (when capacity is moderately large, but tight enough to ration spot demand at low prices); (5) advance sell to the entire market at $\bar{w}(\gamma)$, the willingness to pay of regretful buyers (when capacity is ample).

Without capacity constraints ($C = 1$), or if the optimal policy does not clear capacity, we show in the appendix that profits decrease in the fraction of regretful consumers, α . This is not necessarily true when capacity is tight. Indeed, when discount advance selling clears capacity, more customers

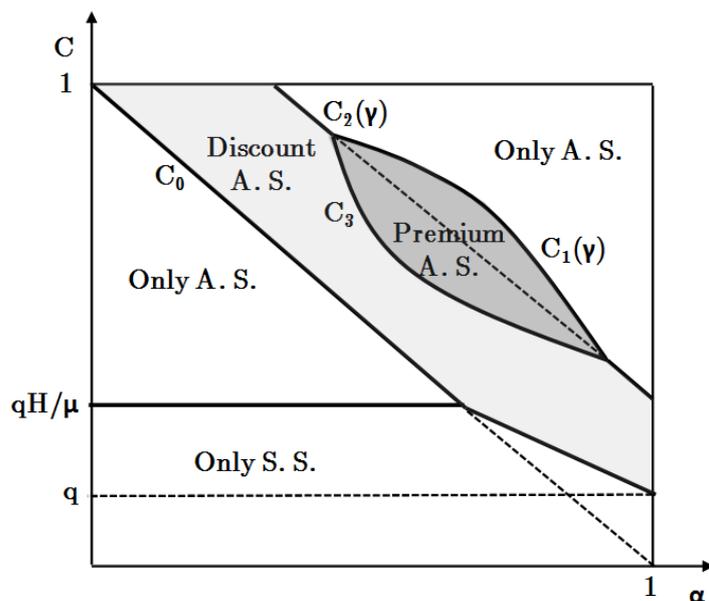


Figure 2 The optimal policy in markets where a fraction α of customers regret, $q \leq L/H$ and $\gamma \leq \bar{\gamma}(L, H, q)$; A.S.=advance selling, S.S.= spot selling. See Appendix for boundary equations. $C_i(\gamma)$ are increasing in γ , for $i = 1, 2$, and independent of γ otherwise, so the Premium A.S. area grows with γ .

buy at spot prices on spot as α increases, resulting in higher profits. Profits decrease in α under premium advance selling, and do not depend on α when the firm sells in one period. These results quantify and condition the intuition regarding the breadth of inertia presented in Su (2009a).

Unlike homogeneous markets, Proposition 7 shows that, for intermediate levels of capacity, the firm segments the market based on regret, and offers the product in both periods. In this case, the pricing policy induces regretful customers to wait, while non-regretful customers buy in advance. When available capacity cannot be cleared at high spot prices, the firm can be better off advance selling at a premium, and clearing the remaining capacity at low spot prices. Unemotional customers pay a premium in advance to avoid being rationed on spot due to competition with regretful buyers. As γ increases, premium advance selling policies become more prevalent, i.e. the corresponding area in Figure 2 becomes larger.

Limiting advance sales does not change our insights, except that profits decrease in α , whereas the booking limit is non-monotone in α . Spot selling and premium advance selling regions shrink in Figure 2, and pure advance selling is suboptimal. Thus, booking limits make discount advance selling policies more prevalent, enabling the firm to clear capacity more often; their benefit and

magnitude are diminished by regret, confirming the insights in Section 4.3.

To summarize, our results in this section propose that regret heterogeneity provides an alternative explanation for premium advance selling. In particular, we obtain this result without assuming aggregate demand uncertainty (Nocke and Peitz 2007), heterogeneity in arrivals (Su 2007) or in valuation distributions (Möller and Watanabe 2009).

6.2. Action Regret May Benefit the Firm

Suppose now that, regrets aside, Professor Regrette has a higher value for attending the INFORMS conference than her über-rational colleague Ilia. She is also more keen on skiing than he is, although unlike him, she has not yet booked her next ski trip; she would get a hard time for doing so if there's not enough snow. How can INFORMS, or the ski resort, leverage this situation when targeting both consumers? In settings where buyers with high valuation regret (purchases) more, we show that action regret can actually benefit the firm, in contrast with our results so far. A similar result can be shown to hold if buyers with low valuation are myopic.

Assume for simplicity that a fraction α of regretful buyers have $\mathbf{v} = (H, q; 0, 1 - q)$, $\gamma > 0$, and the remaining non-regretful buyers ($\gamma = 0$) have a lower valuation $\mathbf{v} = (L, q; 0, 1 - q)$, $H > L$. Our qualitative insights extend for general valuations ordered by first order dominance.

PROPOSITION 8. *The optimal expected profit is increasing in the regret factor γ , and in the proportion of regretful customers α . In particular, for $\gamma > \frac{H-L}{L(1-q)}$, the optimal policy is to sell to low valuation consumers in advance at $p_1 = qL$ and to high valuation regretful consumers on spot at $p_2 = H$; otherwise the firm sells only in one period.*

If high valuation customers regret purchases more, the firm advance sells to non-regretful buyers at a discount while regretful customers wait to purchase on spot. By contrast, in absence of regret, Nocke and Peitz (2008) show that firms advance sells at a discount to high valuation customers and spot sell to low valuation customers. Regret makes high valuation customers delay purchase in our setting — this segmentation extracts the maximum profit potential from the market.

These results suggest that it is important for firms to understand how regret varies across market segments, in order to understand its effect on profits. Organizations such as INFORMS, airlines, or tour operators may actually benefit from consumers' regret, if high valuation buyers regret more. In particular, this advises against offering regret mitigating mechanisms, such as refunds, options or allowing resale markets in these settings.

7. Conclusions

In this paper, we developed a model where strategic customers anticipate regret when deciding whether or not to advance purchase, while uncertain about their true valuations. Our results provided answers to the three questions raised in the Introduction, as follows:

(1) We showed that the purchase behavior of regret-averse consumers with uncertain valuations is characterized by a single regret parameter, γ , which measures the relative strength of action and inaction regrets. Our consumer behavior model explains two behavioral regularities observed in buy-or-wait contexts: inertia (delayed purchase) and frenzies (buying in advance at negative surplus).

(2) We characterized the effect of regret on the firm's policies and profits, and identified a normative threshold above which regret changes the structure of optimal sales policies. In general, we found that action regret reduces the benefits and prevalence of advance selling and booking limit policies, leading to lower advance prices, booking limits and profits for a price-setting firm. Inaction regret has the opposite effects than action regret; when it is dominant, firms can create frenzies by advance selling at high prices even in absence of capacity constraints. These effects are diminished by capacity constraints, and can be reversed when the firm faces price pressure in the advance period, or if regretful buyers have higher valuations. Differences in regret trigger premium advance selling by capacitated firms, who may actually benefit from larger shares of regretful buyers.

(3) Finally, we showed when and how firms should leverage or mitigate regrets. Our results explain the profitability of marketing campaigns that induce inaction regret (e.g. using "buy now or regret later"-type advertising, emphasizing a foregone discount or a limited offer), for firms with

full price flexibility. Such practices may not be beneficial for firms such as airlines, who face price pressure in advance, or if low valuation buyers are myopic (or regret less). On the other hand, we showed how firms can recover a large fraction of the profit lost to action regret by selling in advance with partial refunds, offering options or allowing resales. Our results offer a new explanation for the profitability of these practices, in that they mitigate regret.

Given the limited empirical work on measuring regret (Filiz-Ozbay and Ozbay 2007, Engelbrecht-Wiggans and Katok 2008, Bleichrodt et al. 2010), our results emphasize the importance of measuring the relative strength of anticipated regrets, γ , within and across market segments. Finally, it is critical to understand the type of uncertainty underlying customer valuations, which in turn affects the regret thresholds relevant for the firm's decisions.

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Appendix: Proofs

The following properties of $R(x) \triangleq E[\mathbf{v} - x]^-$ will be useful throughout our analysis.

LEMMA 2. (a) $R(x) \leq 0$ is decreasing and concave in x , in particular if F is continuous then $R'(x) = -F(x)$. (b) $x + R(x)$ is increasing and concave in x . (c) $0 \leq x\bar{F}(x) \leq x + R(x) \leq \mu$, for all $x \geq 0$.

Proof: Part (a) follows because $(v-x)^-$ is decreasing and concave in x , and expectation preserves monotonicity and concavity. The derivative follows from Leibniz rule. Parts (b) and (c) follow by writing $x + R(x) = E[\min(\mathbf{v}, x)]$, which is increasing and concave in x , and moreover $\mu = E[\mathbf{v}] \geq E[\min(\mathbf{v}, x)] = x\bar{F}(x) + E[\mathbf{v}|\mathbf{v} < x]F(x) \geq x\bar{F}(x)$. \square

Proof of Lemma 1: From (3) we can write

$$\bar{\gamma}(p_1, p_2, k) = -\frac{\mu - p_1 - kE[\mathbf{v} - p_2]^+}{E[\mathbf{v} - p_1]^- + k(E[\min(\mathbf{v}, p_2) - p_1]^- - E[\mathbf{v} - p_1]^-)}.$$

Writing $E[\mathbf{v} - p_2]^+ = E[\mathbf{v}] - E[\min(\mathbf{v}, p_2)] = \mu - p_2 - R(p_2)$ gives the nominator in (4). Further use $E[\min(\mathbf{v}, p_2) - p_1]^- = E[\min(\mathbf{v}, p_2, p_1)] - p_1$, to write $E[\mathbf{v} - p_1]^- - E[\min(\mathbf{v}, p_2) - p_1]^- = E[\min(\mathbf{v}, p_1)] - E[\min(\mathbf{v}, p_2, p_1)] = p_1 + R(p_1) - (p_2 + R(p_2))$, if $p_1 > p_2$, and zero otherwise. Because $x + R(x)$ is increasing by Lemma 2b, this gives the desired expression for the denominator. To show monotonicity, after some algebra, we can write:

$$\bar{\gamma}(p_1, p_2, k) = \frac{(1-k)E[\mathbf{v} - p_1]^+ + kE[\min(\mathbf{v}, p_2) - p_1]^+}{-(1-k)E[\mathbf{v} - p_1]^- - kE[\min(\mathbf{v}, p_2) - p_1]^-} - 1.$$

It is easy to see that both nominator and denominator are positive. The nominator is decreasing in k and p_1 and increasing in p_2 , and vice versa for the denominator. This shows the desired result.

Proof of Proposition 1: (a) Suppose $d = \frac{p_1}{p_2} \leq \bar{F}(p_2)$, i.e. $p_1 \leq p_2\bar{F}(p_2)$. Therefore, the firm obtains higher profits if customers wait, i.e. $\Delta S < 0$, or equivalently $\gamma > -\frac{\Delta S(0; p_1, p_2)}{R(p_1)}$. The result then follows because γ is increasing in ρ , and decreasing is δ . The other part is proved similarly.

(b) Suppose the spot price p_2 is fixed. The maximum price that induces consumers to advance purchase is $w(\gamma; p_2)$, which solves $\Delta S = 0$, and hence is decreasing in γ cf. (3). Therefore, so is the optimal profit $\pi^*(\gamma; p_2) = \max\{w(\gamma; p_2), p_2\bar{F}(p_2)\}$.

Consider now the case where the advance price p_1 is exogenously fixed. Assume that $p_1 \leq \bar{w}(\gamma)$, otherwise advance selling at p_1 is not feasible (all consumers wait, and the firm spot sells at p_0 with profit $\bar{\pi}$, independent of γ). The spot price that makes customers indifferent between buying early and waiting, $p_2(\gamma)$, solves (4), and increases in γ , by Lemma 1. The optimal spot selling profit is $\bar{\pi}$ if $p_0 \leq p_2(\gamma)$, and $p_2(\gamma)\bar{F}(p_2(\gamma))$ otherwise. Alternatively, advance selling fetches profit p_1 . If

$p_0 \leq p_2(\gamma)$, optimal profit is $\pi^* = \max(p_1, \bar{\pi})$, independent of γ . On the other hand, if $p_0 > p_2(\gamma)$, optimal profit is $\pi^* = \max(p_1, p_2(\gamma)\bar{F}(p_2(\gamma)))$. This increases in γ because $p\bar{F}(p)$ is unimodal, hence increases to the left of p_0 .

Proof of Proposition 2: The proof follows the logic in the text, and is omitted for conciseness.

Proof of Remark 1: Lemma 3 characterizes the necessary and sufficient conditions for customers to advance purchase at a premium.

LEMMA 3. a) A necessary condition for a markdown pricing policy $p_1 \geq p_2$ to induce advance purchasing is $p_2 \leq p_2^C$, where p_2^C is the unique solution of $Q(p) \triangleq \mu - p + \gamma R(p) - \frac{C}{\bar{F}(p)}E[\mathbf{v} - p]^+ = 0$.

b) For any pricing policy (p_1, p_2) such that $p_2 < p_2^C$, and $p_2 < p_1 < w(\gamma; p_2, C)$ customers advance purchase at a premium.

Proof: For part (a), observe first that the solution must satisfy $\bar{F}(p) > C$, i.e. should induce some rationing. In this case $Q(p) = \mu + \gamma R(p) - (1 - C)p - C \frac{E[\max(\mathbf{v}, p)]}{\bar{F}(p)}$ is strictly decreasing in p , so p_2^C is unique. Further, $Q(p=0) > 0$, and $Q(p) < 0$ for large enough p .

b) For a markdown policy, $p_1 \geq p_2$, to induces advance purchasing, it must be that $w(\gamma; p_2, C) \geq p_2$, i.e. customers' maximum wtp in the advance period must exceed the spot price. Because w solves $\Delta S(\gamma; w, p_2, C) = 0$, the LHS of which is decreasing in p , we have $Q(p_2) \geq 0$. Because $Q(p)$ is decreasing in p (see proof of part a), we conclude that $p_2 \leq p_2^C$. \square

Proof of Remark 2: Follows from Lemma 1 because $\bar{\gamma}(p_1, p_2, k)$ is increasing in k , hence in C .

Proof of Remark 3: Figure 3 depicts the maximum wtp for a two-point valuation distribution which is non-monotone in spot price p_2 .

Proof of Proposition 3: We first show that the firm cannot extract more than $\bar{w}(\gamma)$ in advance by exploiting consumer regret and the threat of rationing risk. Given a spot price p_2 , consumers wtp in advance, $w(\gamma; p_2, C)$, solves $\Delta S = 0$ (eq. 3) and is unique, because ΔS is strictly decreasing in p_1 , non-negative at $p_1 = 0$ (because $\mu - kE[\mathbf{v} - p_2]^+ \geq 0$), and negative at a sufficiently high p_1 . Consider first the case $w = w(\gamma; p_2, C) \leq p_2$, so by (3), $w = p_1$ solves $\mu - p_1 + \gamma E[\mathbf{v} - p_1] = kE[\mathbf{v} - p_2]^+$. The LHS is strictly decreasing in p_1 , so w is maximized by setting $p_2 > v_{max}$, which yields $w = \bar{w}(\gamma)$. It

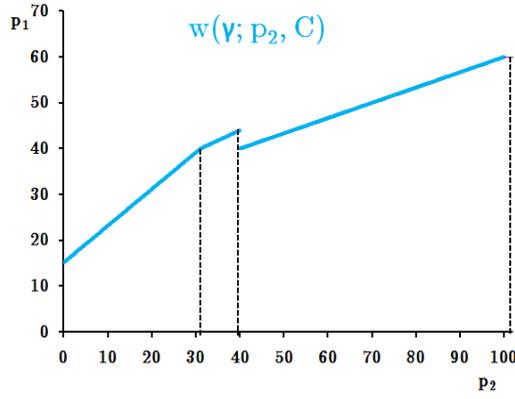


Figure 3 Maximum wtp $w(\gamma; p_2, C)$ is non-monotone in the spot price p_2 ; $\mathbf{v} \sim (100, \frac{3}{5}; 40, \frac{2}{5})$, $\gamma = 2$, $C = \frac{4}{5}$.

remains to show that the firm cannot extract more than $\bar{w}(\gamma)$ by advance selling at a premium, i.e. if $w \geq p_2$. In this case, w solves, by (3): $\mu - p_1 + \gamma E[\mathbf{v} - p_1]^- = kE[\mathbf{v} - p_2]^+ + \gamma kE[\min(\mathbf{v}, p_1) - p_2]^+$, the solution of which cannot exceed $\bar{w}(\gamma)$, because the RHS is always positive. We conclude that $\max_{p_2} w(\gamma; p_2, C) = \bar{w}(\gamma)$.

For any pricing policy, all customers either advance purchase, or wait. By selling exclusively in advance at $\bar{w}(\gamma)$, the firm obtains $\pi = C\bar{w}(\gamma)$. The optimal spot selling strategy solves $\max\{p\bar{F}(p); \bar{F}(p) \leq C\}$. Because $p\bar{F}(p)$ is unimodal, it follows that the profit maximizing spot price is $\max(p_0, \bar{F}^{-1}(C))$ and the optimal profit is $C\bar{\pi}_C$. The optimal policy follows by comparing optimal advance and spot selling profits; $\bar{\gamma}(C)$ is the regret threshold that makes the firm indifferent between the two, solving $\bar{w}(\gamma) = \bar{\pi}_C$, i.e. $\mu - \bar{\pi}_C + \gamma R(\bar{\pi}_C) = 0$, giving the desired result.

The optimal profit $\pi^*(C) = \max(C\bar{w}(\gamma), C\bar{\pi}_C)$ increases in C , because $C\bar{w}(\gamma)$ and $C\bar{\pi}_C$ both increase in C . The latter follows because for $C > \bar{F}(p_0)$, $C\bar{\pi}_C = \bar{\pi}$, is independent of C , and otherwise $C\bar{\pi}_C = C\bar{F}^{-1}(C)$ which is increasing in C , because $p\bar{F}(p)$ is increasing to the left of p_0 by the unimodality assumption. Finally, because $\bar{\pi}_C$ decreases in C and $R(x)$ is negative and decreasing in x (Lemma 2), it follows that $\bar{\gamma}(C) = \frac{\mu - \bar{\pi}_C}{R(\bar{\pi}_C)}$ increases in C . \square

Proof of Proposition 4: We first characterize the policy for a two-point distribution, then for the continuous distribution case.

LEMMA 4. *Suppose that $\mathbf{v} = (H, q; L, 1 - q)$. If $C \leq q$, or $C > q$ and $\gamma > \frac{L}{Hq-L} > 0$, then the firm only spot sells at $p_2 = H$. Otherwise, the optimal policy is to sell in advance at $p_1 = \bar{w}(\gamma)$ with a booking limit $B = \frac{C-q}{1-q}$, and on spot at $p_2 = H$.*

Proof of Lemma 4: For a given policy (p_1, p_2) , customers' wtp in advance, $w(\gamma; p_2, k(p_2, B))$ solves (3) with $k = k(p_2, B)$. For $p_2 = H$, (3) gives $w(\gamma; H, k(H, B)) = \bar{w}(\gamma) = L + \frac{q(H-L)}{1+\gamma(1-q)}$. For $p_2 = L$, (3) simplifies as $\mu - p - kE[\mathbf{v} - L]^+ + \gamma((1-k)E[\mathbf{v} - p]^- + k(L-p)^-) = 0$. Solving this for p gives $w(\gamma; L, k(L, B)) = L + \frac{q(1-k)(H-L)}{1+\gamma(1-q)(1-k)} \leq \bar{w}(\gamma)$. So, for a booking limit B , the maximum price to induce advance purchasing is $\bar{w}(\gamma) = L + \frac{q(H-L)}{1+\gamma(1-q)}$, corresponding to $p_2 = H$. In particular, premium advance selling (P.A.S.) is dominated by advance selling only (A.S.) at $\bar{w}(\gamma)$, which yields $\pi_{AS} = C\bar{w}(\gamma)$. By contrast, pure spot selling (S.S.) at $p_2 = H$ or $p_2 = L$ yields $\pi_{SS} = \max(qH, CL)$.

Obviously, if $C \leq q$, the firm sells only on spot at $p_2 = H$ which yields $\pi^* = CH$. For $C > q$, discount advance selling (D.A.S.) with booking limits can be optimal. In this case, $p_2 = H$ and $p_1 = w(\gamma; H, k(H, B)) = \bar{w}(\gamma)$. The firm profit is: $\pi_B = B\bar{w}(\gamma) + \min(C - B, (1 - B)q)H$. It is then easy to verify that, if $\gamma > \frac{L}{Hq-L} > 0$, then $B = 0$ and $\pi_B = qH$ (i.e. spot selling at $p_2 = H$); otherwise, $B = \frac{C-q}{1-q}$, and $\pi_B = \frac{C-q}{1-q}\bar{w}(\gamma) + \frac{q(1-C)}{1-q}H$. The result follows by comparing profits case by case. \square

We proceed to prove the result for log-concave distributions. Similar to two-point distributions, we show that P.A.S policies are suboptimal and therefore, the optimal policy is either spot sell or D.A.S (advance selling is a special case of D.A.S with $B = C$).

To implement a P.A.S. policy there must be a positive rationing probability on spot, i.e. $k(p_2, B) = \frac{C - B}{(1 - B)\bar{F}(p_2)} < 1$, as otherwise, customers facing a mark-down policy will wait. Therefore, from (3) we obtain:

$$\mu - p_1 - kE[\mathbf{v} - p_2]^+ - \frac{\gamma}{1 + \gamma}(1 - k)E[\mathbf{v} - p_1]^+ = 0. \tag{9}$$

The firm profit under a P.A.S policy is $\pi = p_1B + (C - B)p_2$, i.e. a P.A.S policy clears the capacity over two periods. We next show that π is increasing in B and therefore, at optimality, $B = C$ (implying that pure advance selling is more profitable). Indeed, for a given p_2 , because $p_1 > p_2$, implicit

differentiation on (9) gives: $\frac{\partial p_1}{\partial B} = \frac{1-C}{(1-B)^2 \bar{F}(p_2)} \frac{(1+\gamma)E[\mathbf{v}-p_2]^+ - \gamma E[\mathbf{v}-p_1]^+}{1+\gamma+\gamma(1-k)\bar{F}(p_1)} > 0$. Therefore, π increases in B and $B=C$ at optimality. The optimal advance price solves (substituting $k=0$ in (9)) $\mu - p_1 - \frac{\gamma}{1+\gamma}E[\mathbf{v}-p_1]^+ = 0$, or $p_1 = \bar{w}(\gamma)$. The optimal profit is $\pi = C\bar{w}(\gamma)$.

Next, we investigate D.A.S policies, i.e. $p_1 < p_2$. Under an optimal D.A.S policy the firm must clear the capacity over the two period, i.e. $k \leq 1$ or $C - B \leq (1-B)\bar{F}(p_2)$. To see this, assume the contrary. The firm profit function is then $\pi = p_1 B + (1-B)\bar{F}(p_2)p_2$. It follows that at optimality $p_1 = p_2\bar{F}(p_2)$, as otherwise, the firm can do better by adjusting the booking limit without violating the constraint. For $p_1 = p_2\bar{F}(p_2)$, the objective function becomes $\pi = p_2\bar{F}(p_2)$, which must be optimized subject to $\Delta S = 0$ or $\mu - p_1 + \gamma R(p_1) = E[\mathbf{v}-p_2]^+$. The solution to this problem, whatever it is, is inferior to the profits from a pure spot selling policy.

With $k \leq 1$, the profit function is $\pi = p_1 B + (C-B)p_2$ and the optimal D.A.S policy solves:

$$\max_{p_1, p_2, B} \pi = p_1 B + (C-B)p_2, \quad (10)$$

$$C - B \leq (1-B)\bar{F}(p_2), \quad (11)$$

$$\mu - p_1 + \gamma R(p_1) = \frac{C-B}{(1-B)\bar{F}(p_2)} E[\mathbf{v}-p_2]^+. \quad (12)$$

Because \mathbf{v} is log-concave and hence $\frac{E[\mathbf{v}-p_2]^+}{F(p_2)}$ is decreasing (Lemma 2, Bagnoli and Bergstrom 2005), it follows that $k=1$, i.e. $B = \frac{C-\bar{F}(p_2)}{F(p_2)}$; otherwise, the firm can increase p_2 , resulting in an increase in p_1 (the LHS of (12) is decreasing in p_1) and therefore improve π for the same B .

From (12), we have $p_1 = w(\gamma; p_2)$. Therefore, we can simplify the optimization problem (10) to an unconstrained optimization problem $\max_{p_2} \pi = (w(\gamma; p_2) - p_2) \frac{C-\bar{F}(p_2)}{F(p_2)} + C p_2$. To show that the optimal booking limit decreases in γ , it suffices to show that π is submodular in (γ, p_2) . This is indeed the case because $\frac{\partial B}{\partial p_2} = (1-C) \frac{f(p_2)}{[F(p_2)]^2} > 0$, $\frac{\partial p_1}{\partial p_2} = \frac{\bar{F}(p_2)}{1+\gamma F(p_1)} > 0$ and

$$\begin{aligned} \frac{\partial^2 \pi}{\partial \gamma \partial p_2} &= \frac{\partial}{\partial p_2} \frac{\partial p_1}{\partial \gamma} B + \frac{\partial p_1}{\partial \gamma} \frac{\partial B}{\partial p_2} \\ &= \frac{\partial p_1}{\partial p_2} \frac{-F(p_1)(1+\gamma F(p_1)) - \gamma R(p_1)f(p_1)}{(1+\gamma F(p_1))^2} B + (1-C) \frac{R(p_1)}{1+\gamma F(p_1)} \frac{f(p_2)}{[F(p_2)]^2} < 0. \end{aligned}$$

This always holds for $\gamma < 0$. On the other hand, because f is log-concave, so is $-R(p_1) = \int_0^{p_1} F(t) dt$ (Bagnoli and Bergstrom 2005), i.e. $\frac{\partial \log(-R(p_1))}{\partial p_1} = \frac{F(p_1)}{-R(p_1)}$ is decreasing, or $F^2(p_1) + f(p_1)R(p_1) > 0$.

So the first term is negative for $\gamma \geq 0$. Thus the optimal B, p_1, p_2 all decrease in γ . This implies the existence of the threshold $\bar{\gamma}_B(C)$, which exceeds $\bar{\gamma}(C)$. Indeed, if it is optimal to spot sell when booking limits are allowed, it is also optimal to do so when they are not allowed. \square

Proof of Proposition 5: We first show that $\bar{w}(\gamma; r)$ is increasing in r and decreasing in γ . These follow by differentiating the characteristic equation of $\bar{w}(\gamma; r)$, i.e.

$$\mu - p + \gamma R(p) = (1 + \gamma)R(r), \tag{13}$$

and using $R'(x) = -F(x)$, by Lemma 2(a), as follows:

$$\frac{\partial}{\partial r} \bar{w}(\gamma; r) = \frac{(1 + \gamma)F(r)}{1 + \gamma F(\bar{w}(\gamma; r))} \geq 0, \tag{14}$$

$$\frac{\partial}{\partial \gamma} \bar{w}(\gamma; r) = \frac{R(\bar{w}(\gamma; r)) - R(r)}{1 + \gamma F(\bar{w}(\gamma; r))} \leq 0, \tag{15}$$

where the last inequality holds because $\bar{w}(\gamma; r) \geq r$ and R is decreasing.

The optimal solution, if interior, satisfies the first order condition with respect to r on $\pi_s(\gamma; r) = \bar{w}(\gamma; r) - (r - s)F(r)$, which gives precisely (8). We next rule out boundary solutions, in particular we argue that full refunds are not optimal, i.e. $r^* < p^*$. Indeed, if $r^* = p^*$, substituting in (13) gives $\mu - r^* E[\mathbf{v} - r^*]^- = E[\mathbf{v} - r^*]^+ = 0$, implying $r^* = p^* = v_{\max}$. This policy yields zero profit, so cannot be optimal. Further, $r^* > 0$, because $\pi_s(\gamma; r = 0) = \bar{w}(\gamma) < \pi_s(\gamma; r = s) = \bar{w}(\gamma; s)$. The inequality holds because $\bar{w}(\gamma; s)$ solves (13) the LHS of which is decreasing in p .

To show that $r^*(\gamma)$ is increasing in γ , it suffices to show that $\pi(\gamma, r) = \bar{w}(\gamma; r) - (r - s)F(r)$ is supermodular in (γ, r) . This follows because from (14), by writing

$$\frac{\partial \pi^2}{\partial r \partial \gamma} = \frac{\partial}{\partial \gamma} \frac{\partial \bar{w}(\gamma; r)}{\partial r} = \frac{F(r)}{(1 + \gamma F(\bar{w}))^2} \left(1 - F(\bar{w}) - (1 + \gamma) \gamma f(\bar{w}) \frac{\partial \bar{w}(\gamma; r)}{\partial \gamma} \right) \geq 0 \tag{16}$$

for all $\gamma \geq 0$, because $p(\gamma; r)$ is decreasing in γ by (15).

Writing (8) as $(r - s) \frac{f(r)}{F(r)} = \frac{\gamma \bar{F}(\bar{w})}{1 + \gamma F(\bar{w})}$ shows that $r^*(\gamma = 0) = s$, and $r^*(\gamma) > s$ for $\gamma > 0$.

Finally, because $\bar{w}(\gamma; r)$ is decreasing in γ , so is $\pi^*(\gamma) = \max_r \bar{w}(\gamma; r) - (r - s)F(r)$, as the maximum of decreasing functions. \square

Full refunds vs. no refunds. For a full refund policy, the objective function is $\pi(r; s) = (r - s)\bar{F}(r)$ (see (7)) which is the profit from a pure spot selling policy. The optimal profit is $\pi^*(s) = (r^*(s) - s)\bar{F}(r^*(s))$. On the other hand, the optimal profit without a refund policy is $\bar{w}(\gamma) - s$ where $\bar{w}(\gamma)$ is as defined in (6). Full refunds dominate no refunds whenever $\pi^*(s) \geq \bar{w}(\gamma) - s$. In particular, this is true whenever $\pi^*(s) - s \geq \mu$, because $\bar{w}(\gamma) \leq \mu$. Note that the function $s + \pi^*(s)$ is increasing in s , because it's derivative is $F(r^*(s))$. So we can uniquely define \bar{s} such that $\bar{s} + \pi^*(\bar{s}) = \mu$, and $s + \pi^*(s) \geq \mu \geq \bar{w}(\gamma)$ for all $s \geq \bar{s}$. In this case full refunds dominate no refunds regardless of regret. Otherwise, for any $s \leq \bar{s}$, $\pi^*(s) + s = \bar{w}(\gamma)$ defines the threshold on γ above which full refunds dominate.

Unimodal objective. We show that the objective function $\pi_s(\gamma; r) = \bar{w}(\gamma; r) - (r - s)F(r)$ is unimodal for uniform valuation distributions. The result can be shown similarly for an exponential valuation distribution. Assume the valuation distribution is uniform $U[0, b]$. For a refund level r , the maximum price inducing advance purchases $\bar{w}(\gamma; r)$ solves (13), so $\bar{w}(\gamma; r) = (\sqrt{(1 + \gamma)(b^2 + \gamma r^2)} - b)/\gamma$. The objective function becomes: $\pi(r) = \frac{\sqrt{(1 + \gamma)(b^2 + \gamma r^2)} - b}{\gamma} - (r - s)\frac{r}{b} - s$, and its first and second derivatives are: $\pi'(r) = \sqrt{\frac{1 + \gamma}{b^2 + \gamma r^2}}r - \frac{2r - s}{b}$, $\pi''(r) = \sqrt{\frac{1 + \gamma}{b^2 + \gamma r^2}}\frac{b^2}{b^2 + \gamma r^2} - \frac{2}{b}$. The latter is decreasing in r , so π is convex-concave. At any r satisfying the FOC, $\pi''(r) = -\frac{b^2 s + 2\gamma r^3}{br(b^2 + \gamma r^2)} < 0$. Therefore the FOC gives the unique maximizer of π , which must be unimodal. \square

Proof of Proposition 6: The firm optimizes the option price x , exercise price r and the number of options to sell X , by solving:

$$\begin{aligned} \max_{X, x, r} \quad & X(x + r\bar{F}(r)) \\ \text{s.t.} \quad & X \leq 1 \\ & X\bar{F}(r) \leq C \\ & \mu - (x + r) + \gamma R(x + r) - (1 + \gamma)R(r) = 0. \end{aligned} \tag{17}$$

Define $x(\gamma; r)$ to solve the last equation, so $x(\gamma; r) = \bar{w}(\gamma; r) - r$ where $\bar{w}(\gamma; r)$ solves (13). From (14) and (15) it follows that $x(\gamma; r)$ is decreasing in r and γ .

Obviously, at optimality $X = \min(1, \frac{C}{\bar{F}(r)})$. If $r \leq r(C) = \min\{r \geq 0 : \bar{F}(r) \leq C\}$, the objective is $\pi_C(r) = C(\frac{x(\gamma; r)}{\bar{F}(r)} + r)$. This is increasing, so maximized at $r(C)$ (over $r \leq r(C)$), because

$$\pi'_C(r) = 1 + \frac{1}{\bar{F}(r)} \frac{\partial x}{\partial r} + \frac{x f(r)}{(\bar{F}(r))^2} = \frac{F(r)}{\bar{F}(r)} \frac{\gamma \bar{F}(x + r)}{1 + \gamma F(x + r)} + \frac{x f(r)}{(\bar{F}(r))^2} \geq 0.$$

So, at optimality, $X = 1$ and the problem reduces to $\max\{\pi(r) = x(\gamma; r) + r\bar{F}(r); r \geq r(C)\}$. The optimal profit decreases in γ because $x(r; \gamma)$ does.

For $C < 1$, $r(C) > 0$, so the optimal solution, $r^*(\gamma) > 0$. Otherwise, for $C = 1$, $r(C) = 0$ so the problem is unconstrained. Observe that $\pi(r)$ is decreasing in r for $\gamma \leq 0$, because $\pi'(r) = \frac{\gamma\bar{F}(x(\gamma; r) + r)F(r)}{1 + \gamma F(x(\gamma; r) + r)} - rf(r)$, so $r^*(\gamma) = 0$. So in absence of capacity constraints, the firm only advance sells to consumers who regret inactions more than actions. For sufficiently large $\gamma > 0$, such that $\gamma(1 - 2F(\bar{w}(\gamma))) > 1$ the second order condition at 0 ensures that $r = 0$ is suboptimal, so $r^*(\gamma) > 0$.

To show that $r^*(\gamma)$ increases in γ , it suffices to show that $\pi(r)$ is supermodular in (γ, r) . Indeed, using $\frac{\partial}{\partial r}x(\gamma; r) = \frac{(1 + \gamma)F(r)}{1 + \gamma F(x + r)} - 1 \leq 0$, and because $R(x)$ decreases in x we have:

$$\frac{\partial^2}{\partial \gamma \partial r}(x(\gamma; r) - r\bar{F}(r)) = F(r)\bar{F}(x + r) - \gamma(R(x + r) - R(r))f(x + r)(1 + \frac{\partial x}{\partial r}) > 0.$$

Finally, we argue that $x(\gamma; r^*) > 0$, i.e. spot selling is suboptimal. It suffices to show that $\bar{w}(\gamma; r) > r$ at r^* . Because the LHS in (13) is decreasing, this amounts to $\mu - r + \gamma R(r) > (1 + \gamma)R(r)$, or equivalently $\mu > E[\min(\mathbf{v}, r)]$. This holds whenever $r < v_{max}$. But $x^* = 0, r^* = v_{max}$ cannot be optimal, which concludes the proof. \square

Proofs of the Results in Section 5.3: Because $\mathbf{w} = \max(\mathbf{v}, \mathbf{s})$, it follows that $F_{\mathbf{w}} = F_{\mathbf{s}}F_{\mathbf{v}}$, so $\mathbf{w} \succeq_{SD_1} \mathbf{v}$. This implies that: (i) $\bar{F}_{\mathbf{v}}(p) \leq \bar{F}_{\mathbf{w}}(p)$, (ii) $\mu_{\mathbf{v}} \leq \mu_{\mathbf{w}}$, and (iii) $R_{\mathbf{v}}(p) \leq R_{\mathbf{w}}(p)$. Notation follows that in previous sections, indexed here by the corresponding valuation distribution, \mathbf{v} or \mathbf{w} . The results stated in the text can be summarized by the following result.

CLAIM 1. (a) $\bar{w}^{\mathbf{v}}(\gamma) \leq \bar{w}^{\mathbf{w}}(\gamma)$, (b) $\bar{\pi}_{\mathbf{v}} \leq \bar{\pi}_{\mathbf{w}}$, and (c) $\pi_{\mathbf{v}}^*(\gamma) \leq \pi_{\mathbf{w}}^*(\gamma)$.

Parts (a) and (b) are immediate from (i), respectively (iii) above. Finally, (c) follows because $\pi_{\mathbf{v}}^*(\gamma) = \max(\bar{w}^{\mathbf{v}}(\gamma), \bar{\pi}_{\mathbf{v}}) \leq \max(\bar{w}^{\mathbf{w}}(\gamma), \bar{\pi}_{\mathbf{w}}) = \pi_{\mathbf{w}}^*(\gamma)$.

Proof of Proposition 7: Define $C_0 = 1 - \alpha$. For $C \leq C_0$, sales can happen only in one period. The optimal policy in this case is (1) If $C \leq q$, spot selling at $p_2 = H$ and $\pi = CH$, (2) If $q < C \leq q\frac{H}{\mu}$, spot selling at $p_2 = H$ and $\pi = qH$, (3) If $C > q\frac{H}{\mu}$, advance selling at $p_1 = \mu$ and $\pi = C\mu$.

If $C > C_0$, the firm can implement four possible strategies:

Pure advance selling (A.S). The firm can offer the product at $p_1 = \bar{w}(\gamma) = \frac{\mu + \gamma(1-q)L}{1 + \gamma(1-q)}$ upon which all customers are willing to buy and firm profit is: $\pi_{AS} \triangleq C \frac{\mu + \gamma(1-q)L}{1 + \gamma(1-q)}$. Alternatively, the firm can charge $p_1 = \mu$ upon which only non-regretful customers buy and the firm profit is $\pi = (1 - \alpha)\mu$; however we will show that this is never optimal.

Pure spot selling (S.S). It is straightforward to verify that (1) If $C \leq q$, then $p_2 = H$ and $\pi = CH$, (2) If $q < C \leq q\frac{H}{L}$, then $p_2 = H$ and $\pi = qH$, (3) If $C > q\frac{H}{L}$, then $p_2 = L$ and $\pi = CL$.

Premium advance selling (P.A.S). This policy is implementable only if $C > C_0$, so that sales can happen in both periods. In this case, $p_2 = L$ and $p_1 = w(p_2 = L, \gamma = 0, k = \frac{C - (1-\alpha)}{\alpha})$, i.e. $\mu - p_1 = \frac{C - (1-\alpha)}{\alpha}(\mu - L)$, or $p_1 = L + (\mu - L)\frac{1-C}{\alpha}$. The firm profit is: $\pi_{PAS} \triangleq \frac{1-\alpha}{\alpha}(\mu - L)(1 - C) + CL$.

Discount advance selling (D.A.S). This policy is implementable only if $C > C_0$, so that sales can happen in both periods. In this case, the optimal prices are $p_2 = H$ and $p_1 = \mu$, and the firm profit is: $\pi = \mu(1 - \alpha) + H \min(C - (1 - \alpha), \alpha q)$.

Putting it together, the optimal policy (for $C > C_0$) follows by comparing the optimal profits from the above policies. First, if $C_0 < C \leq (1 - \alpha) + \alpha q = C_5$, D.A.S yields $\pi = \mu(1 - \alpha) + H(C - (1 - \alpha))$, which dominates A.S. and PAS. Therefore, the optimal policy is S.S. at $p_2 = H$ if $C < C_4 = q + (1 - \alpha)\frac{H - \mu}{H}$, otherwise D.A.S. Second, if $C > C_5 = (1 - \alpha) + \alpha q$, D.A.S yields $\pi_{DAS} \triangleq \mu(1 - \alpha) + qH\alpha = (\mu - \alpha(1 - q)L)$. This dominates S.S. and A.S. at μ , which yields $\pi = (1 - \alpha)\mu$. Therefore, we need to only compare A.S. at $\bar{w}(\gamma)$, D.A.S. and P.A.S. The following thresholds emerge from comparing the corresponding profits, π_{AS}, π_{DAS} and π_{PAS} :

- $C_1(\gamma) = \frac{(\mu - L)(1 - \alpha)}{\mu - L + \alpha(\bar{w}(\gamma) - \mu)} = 1 - \frac{\alpha}{1 + \gamma(1 - \alpha)(1 - q)}$ results from $\pi_{AS} = \pi_{PAS}$. In particular, C_1 is decreasing in α and increasing in γ ; the points where C_1 crosses the box boundaries in the figure are $C_1(\alpha = 1) = 0$ and $C_1(\alpha = 0) = 1$.
- $C_2(\gamma) = \frac{\mu - \alpha(1 - q)L}{\bar{w}(\gamma)} = (1 + \gamma(1 - q))\frac{\mu - \alpha(1 - q)L}{\mu + \gamma(1 - q)L}$, from $\pi_{DAS} = \pi_{AS}$. In particular, C_2 is increasing in γ and $C_2(\alpha = 1) = qH/\bar{w}(\gamma)$, and $C_2 = 1$ for $\bar{\alpha}_2 = \frac{\gamma}{1 + \gamma(1 - q)}(\frac{\mu}{L} - 1)$.
- $C_3 = 1 - \alpha + \frac{\alpha^2 q H}{L - (1 - \alpha)\mu} = 1 - \alpha \frac{L + \alpha(1 - q)L - \mu}{L - (1 - \alpha)\mu}$ from equating $\pi_{PAS} = \pi_{DAS}$; this equation has no solution for $\alpha = 1 - \frac{L}{\mu}$. This is not relevant for $\alpha < 1 - \frac{L}{\mu}$, because $C_3 \leq C_5$ (with equality at $\alpha = 0$

and $\alpha = -\frac{q}{1-q} < 0$). For $\alpha > 1 - \frac{L}{\mu}$, $C_3 > C_5$, in particular, $C_3 = 1$ for $\bar{\alpha}_3 = (\frac{\mu}{L} - 1)/(1 - q) > \bar{\alpha}_2$, and $C_3 = qH/L > C_2$ at $\alpha = 1$.

By definition, if $C_1(\gamma)$, $C_2(\gamma)$ and C_3 intersect, they do so at the same two points that satisfy $\pi_{AS} = \pi_{DAS} = \pi_{PAS}$; these points identify the area for premium advance selling. Because both C_1 and C_2 are increasing in γ , this area, if it exists, expands with γ . Premium advance selling is optimal for $\alpha \in [\alpha_1, \alpha_2]$, where $\alpha_{1,2}$ solve $C_1(\gamma) = C_2(\gamma)$, or equivalently:

$$L(1 - q)(\mu - \bar{w}(\gamma))\alpha^2 - ((2\mu - L)(\mu - \bar{w}(\gamma)) - (\mu - (1 - q)L)(\mu - L))\alpha + (\mu - \bar{w}(\gamma))(\mu - L) = 0.$$

For $\gamma = 0$ this implies $\alpha = 0$. Otherwise, using $\mu - \bar{w}(\gamma) = (\mu - L)/(1 + \frac{1}{\gamma(1-q)})$, we rewrite this as

$$L(1 - q)\alpha^2 - \left(\mu - qL - \frac{\mu - (1 - q)L}{\gamma(1 - q)} \right) \alpha + \mu - L = 0. \quad (18)$$

A necessary and sufficient condition for P.A.S. is that this equation admits solutions in $[0, 1]$. A necessary condition is that $\alpha_1\alpha_2 = \frac{\mu - L}{L(1 - q)} \in [0, 1]$, or $qH \leq L$. Further, $\alpha_1 + \alpha_2 \geq 0$, or equivalently, $\gamma \geq \frac{\mu - (1 - q)L}{(1 - q)(\mu - qL)}$. Because $(1 - \alpha_1)(1 - \alpha_2) = \frac{qH}{\gamma(1 - q)^2 L} > 0$, the above conditions guarantee that, if (18) has real roots, then both are in $[0, 1]$.

It remains to determine the conditions under which (18) has real roots. The discriminant can be written as: $\left(\mu - qL - \frac{\mu - (1 - q)L}{\gamma(1 - q)} \right)^2 - 4(\mu - L)(1 - q)L$. This is non-negative for $\gamma \geq \frac{\mu - (1 - q)L}{(1 - q)(\mu - qL)}$ if and only if $\mu - qL - \frac{\mu - (1 - q)L}{\gamma(1 - q)} \geq 2\sqrt{(\mu - L)(1 - q)L}$, i.e. whenever $\gamma \geq \frac{\mu - (1 - q)L}{(1 - q)(\mu - qL - 2\sqrt{(\mu - L)(1 - q)L})} = \frac{qH}{1 - q} \left(\frac{\sqrt{q(H - L)} + \sqrt{(1 - q)L}}{qH - L} \right)^2 \triangleq \bar{\gamma}(L, H, q)$. This identifies the bound on γ in the proposition, which together with $qH \leq L$, ensures that the PAS occurs for a range of α between $\alpha_{1,2}(\gamma)$ solving (18).

The optimal profit is decreasing in γ because it is the maximum of functions that are decreasing in γ ; similarly, it is increasing in C . We have argued in the text why it is non-monotone in α when $C < 1$; indeed, the optimal profits $\pi = CH - (1 - \alpha)(H - \mu)$ increase in α for $C_4 < C < C_5$ (discount advance selling). \square

We conclude by proving the statements following Proposition 7, concerning the uncapacitated case $C = 1$ and the effect of booking limits.

Uncapacitated case. For $C = 1$, the optimal policy is either D.A.S. ($p_1 = \mu$ and $p_2 = H$) or exclusively advance sell at $\bar{w}(\gamma)$ and the optimal profit is $\max(\bar{w}(\gamma), (1 - \alpha)\mu + \alpha qH)$. Therefore, profits decrease in γ and the share of the regretful customers α . In Figure 2, $C_2(\gamma)$ intersects with $C = 1$ at $\alpha = \frac{\mu - \bar{w}(\gamma)}{\mu - qH}$ which is decreasing in γ . Therefore, above a threshold, the firm segments the market based on customers' regret, selling to non-regretful customers in advance and to regretful customers on spot. \square

Booking Limits. We finally discuss the effect of booking limits on the optimal policy in heterogeneous markets. It is easy to observe that booking limits only affect optimal D.A.S. policies. Sales can happen in two periods under a D.A.S. policy at ($p_1 = \mu, p_2 = H$) and, unlike the case without booking limits, at ($p_1 = \bar{w}(\gamma), p_2 = H$). The firm profit is $\pi = \min(B, 1 - \alpha)\mu + \min(C - B, (1 - \min(B, 1 - \alpha))q)H$, respectively, $\pi = B\bar{w}(\gamma) + \min(C - B, (1 - B)q)H$. Under pricing policy ($p_1 = \bar{w}(\gamma), p_2 = H$), the optimal booking limit is $B = \frac{C-q}{1-q}$ and the optimal profit is $\pi_{DAS}^{(2)} = \frac{C-q}{1-q}\bar{w}(\gamma) + \frac{q(1-C)}{1-q}H$. To derive the optimal D.A.S. policy under pricing policy ($p_1 = \mu, p_2 = H$), notice that for $p_2 = H$, customers' maximum wtp in advance is independent of B , and therefore it must be that $\min(B, 1 - \alpha) = B$ as otherwise the firm can always do better by lowering B to make more capacity available on spot without any effect on advance sales. Therefore, the profit function simplifies as $\pi = B\mu + \min(C - B, (1 - B)q)H$. Upon optimizing subject to $B \leq (1 - \alpha)$, we find that (1) If $C \leq C_5$, then $B = \frac{C-q}{1-q}$ and $\pi_{DAS}^{(3)} = \frac{C-q}{1-q}\mu + \frac{q(1-C)}{1-q}H$, (2) If $C > C_5$, then $B = (1 - \alpha)$ and $\pi_{DAS} = (1 - \alpha)\mu + \alpha qH$.

Define $C_3(\gamma)$ to solve $\pi_{DAS} = \pi_{DAS}^{(2)}$. The optimal D.A.S. policy then is (1) If $C \leq C_5$, then ($p_1 = \mu, p_2 = H$), $B = \frac{C-q}{1-q}$ and the optimal profit is $\pi_{DAS}^{(3)}$, (2) If $C_5 < C \leq C_3(\gamma)$, then ($p_1 = \mu, p_2 = H$), $B = (1 - \alpha)$ and the optimal profit is π_{DAS} , (3) If $C > C_3(\gamma)$, then ($p_1 = \bar{w}(\gamma), p_2 = H$), $B = \frac{C-q}{1-q}$ and the optimal profit is $\pi_{DAS}^{(2)}$.

The optimal policy then follows by comparing the profits obtainable under optimal D.A.S., P.A.S., S.S. and A.S. We omit details for conciseness and conclude with the following insights:

1. Given $C < 1$, an exclusive A.S. policy is never optimal. Such a policy generates $\pi = \max(C\bar{w}(\gamma), (1 - \alpha)\mu)$. D.A.S. policy ($p_1 = \bar{w}(\gamma), p_2 = \mu$) and $B = \frac{C-q}{1-q} < C$ generates $\pi_{DAS}^{(2)} > C\bar{w}(\gamma)$.

On the other hand, if the optimal A.S. policy does not clear the capacity; i.e. $\pi = (1 - \alpha)\mu$, a D.A.S. policy $(p_1 = \mu, p_2 = H)$ generates $\pi = (1 - \alpha)\mu + \min(C - (1 - \alpha), qH\alpha)$ which better utilizes the capacity and yields higher profits.

2. The firm optimal profit decreases in α , the fraction of regretful customers. This is because the firm profit depends on regret only when the firm cannot clear the capacity through a D.A.S. policy, i.e. $(p_1 = \mu, p_2 = H)$ with profits π_{DAS} , or implements a P.A.S. policy with profits π_{PAS} . In both cases, profits decrease in α .

3. The optimal booking limit decreases in γ and is non-monotone in α . The former is because the boundaries, $C_i(\gamma)$ are increasing in γ and else constant. The latter follows because the optimal booking limit for a capacity level $C > q$ is $\frac{C-q}{1-q}$ (and independent of α) if α is sufficiently large or small, and $(1 - \alpha)$ for intermediate α , which is decreasing in α .

Proof of Proposition 8: Obviously, the optimal spot price is $p_2 \in \{L, H\}$. For each case, we determine the optimal advance price, and obtainable profits. The proposition then follows by comparing the profits in the cases.

Case 1: $p_2 = L$. The maximum wtp for type A, respectively type B, customers is $w^A(\gamma; p_2) = \frac{qL}{1+\gamma(1-q)}$, and $w^B(\gamma = 0; p_2) = qL$ (see (6)). Because $w^A(\gamma; p_2) \leq w^B(\gamma = 0; p_2)$, it follows that the optimal advance price is either $p_1 = qL$ or $p_1 > H$ (i.e. not advance selling). The corresponding profit is $\pi = qL$.

Case 2: $p_2 = H$. The maximum wtp for type A, respectively type B, customers is $w^A(\gamma; p_2) = \frac{qH}{1+\gamma(1-q)}$, and $w^B(\gamma = 0; p_2) = qL$. Thus, the optimal advance price is $p_1 \in \{qL, \frac{qH}{1+\gamma(1-q)}, p_1 > H\}$. If $w^A(\gamma; p_2) \geq w^B(\gamma = 0, p_2)$, i.e. $\gamma \leq \frac{H-L}{L(1-q)}$, then $\pi = \max\{qL, \alpha \frac{qH}{1+\gamma(1-q)}, \alpha qH\} = \max\{qL, \alpha qH\}$. In this case, if $\alpha > \frac{L}{H}$, then the optimal profit is $\pi = \alpha qH$, and otherwise $\pi = qL$. Otherwise, $\pi = \max\{(1 - \alpha)qL + \alpha qH, \frac{qH}{1+\gamma(1-q)}, \alpha qH\} = (1 - \alpha)qL + \alpha qH$.

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