Research Note: Additional Learning and Implications on the Role of Informative Advertising

by

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Abstract

Observers argue that evidence for the persuasive role of advertising comes from competitive categories where increases in advertising lead to higher average prices. Conversely, others claim that advertising serves a purely informational role. Here, higher levels of advertising lead to better-informed consumers, increased competition and hence, lower prices. The objective of this study is to neither confirm nor refute either of these perspectives. It is rather to show that increases in informative advertising alone can lead to higher or lower prices. I further show that the direction of this relationship depends on the level of differentiation between competing firms. Similar to Grossman and Shapiro (1984), I examine conditions where the differences between competing products are small. But I also examine conditions where the differences are significant. Advertising’s role is to inform consumers about individual products and higher advertising for a product means that more of the potential market knows about it. Higher advertising increases the relative importance of fully informed consumers compared to partially informed consumers. This dynamic is the basis for explaining why informative advertising can either push prices up or down in a uniformly distributed spatial market.

Key Words: advertising/price competition, persuasive advertising, spatial competition, differentiation.
1 Introduction

A primary role of advertising is to generate awareness for products and to inform consumers of how competing products are different. In many markets, differences between products mean that some products are better for some consumers than for others. This is often called horizontal differentiation. The relationship between horizontal differentiation and pricing has been examined in detail but less attention has been devoted to understanding how differentiation interacts with both advertising and pricing. One exception is the model of Grossman and Shapiro (1984) where competing firms make decisions about advertising and pricing in a market of horizontally differentiated firms. However, Grossman and Shapiro (G&S) restrict their analysis to conditions where the differences between products are small.

The objective of this note is to further our understanding of how horizontal differentiation interacts with advertising and pricing by extending the analysis of G&S to situations where the differences between products are stronger. The major insight is that when the level of differentiation between products is high, increased advertising leads to higher prices. This is the opposite of the relationship observed when the level of differentiation between products is low (as in G&S). These findings are explained by considering how informative advertising works and how higher levels of advertising affect the marketing landscape. In its simplest form, informative advertising creates awareness of products (and their attributes) so that a consumer can identify the product (if any) which best meets her needs. Higher advertising for an individual product means that a greater fraction of the potential market knows about it. But higher advertising also increases the relative importance in the market of consumers with complete information compared to consumers with partial information. This dynamic provides a basis to explain the relationship that is observed between advertising and prices.

Before presenting the analysis, I provide a brief review of the literature that examines the relationship between advertising and prices.

2 Literature Review

In many models, advertising is an instrument which increases either the intensity of demand (at all price levels) or the amount consumers are willing to pay for a product. Given this representation of advertising, many models predict a positive relationship between advertising and prices. First, there are models based on the idea that consumers lack information on
product quality (Nelson 1974). These models predict that high quality products have higher levels of advertising and higher prices. The idea is that the quantity of advertising (not its content) signals quality to consumers (Nelson 1974, Schmalensee 1978, Klein and Leffler 1981, Milgrom and Roberts 1986, Bagwell and Ramey 1994). A second set of models based on advertising increasing the benefits associated with the consumption of products also predicts a positive correlation between advertising and prices (Ehrlich and Fisher 1982 and Becker and Murphy 1993). In contrast to the first set of models, the content of advertising plays a role in creating the positive correlation. A third set of models based on psychological theory also supports the existence of a positive relationship between advertising and prices. Experiments have shown that advertising can lead to higher brand evaluations because of familiarity and “mere exposure” (Anand, Holbrook and Stephens 1988, Heath 1990). Another argument to explain why consumers pay more for advertised products is enhancing product value by creating pleasurable associations with consuming the product (Cafferata and Tybout 1989).

The idea that consumers pay more for products because of “familiarity” or “pleasurable associations” due to advertising is perhaps the basis for the “adverse view” of advertising. It contends that advertising persuades consumers into perceiving significant differences between products that are physically similar (Bain 1956, Galbraith 1967, Solow 1967). Comanor and Wilson (1974) argue that advertising leads consumers to pay premiums for products that are physically identical.

This view of advertising is countered by the “partial view” which asserts that advertising provides factual information to consumers about product attributes including price (Telser 1964). The “partial view” argues that advertising reduces product differentiation (created by a lack of information) and leads to lower prices. Studies in several industries show that prices are significantly higher in regions where advertising is prohibited (Benham 1972, Cady 1976 and Steiner 1973, Milyo and Waldfogel 1999). A number of analytical models based on advertising providing uninformed consumers with price information also predict an inverse relationship between advertising levels and prices (Robert and Stahl 1993, Bester and Petrakis 1995 and Grossman and Shapiro 1984).

The above controversy has not been resolved yet a negative correlation between advertising and pricing is common in models where price is part of the advertising message. Con-

\footnote{The same effect is generated in Lynch and Ariely (2000) with products that are different but without advertising, are perceived to be identical.}
versely, models of advertising that predict a positive correlation between advertising levels and pricing, generally involve messages that enhance the value of the product for consumers (by signalling higher quality for example).

My objective is not to disprove either of these views. In fact, a significant number of researchers propose dichotomous models of advertising in which both mechanisms of advertising are represented (enhancing the willingness to pay and providing better information). My objective is to show that informative advertising alone can lead to a positive relationship between advertising and price even when the ads contain pricing information. This was first demonstrated by Meurer and Stahl (1994) using discrete or non-uniform distributions of consumers. The contribution of this paper is to show that a positive relationship can be obtained using a standard model of spatial differentiation where consumers are distributed uniformly. In addition, the model identifies the precise conditions where the positive relationship between advertising and prices disappears and a negative correlation, typical of models of informative advertising, is recovered.

The analysis highlights the danger of backward induction when positive correlations between advertising and price are observed. One cannot assume that advertising enhances a consumer’s willingness to pay for a product simply because a positive relationship between advertising levels and prices is observed.

Similar to Grossman and Shapiro (1984), the starting point for my model is the idea that awareness (created by advertising) is a key determinant of demand for consumer products. Empirical work by Kwoka (1993) shows that the impact of advertising tends to be short-lived. This underlines the importance of on-going advertising to create awareness. There is also behavioral literature which demonstrates the role of consideration sets in a consumer’s decision-making process. A key message is that awareness is critical for a product to be included in a consumer’s consideration set (Nedungadi 1990, Mitra and Lynch 1995). Moreover, Dickson and Sawyer (1990) find that the average consumer spends less than 30 seconds making most grocery shopping choices. In these situations, the awareness of brands and their key attributes is an excellent predictor of brand choice. In Grossman and Shapiro (1984), advertising makes consumers aware of products and their characteristics (including price) in a horizontally differentiated market. However, their analysis is restricted to situations

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2 The model of Boyer (1974) incorporates informative advertising (which generates awareness) and goodwill advertising which leads to increases in the “valuation” of products. Similar approaches are employed in Kotowitz and Mathewson (1979), Farris and Albion (1980) and Krisnamurhti and Raj (1985).
where differentiation is low. My contribution is to better understand the relationship between advertising levels and pricing by extending the analysis of G&S to conditions where differentiation is high.

3 The Model

The model consists of two firms that compete in a circular market with a circumference of 2. Following Salop (1979), consumers are uniformly distributed around the market. The distribution of consumers implies that consumers have a range of tastes along a single attribute: each consumer is identified by an ideal point on the attribute that corresponds to her preferred brand. To simplify the analysis, I assume that the total mass of consumers is 1.

The two firms are maximally differentiated (Firm 1 is located at the top of the market and Firm 2 is located at the bottom of the market). Each firm produces a single product at a constant marginal cost of production, c. The game consists of two stages. In the first stage, firms choose advertising levels ($\phi_i$) and prices ($p_i$) simultaneously. The advertising level is the fraction of the market that is exposed to Firm i’s advertising. In the second stage, consumers who have been exposed to advertising make purchase decisions.

An informed consumer buys at most one unit of product and places a value $v$ on her ideal product. In general, a consumer cannot obtain her ideal product. A consumer located a distance $x$ from Firm i ($i = 1, 2$) obtains a surplus $v - tx - p_i$ by consuming Firm i’s product: $t$ is the “preference” cost per unit distance and $p_i$ is the price charged by Firm i. The parameter $t$ measures the sensitivity of consumers to the attribute and thus represents the degree of differentiation in the market. Advertising is assumed to be informative and thus, has no effect on $v$ (the consumer’s willingness to pay) or $t$ (the preference cost in the market).

The only way a consumer becomes informed about a firm’s product is through its advertising. In other words, consumers do not search for or experiment with products for which they have not seen advertising. If a consumer knows about more than one product offering positive surplus, she will buy the product offering the greatest surplus. These assumptions do not preclude consumers knowing the structure of the market a priori i.e. that products are located at either end of the linear market. But information in the advertising message is needed to identify the firm that sells the product of a given specification.
The cost of advertising is assumed to be $\alpha \phi^2_i$ for both firms. The convexity of the cost function reflects the fact that some consumers are harder to reach than others i.e., when a firm wishes to increase advertising reach, more of the consumers who are difficult to reach need to see the advertising so the marginal cost of advertising increases. The $\alpha$ parameter represents the relative cost of media in the market. I restrict $\alpha$ to values such that the advertising levels for both firms are less than 1.

An implication of this representation of advertising is that it creates a second dimension of consumer heterogeneity based on the information consumers have about products (the first is location around the circle). In fact, when the firms have conducted advertising at levels $\phi_1$ and $\phi_2$ respectively, there are 4 distinct groups of consumers uniformly distributed around the circular market. First, there are consumers who have seen advertising from both firms (a fraction $\phi_1 \phi_2$ of the market). Second, there are consumers who have not seen any advertising (a fraction $(1 - \phi_1)(1 - \phi_2)$ of the market). Finally, there are two groups of consumers who have seen advertising from only one of the two competing firms (given by $\phi_1(1 - \phi_2)$ and $\phi_2(1 - \phi_1)$ respectively).

The profit of each firm is a function of a) expenditures on advertising, b) the price that it charges and, c) the demand that it realizes from each of the two segments that it serves (consumers who are only informed about the focal firm $x_i$ and consumers who informed about the products of both firms $y_i$).

\begin{align}
\pi_1 &= (p_1 - c)(\phi_1(1 - \phi_2)x_1 + \phi_1\phi_2y_1) - \alpha_1 \phi_1^2 \\
\pi_2 &= (p_2 - c)(\phi_2(1 - \phi_1)x_2 + \phi_1\phi_2y_2) - \alpha_2 \phi_2^2
\end{align}

In the group of consumers who have only seen Firm $i$’s advertising (and not the competitor’s), demand is determined by individual rationality i.e. all consumers in the segment who obtain positive surplus from Firm $i$’s product will buy from Firm $i$. This implies that demand from this segment is $x_i = \frac{v - p_i}{t}$ except if $v - p_i > t$ in which case $x_i = 1$. The derivation of demand from consumers who have seen advertising from both firms depends on the location of the indifferent consumer given prices $p_1$ and $p_2$. It is straightforward to show that $y_1 = \frac{p_2 - p_1 + t}{2t}$ and $y_2 = \frac{p_1 - p_2 + t}{2t}$.

Because firms make simultaneous decisions to choose advertising levels and prices, the game is one of complete but imperfect information. My objective is to compare the symmetric
Nash equilibria in two situations. First, I consider situations where the level of differentiation in the market \((t)\) is low relative to the available surplus \((v)\), i.e. \(t < \frac{v-c}{2}\). This is analogous to the conditions examined by G&S where every product has the potential to provide a consumer in the market with positive surplus. I then consider the outcome when the level of differentiation in the market is stronger, i.e. where \(t > \frac{v-c}{2}\). These situations are mutually exclusive and exhibit different relationships between advertising and prices. Derivations of these limits are available in a technical appendix available from the author.

4 The Relationship between Advertising and Prices when Differentiation is Low: \(t < \frac{v-c}{2}\)

The optimal price for firms is compromise between the prices that would be optimal for each of the segments created by advertising (partially and fully informed consumers). To understand how advertising levels affect prices, I assume that \(v\) and \(t\) are fixed and examine different advertising levels by changing \(\alpha\) (a reduction in \(\alpha\) means that advertising is less expensive so firms will advertise more). When advertising levels are low (\(\alpha\) is high), the equilibrium involves pure price and advertising strategies. However, the optimal price is a corner solution corresponding to the reservation price for the most distant partially informed consumer i.e., \(v - t\). In this situation, the potential increase in demand from fully informed consumers is not sufficient to create an incentive for firms to reduce price from \(v - t\). At higher levels of advertising, the first order conditions are satisfied at a price less than \(v - t\). Unless two conditions are satisfied (in particular a very low transportation cost \(t\) and a low advertising cost parameter \(\alpha\)), this leads to an internal equilibrium in pure price and advertising strategies where \(p = \frac{2t}{\phi} - t + c\) and \(\phi = \frac{2t}{2\sqrt{at+t}}\). These equilibrium values lead to Proposition 1. The derivations of all results are available in the technical appendix. Even when the equilibrium entails mixed pricing strategies, the equilibrium price is decreasing in the advertising levels.

**Proposition 1** When \(\alpha \in \left(\frac{1}{4}t \cdot \frac{v+c+t}{2t+c}, \frac{1}{4}(v-c-t)^2\right)\), the average prices charged by firms are

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3For all conditions examined in the paper, it is easy to show that the second order conditions hold in the neighborhood of the symmetric equilibria. When \(t > \frac{v-c}{2}\), it is also possible to show that the symmetric equilibrium is unique because the best response functions of both firms are contractions (Friedman 1986) for \(p \in [0, v-t]\) and \(\alpha\) large enough such that \(\phi \in (0, 1)\). When \(t < \frac{v-c}{2}\), it is not possible to prove uniqueness due to discontinuities in the profit functions.

4When the differentiation between firms is low and \(\alpha < \frac{1}{8} \cdot \frac{3t^2-4st-t}{\sqrt{5t^2-6st}+s} \cdot \frac{3t^2-4st-t}{\sqrt{5t^2-6st}+s} + s \cdot \sqrt{5t^2-6st} + s\) (\(s\) is the surplus \(v - c\) created by the consumption of an ideal product), the equilibrium involves mixed pricing strategies (there is an incentive for firms to defect from the pure price strategy to the reservation price for partially informed consumers). A complete solution is found in the technical appendix.
strictly decreasing in the level of advertising.

Proposition 1 demonstrates that a negative relationship between advertising levels and price is observed when differentiation is low. This relationship is independent of whether the equilibrium pricing strategy of the firms is pure or mixed. The negative relationship between advertising levels and prices is analogous to the results from the oligopolistic models of Grossman and Shapiro (1984) and Tirole (1988). As noted earlier, G&S restrict their analysis to conditions where the differentiation between firms is low.

The negative relationship is a function of how advertising affects demand for each firm’s product. Increases in advertising have a first order effect on the potential demand for each firm. The more consumers who have seen a firm’s advertising, the higher the potential demand for that firm. But advertising affects the composition as well as the extent of demand. When advertising levels are low, the vast majority of demand comes from partially informed consumers (consumers who have seen one firm’s advertising are unlikely to have seen advertising from the other). Each firm is a de facto monopolist for the majority of consumers informed about its product. Thus, the optimal pricing strategy is close to the pricing strategy that would be chosen by a monopolist located in a circular market. As advertising levels increase however, a greater fraction of demand comes from consumers who are fully informed. At high advertising levels, the majority of consumers have seen advertising from both firms. In contrast to partially informed consumers, fully informed consumers compare the attractiveness of each product (which is a function of each product’s location and price) and make decisions. The optimal price for this group of consumers is different than the optimal price for consumers who are partially informed. A key effect of higher advertising is to make fully informed consumers relatively more important than partially informed consumers in each firm’s demand function. This implies that as advertising levels increase, the average price in the market shifts from close to the theoretical monopoly price towards the competitive price.

When differentiation is low, prices are less than the reservation price for the most distant partially informed consumer. As a result, demand from partially informed consumers cannot be increased by reducing price (demand from partially informed consumers is completely inelastic). In contrast, demand from fully informed consumers depends on price because price affects the relative attractiveness of the products. This means that demand from fully informed consumers is price elastic. As noted earlier, higher advertising increases the importance of fully informed consumers relative to partially informed consumers (for both firms).
other words, higher advertising shifts demand from an inelastic segment to an elastic segment. Consequently, demand becomes more elastic as advertising levels increase and this leads to reductions in the average price. This reasoning applies independent of whether the pricing equilibrium is in pure or in mixed strategies. We now turn to the case of high differentiation.

5 The Relationship between Advertising and Prices when Differentiation is High: \( t \in \left( \frac{v-c}{2}, \frac{2(v-c)}{3} \right) \)

In this section, I analyze conditions where the level of differentiation is higher (i.e. \( t > \frac{v-c}{2} \)).5 Because my interest is the relationship of advertising to prices in competitive conditions, I restrict attention to conditions where under full information, all consumers in the market would buy (i.e. \( t < \frac{2(v-c)}{3} \)). This condition is based on the requirement that a consumer located half way between the firms would buy when \( p = t + c \) (the full information competitive price).

To obtain the objective functions for each firm, I substitute the appropriate demand expressions into equations 1 and 2. In these conditions, \( x_i = \frac{v-p_i}{t} \) because \( v - p_i < t \). The first order conditions for price and symmetry lead to Proposition 2.

**Proposition 2** When differentiation is high, the equilibrium price is

\[
\frac{2\phi v - t\phi + \phi c - 2v - 2c}{3\phi - 4}.
\]

This implies that the equilibrium price is strictly increasing in the level of advertising.

When differentiation is high, the price equilibrium is always in pure strategies because the optimal prices for fully informed and partially informed consumers are relatively close. Proposition 2 also implies that the relationship between advertising levels and prices is positive, the exact opposite of the relationship observed when differentiation is low.

To explain this finding, we consider how increases in advertising affect the composition of demand. Similar to the case of low differentiation, higher advertising makes fully informed consumers relatively more important in each firm’s demand function. However, when differentiation is high, demand from both groups of consumers is sensitive to changes in price. As before, price affects the relative attractiveness of the products so demand from fully informed consumers responds to changes in price.

\[5\]These conditions pertain to categories where a consumer would buy if she were informed about the “right” product. A consumer may know of brands in the UHT milk category but until she learns about a UHT brand that tastes like regular milk, she may be unwilling to buy. Similarly, a consumer who primarily owns furniture that is stained (versus being varnished) may not buy furniture polish until she learns about a brand of polish that is suitable for stained furniture.
The big difference when differentiation is high is that demand from partially informed consumers also responds to changes in price (this is not the case when differentiation is low). Consumers incur a significant taste cost to consume a product that is not suited to their tastes when differentiation is high. As a result, some partially informed consumers find not purchasing at all the better option (the taste costs and price exceed the benefit \( v \) that consumers obtain by consuming their ideal product). In fact, both firms have partially informed consumers located close to the competitor who find the product too expensive. When a firm lowers its price, some of these consumers change their minds and buy the product. This leads to demand from partially informed consumers being price sensitive.

The preceding paragraph explains why demand from partially informed consumer is price sensitive when differentiation is high. However, the relationship between advertising levels and prices is not explained by the simple observation that demand from partially informed consumers is price sensitive. The relationship is driven by the fact that demand from partially informed consumers has become more sensitive to changes in price that demand from fully informed consumers. How is this possible? The answer is that to attract a fully informed consumer who is now buying from the competition, a firm must do more than compensate that consumer for increased taste costs. The firm must also compensate for the fact that as it increases demand from fully informed consumers who are further away, those same consumers are proportionally closer to the competitor. This makes the required price reduction to attract those consumers high. In contrast, to increase demand from partially informed consumers, the firm need only compensate more-distant consumers for increased taste costs. The firm does not need to worry that more-distant partially informed consumers are closer to the competitor. Partially informed consumers (by definition) are not informed about the competitor and do not consider it.

Mathematically, these observations can be demonstrated by examining the derivatives of demand with respect to price for each segment. For partially informed consumers, \( \frac{\partial x_i}{\partial p_i} \) equals \( \frac{1}{2} \). This is twice the sensitivity of demand from fully informed consumers \( \frac{\partial y_i}{\partial p_i} \), which equals \( \frac{1}{2T} \). Since demand from partially informed consumers is more sensitive to price than demand from fully informed consumers, the optimal price for partially informed consumers is lower than the optimal price for fully informed consumers. Because higher advertising increases...
the relative importance of fully informed consumers (as a fraction of total demand), higher advertising leads to higher prices.

Regardless of whether differentiation is high or low, equilibrium pricing is a compromise between the prices that are optimal for each of the segments created by advertising (partially informed consumers and fully informed consumers). However, when differentiation is high, advertising shifts demand from a segment with a low optimal price (partially informed consumers) to a segment with a high optimal price (fully informed consumers).

6 Conclusion

The primary message of this analysis is that the relationship between the level of informative advertising and prices can be either negative or positive. Most models that exhibit both positive and negative relationships between advertising and prices assume that advertising has a dichotomous character (advertising informs but also increases a consumer’s willingness to pay for a product). In this model, advertising is purely informative and has no effect on a consumer’s willingness to pay.

As noted earlier there are models of informative advertising that generate both negative and positive relationships between advertising levels and pricing. However, these models are based on markets with discrete segments or a non-uniform distribution of consumers. The primary contribution of the study is to show that positive and negative relationships can be found in a uniformly distributed market. The model builds on the analysis of G&S and demonstrates that the valence of the relationship depends on the level of differentiation between products.

Both relationships are found because of how informative advertising affects the composition of firm demand. Independent of whether the level of differentiation between the firms is high or low, the main effect of higher advertising is to increase the importance of fully informed consumers (who have seen advertising from both alternatives) relative to the importance of partially informed consumers (who have seen advertising from only one firm).

When differentiation is low, demand from partially informed consumers is completely inelastic. Conversely, demand from fully informed consumers depends on price. As advertising increases, demand is shifted from an completely inelastic segment to an elastic segment so prices fall.

In contrast, when differentiation is high, demand from both groups of consumers depends
on price. Here, demand from partially informed consumers is affected by prices because some partially informed consumers find the product they know about too expensive. In fact, demand from partially informed consumers is more responsive to changes in price than demand from fully informed consumers. This means that the optimal price for partially informed consumers is lower than the optimal price for fully informed consumers. Higher advertising shifts demand from a segment with a lower optimal price to a segment with a higher optimal price so a positive relationship between advertising levels and prices is observed.
References


Technical Appendix

Derivation of the Limits for the Region of Low Differentiation

In order for demand from partially informed consumers to be inelastic, the condition \(v - p - t > 0\) must be satisfied. For this to be true, the maximum price observed in conditions of low differentiation must be less than \(v - t\). If \(v - t\) is the maximum price then \(\frac{\partial \pi_1}{\partial p_1}\) must be strictly negative for any price higher than \(v - t\). Substituting the appropriate demand expressions into equation 1 in the text when \(p_1 > v - t\) and differentiating, I obtain \(\frac{\partial \pi_1}{\partial p_1} = \frac{1}{2} \phi_1 \frac{2v - 4p_1 - 2\phi_2 v + 2\phi_2 p_1}{t} + \phi_2 p_2 + \phi_2 p_1 + \phi_2 + 2c - \phi_2 c\). Note that because \(\frac{\partial^2 \pi_1}{\partial p_1^2} = \frac{1}{2} \phi_1 \frac{-4 + 2\phi_2}{t} < 0\), the maximum value of \(\frac{\partial \pi_1}{\partial p_1}\) in the range \((v - t, \infty)\) is at \(p = v - t\). When both firms choose, \(p = v - t\), then \(\frac{\partial \pi_1}{\partial p_1} = \frac{1}{2} \phi_1 \frac{2v - 4t - \phi_2 v + 2t \phi_2 - 2c + \phi_2 c}{t}\). In order for this to be negative, \((2 - \phi) (v - 2t - c) > 0\). This is true if and only if \(v - 2t - c > 0\) or \(\frac{v - c}{2} > t\). As a result, when \(\frac{v - c}{2} > t\), prices greater than \(v - t\) are not observed and the condition \(v - p - t \geq 0\) is always satisfied.

Proof of Proposition 1

When \(t < \frac{v - c}{2}\), the objective functions when prices are marginally less than \(v - t\) are:

\[
\pi_1 = (p_1 - c) \left( \phi_1 (1 - \phi_2) + \phi_1 \phi_2 \frac{p_2 - p_1 + t}{2t} \right) - \alpha \phi_1^2
\]

(1)

\[
\pi_2 = (p_2 - c) \left( \phi_2 (1 - \phi_1) + \phi_2 \phi_1 \frac{p_1 - p_2 + t}{2t} \right) - \alpha \phi_2^2
\]

(2)

The first order conditions for prices are:

\[
\frac{\partial \pi_1}{\partial p_1} = \frac{1}{2} \phi_1 \frac{2t - t \phi_2 + \phi_2 p_2 - 2 \phi_2 p_1 + \phi_2 c}{t} = 0
\]

(3)

when \(p_1 = p_2 = v - t \Rightarrow \frac{\partial \pi_1}{\partial p_1} = \frac{1}{2} \phi_1 \frac{2t - \phi_2 v + \phi_1 c}{t}.

\[
\frac{\partial \pi_2}{\partial p_2} = \frac{1}{2} \phi_2 \frac{2t - \phi_1 t + \phi_1 p_1 - 2p_2 \phi_1 + \phi_1 c}{t} = 0
\]

(4)

when \(p_1 = p_2 = v - t \Rightarrow \frac{\partial \pi_2}{\partial p_2} = \frac{1}{2} \phi_2 \frac{2t - \phi_1 v + \phi_1 c}{t}.\) Therefore \(\frac{\partial \pi_1}{\partial p_1}, \frac{\partial \pi_2}{\partial p_2} > 0\) for \(\phi\) sufficiently low. The objective functions when prices are marginally greater than \(v - t\) are:

\[
\pi_1 = (p_1 - c) \left( \phi_1 (1 - \phi_2) \frac{v - p_1}{t} + \phi_1 \phi_2 \frac{p_2 - p_1 + t}{2t} \right) - \alpha \phi_1^2
\]

(5)

\[
\pi_2 = (p_2 - c) \left( \phi_2 (1 - \phi_1) \frac{v - p_2}{t} + \phi_1 \phi_2 \frac{p_1 - p_2 + t}{2t} \right) - \alpha \phi_2^2
\]

(6)

The first order conditions for prices are:

\[
\frac{\partial \pi_1}{\partial p_1} = \frac{1}{2} \phi_1 \frac{2v - 4p_1 - 2\phi_2 v + 2\phi_2 p_1 + \phi_2 p_2 + t \phi_2 + 2c - \phi_2 c}{t} = 0
\]

(7)

\[
\frac{\partial \pi_2}{\partial p_2} = \frac{1}{2} \phi_2 \frac{2v - 4p_2 - 2\phi_1 v + 2p_2 \phi_1 + \phi_1 p_1 + \phi_1 t + 2c - \phi_1 c}{t} = 0
\]

(8)
when \( p_1 = p_2 = v - t \Rightarrow \frac{\partial \pi_1}{\partial p_1} = -\frac{1}{2} \phi_1 \frac{2\phi_4 - \phi_2^2 + 2t \phi_2 - 2c + \phi_2 c}{t} \). It is straightforward to show that 
\( -\frac{1}{2} \phi_1 \frac{2\phi_4 - \phi_2^2 + 2t \phi_2 - 2c + \phi_2 c}{t} < 0 \) for all \( t < \frac{v - c}{2} \), i.e., when \( \phi \) is low enough, \( v - t \) is the price equilibrium because \( \frac{\partial \pi}{\partial p} \bigg|_{v-t} < 0 \) and \( \frac{\partial \pi}{\partial p} \bigg|_{v-t} > 0 \). At this price, the objective functions for firms are:

\[
\begin{align*}
\pi_1 &= \phi_1 v - \frac{1}{2} v \phi_1 \phi_2 - \phi_1 t + \frac{1}{2} t \phi_1 \phi_2 - \phi_1 c + \frac{1}{2} c \phi_1 \phi_2 - \alpha \phi_1^2 \\
\pi_2 &= \phi_2 v - \frac{1}{2} v \phi_1 \phi_2 - t \phi_2 + \frac{1}{2} t \phi_1 \phi_2 - \phi_2 c + \frac{1}{2} c \phi_1 \phi_2 - \alpha \phi_2^2
\end{align*}
\]

The solution to these conditions is:

\[
\begin{align*}
\phi_1 &= 2 \frac{v + t + c}{\alpha_1 + \alpha_2}, \quad \text{and} \quad \phi_2 = 2 \frac{v + t + c}{\alpha_1 + \alpha_2} > 0 \text{ for a corner solution in prices. Substituting implies that } \alpha > \frac{1}{4} \frac{v^2 - 2ct - 2v^2 + 2t^2 c + c^2}{t^2 + 2t c + c^2} \text{ for } \frac{\partial \pi}{\partial p} > 0. \\
\text{Equations 3 and 4 are negative at a price marginally less than } v - t. \text{ Therefore, in a symmetric equilibrium, the prices must be less than } v - t. \text{ The four first order conditions are equations 3 and 4 and:}
\end{align*}
\]

\[
\begin{align*}
\frac{\partial \pi_1}{\partial \phi_1} &= \frac{1}{2} \frac{2p_1 t - p_1 \phi_2 t + p_1 \phi_2 p_2 - \phi_2 p_1^2 - 2ct + c \phi_2 t - c \phi_2 p_2 + c \phi_2 p_1 - 4 \phi_1 t}{t} \\
\frac{\partial \pi_2}{\partial \phi_2} &= \frac{1}{2} \frac{-2p_2 t + p_2 \phi_2 t - p_2 \phi_1 p_1 + \phi_1 p_2^2 + 2ct - c \phi_1 t + c \phi_1 p_1 - c \phi_1 p_2 + 4 \phi_2 t}{t}
\end{align*}
\]

Solving symmetrically yields a pure strategy equilibrium of \( p = 2\sqrt{\alpha l} + c, \phi = \frac{2t}{2\sqrt{\alpha l} + t} \), and \( \pi = 4t^2 \frac{\alpha}{(2\sqrt{\alpha l} + t)^2} \). Because \( \frac{\partial \pi}{\partial \alpha} = \frac{4t^2}{(2\sqrt{\alpha l} + t)^2} > 0 \), the lower is \( \alpha \), the lower are profits. Note that \( p = \frac{2t}{\phi} - t + c \) implying that \( \frac{\partial \pi}{\partial \phi} = \frac{-2t}{\phi} < 0 \) (the main result of Proposition 1 when the equilibrium involves pure pricing strategies).

The pure strategy equilibrium must be stable to defections by either firm to \( v - t \). The profits earned by charging \( v - t \) to partially informed consumers is \( \phi (1 - \phi) (v - t - c) - \alpha \phi^2 \). Therefore, \( 4t^2 \frac{\alpha}{(2\sqrt{\alpha l} + t)^2} > \phi (1 - \phi) (v - t - c) - \alpha \phi^2 \) is necessary for a pure strategy in prices. Substituting the equilibrium value for \( \phi \) yields a cubic equation in \( \alpha \). The largest root of the equation is \( \alpha^* = \frac{1}{8} \frac{3t^2 - 4(v - c)t - t\sqrt{(5t^2 - 6(v-c)t + (v-c)^2) + (v-c)^2}}{t} \). When \( \alpha < \alpha^* \), firms have an incentive to defect to \( v - t \). Note that \( \alpha < \alpha^* \) that satisfy second order conditions are only possible when \( t < \frac{v - c}{5} \). When \( \alpha < \alpha^* \), the best response to a price of \( v - t \) is for the competitor to set price at \( v - 2t \) and capture demand from the groups of fully informed and partially informed consumers. Here, each firm effectively faces demand from two discrete groups of consumers with very different reservation prices. Following Shilony (1977) and Narasimhan (1988), the equilibrium is in mixed pricing strategies.
Let $F(p)$ and $f(p)$ be the c.d.f. and p.d.f. respectively of the mixed pricing strategy. The objective functions for each firm are:

\[
\pi_1 = (p_1 - c) \left( \phi_1(1 - \phi_2) + \phi_1 \phi_2 \left( \int_{p_1 - t}^{p_1 + t} \frac{p_2 - p_1 + t}{2t} f(p_2) \, dp_2 + (1 - F(p_1 + t)) \right) \right) - \alpha \phi_1^2
\]

\[
\pi_2 = (p_2 - c) \left( \phi_2(1 - \phi_2) + \phi_1 \phi_2 \left( \int_{p_2 - t}^{p_2 + t} \frac{p_1 - p_2 + t}{2t} f(p_1) \, dp_1 + (1 - F(p_2 + t)) \right) \right) - \alpha \phi_2^2
\]

For Firm 1, I take the derivative with respect to advertising:

\[
\frac{\partial \pi_1}{\partial \phi_1} = (p_1 - c) \left( (1 - \phi_2) + \phi_2 \left( \int_{p_1 - t}^{p_1 + t} \frac{p_2 - p_1 + t}{2t} f(p_2) \, dp_2 + (1 - F(p_1 + t)) \right) \right) - 2\alpha \phi_1 = 0
\]

\[
\Rightarrow 2\alpha \phi_1 = (p_1 - c) \left( (1 - \phi_2) + \phi_2 \left( \int_{p_1 - t}^{p_1 + t} \frac{p_2 - p_1 + t}{2t} f(p_2) \, dp_2 + (1 - F(p_1 + t)) \right) \right)
\]

Substituting into equation 15, I obtain:

\[
\pi_1 = 2\alpha \phi_1^2 - \alpha \phi_1^2 = \alpha \phi_1^2
\]

By the equal profit condition, $\pi_1$ is a constant so Firm 1 employs a pure strategy in advertising. The same reasoning applies to Firm 2’s advertising strategy.

Due to symmetry, I focus on the objective function of one firm. In equation 15, demand from consumers who are aware of both firms is $\phi_1 \phi_2 \int_{p_1 - t}^{p_1 + t} \left( \frac{p_2 - p_1 + t}{2t} f(p_2) \right) \, dp_2$. This can be rewritten using integration by parts. Let $du = f(p_2) \, dp_2$ and $w = \frac{p_2 - p_1 + t}{2t}$. Then $u = F(p_2)$ and $dw = \frac{1}{2t} dp_2$. Using the rule $\int_a^b f(x) g(x) \, dx = f(x) g(x) \big|_a^b - \int_a^b f(x) g'(x) \, dx$, I write:

\[
\int_{p_1 - t}^{p_1 + t} \frac{p_2 - p_1 + t}{2t} f(p_2) \, dp_2 = \left[ F(p_2) \frac{p_2 - p_1 + t}{2t} \right]_{p_1 - t}^{p_1 + t} - \int_{p_1 - t}^{p_1 + t} F(p_2) \frac{1}{2t} \, dp_2
\]

The first term is $\frac{F(p_1 + t) - F(p_1 - t)}{2}$ and the second term $\int_{p_1 - t}^{p_1 + t} F(p_2) \frac{1}{2t} \, dp_2$ can be approximated as $\frac{1}{2t}$ multiplied by the area of a trapezoid with a base of $2t$ and sides of $F(p_1 + t)$ and $F(p_1 - t)$ respectively. Substituting I obtain:

\[
F(p_1 + t) - \frac{1}{2} \left( \frac{2t}{F(p_1 + t) + F(p_1 - t)} \right) = F(p_1 + t) - \frac{F(p_1 + t) - F(p_1 - t)}{2}
\]

which is $\frac{F(p_1 + t) - F(p_1 - t)}{2}$ i.e. half of the consumers in that range. This allows me to approximate the objective function for Firm 1 as:

\[
\pi_1 = (p_1 - c) \left( \phi_1(1 - \phi_2) + \phi_1 \phi_2(1 - F(p_1)) \right) - \alpha \phi_1^2
\]

The maximum price in the mixed pricing strategy support is $v-t$ ($\frac{\partial \pi_1}{\partial \phi} < 0$, when $p < v-t$). I assume that when this price is chosen, the firm collects negligible demand from fully informed consumers (i.e. $F(v-t) = 0$). Because $\phi_1 = \phi_2$, $\pi_1 = (v-t-c) \left( \phi - \phi^2 \right) - \alpha \phi^2$. Substituting into equation 15, I obtain $F(p) = \frac{1}{\phi} \cdot \frac{v-t + \phi t - \phi^2}{p-c}$. Therefore, $\frac{\partial F(p)}{\partial \phi} = \frac{(v-t-p)(p-c)\phi^2}{(p-c)p^{\phi^2}} > 0$. As a result, when $\phi'' < \phi'$, $F(\phi'')$ first-order stochastically dominates $F(\phi')$ implying that average
price under \( \phi'' \) is strictly greater. In addition, the minimum price in the support for \( F(\phi') \) is strictly less than that of \( F(\phi'') \). Straightforward calculations also imply the following equilibrium decisions for \( \alpha < \alpha^* \): \( \phi = \frac{t-\alpha}{v-t-c+2\alpha} \) and a mixed pricing strategy with c.d.f. \( F(p) = \frac{(p-c)(v-t-c)-2\alpha(v-t-t)}{(p+c)(v-t-c)} \) for \( p \in \left( \frac{2\alpha(v-t-c)+(v-t-c)}{v-t-c+2\alpha}, v-t \right) \). Note that the minimum price in the support cannot be less than \( t + c \) and this places a further constraint on the mixed pricing strategy described above. In order for \( p > t + c \), simple calculations imply that \( \alpha > \frac{1}{2}(t+4c) \). Therefore, the equilibrium described above applies when \( \alpha \in \left( \frac{1}{2}(t+4c), \alpha^* \right) \).

When \( \alpha > \frac{1}{2}(t+4c) \), the pricing equilibrium includes a mass point at \( p = t + c \). Let \( q \) be the probability that each firm chooses a price of \( t + c \) and \( G(p) \) be the c.d.f. of the mixed pricing strategy conditional on the firm not choosing a price of \( t + c \). The profit function for Firm 1 when prices are greater than \( t + c \) can be written as (I use the approximation derived above):

\[
\pi_1 = (p_1 - c)(\phi_1(1 - \phi_2) + \phi_1\phi_2(1 - q)(1 - G(p))) - \alpha\phi_1^2
\]  

(22)

When a price of \( t + c \) is chosen then:

\[
\pi_1 = t\left(\phi_1(1 - \phi_2) + \phi_1\phi_2(1 - q) + \phi_1\phi_2\frac{q}{2}\right) - \alpha\phi_1^2
\]  

(23)

As before, it is straightforward to show that the firms employ pure advertising strategies. At \( v - t \), \( \pi_1 = (v - t - c)\phi_1(1 - \phi_2) - \alpha\phi_1^2 \). Differentiating with respect to \( \phi_1 \), \( \frac{\partial\pi_1}{\partial\phi_1} = v - \phi_2v - t + \phi_2t - c + \phi_2c - 2\alpha\phi_1 = 0 \). Assuming symmetry, \( \phi = \frac{v-t-c}{v-t-c+2\alpha} \). This implies that \( \pi = \alpha\frac{(v-t-c)^2}{(v-t-c+2\alpha)^2} \). Substituting \( \pi \) and \( \phi \) into equation 23, I derive the probability that each firm chooses a price of \( t + c \): \( q = \frac{2\alpha(v-t-c) + t+c - t^2 - 2\alpha c}{t(t-v+c)} \). Straightforward algebra can be used to derive \( G(p) \). However, \( q \) is restricted to values less than 1. This implies that \( \alpha > \frac{1}{3}(t+4c) \). When \( \alpha > \frac{1}{3}(t+4c) \), both firms choose a pure pricing strategy of \( t + c \) and the profit function becomes:

\[
\pi_1 = t\left(\phi_1(1 - \phi_2) + \phi_1\phi_2(1 - q) + \phi_1\phi_2\frac{q}{2}\right) - \alpha\phi_1^2
\]  

(24)

The first order condition for advertising is \( \frac{\partial\pi_1}{\partial\phi_1} = t - \frac{1}{2}\phi_2t - 2\alpha\phi_1 = 0 \). Assuming symmetry, the equilibrium advertising is \( \phi = \frac{1}{2}t \). This implies that \( \phi = 1 \), when \( \alpha \leq \frac{1}{6}t \). Therefore when \( \alpha \in \left( \frac{1}{6}t, \frac{1}{3}(v-t-c) \right) \), \( \pi = 4\alpha\frac{t^2}{t(4t+2c)} \). When \( \alpha \leq \frac{1}{6}t \), the profit for both firms is \( \frac{1}{2}t - \alpha \) and both firms advertise to the entire market. The above analysis implies that price are negatively related to advertising except when advertising levels become high enough to sustain a pure strategy price equilibrium of \( t + c \) i.e. when \( \alpha < \frac{1}{3}(t+4c) \). Q.E.D.

**Derivation of the Limits for the Region of High Differentiation**

In this region, the optimal price for partially informed consumers is lower than the optimal price for fully informed consumers. Thus, the highest price occurs when the market is fully informed i.e., \( p = t + c \). To guarantee that firms always compete for fully informed consumers, the surplus realized by the consumer at \( x = \frac{1}{2}t \) must be positive. This implies that \( v - t - (t + c) > 0 \) or \( \frac{2(v-c)}{3} > t \). This is the upper limit of the range for \( t \).
The lower limit is based on the individual rationality constraint binding for at least one partially informed consumer. The IR constraint is least likely to bind when prices are lowest and the lowest price is observed when the market is almost entirely partially informed. When the market is almost entirely partially informed, \( \pi = (p - c) \frac{v - t}{t} \) and this is maximized at \( p = \frac{v - t}{t} \). For the most distant consumer, this implies that \( CS = v - t - \frac{v - c}{2} < 0 \). Rearranging to obtain \( \frac{v - c}{2} < t \), the lower limit of the range for \( t \).

**Proof of Proposition 2**

When \( t > \frac{v - c}{2} \), the first order conditions are:

\[
\frac{\partial \pi_1}{\partial p_1} = \frac{1}{2} \frac{2v - 4p_1 - 2\phi_2 v + 2\phi_2 p_1 + \phi_2 p_2 + t\phi_2 + 2c - \phi_2 c}{t} = 0 \quad (25)
\]

\[
\frac{\partial \pi_2}{\partial p_2} = \frac{1}{2} \frac{2v - 4p_2 - 2\phi_1 v + 2\phi_2 p_1 + \phi_1 p_1 + \phi_1 t + 2c - \phi_1 c}{t} = 0 \quad (26)
\]

As noted in the text, I assume that \( v - p_i < t \). I later show that the solution satisfies this condition. Solving equations 25 and 26, yields the following expressions for prices in terms of the advertising levels and exogenous parameters:

\[
p_1 = \frac{8v + 8c - 6\phi_2 v + 4t\phi_2 - 2\phi_2 c + 2\phi_3 v\phi_1 - t\phi_2 \phi_1 + \phi_2 c\phi_1 - 4\phi_1 v - 4\phi_1 c}{16 - 8\phi_1 - 8\phi_2 + 3\phi_2 \phi_1} \quad (27)
\]

\[
p_2 = \frac{8v - 6\phi_1 v + 4\phi_1 t + 8c - 2\phi_2 c - 4\phi_2 v + 2\phi_3 v\phi_1 - t\phi_2 \phi_1 - 4\phi_2 c + \phi_2 c\phi_1}{16 - 8\phi_1 - 8\phi_2 + 3\phi_2 \phi_1} \quad (28)
\]

In the symmetric solution \( \phi_1 = \phi_2 \). As a result, equations 27 and 28 imply that \( p_1 = p_2 = \frac{2\phi v - t\phi + \phi c - 2v - 2c}{3\phi - 4} \). Therefore, \( \frac{\partial p}{\partial \phi} = -2 \frac{v - 2t - c}{(3\phi - 4)^2} < 0 \) (because \( t > \frac{v - c}{2} \) when differentiation is high, \( \frac{\partial p}{\partial \phi} > 0 \)). Now I show that \( p > v - t \). If \( p > v - t \) for all \( p \) then the minimum value of \( p \) must also satisfy the condition. Since the minimum value of \( p \) occurs when \( \phi_1 = \phi_2 = 0 \), I substitute to obtain \( p = \frac{v + c}{2} \). If this is less than \( v - t \) then \( \frac{v + c}{2} < v - t \Rightarrow t < \frac{v - c}{2} \) which violates the initial conditions for \( t \). Q.E.D.

**Computation and Comparison of Segment Specific Elasticities when Differentiation is High**

The elasticity for segment \( z \) (partially informed consumers or fully informed consumers) is: \( \varepsilon_z = \frac{p_i \frac{\partial q_i}{\partial p_i}}{\varepsilon_z} \). In equilibrium \( p_i = \frac{2\phi v - t\phi + \phi c - 2v - 2c}{3\phi - 4} \). Also because \( q_i \) is defined as \( q_i = \frac{v - p_i}{t} \) (for partially informed consumers) and \( y_i = \frac{p_i - p_i + t}{2t} \) (for fully informed consumers), \( \frac{\partial q_{\text{partial}}}{\partial p_i} = -\frac{1}{t} \) and \( \frac{\partial q_{\text{full}}}{\partial p_i} = -\frac{1}{2t} \). Moreover, \( q_{\text{partial}} = \frac{\phi v - 2v + t\phi - \phi c + 2c}{(3\phi - 4)t} \) and \( q_{\text{full}} = \frac{1}{2} \). By substituting, \( \varepsilon_{\text{partial}} = -\frac{2v - 2t + \phi c - 2v - 2c}{\phi v - 2v + t\phi - \phi c + 2c} \) and \( \varepsilon_{\text{full}} = -\frac{2v - 2t + \phi c - 2v - 2c}{(3\phi - 4)t} \). This implies that \( \frac{\varepsilon_{\text{partial}}}{\varepsilon_{\text{full}}} = \frac{(4 - 3\phi)t}{2v - \phi v - t\phi + \phi c - 2c} \). If \( \frac{\varepsilon_{\text{partial}}}{\varepsilon_{\text{full}}} > 1 \), then the elasticity of partially informed consumers is higher than that of fully informed consumers. Assume that \( \frac{\varepsilon_{\text{partial}}}{\varepsilon_{\text{full}}} < 1 \). Because both the numerator and denominator are positive, this assumption (with some algebra) implies that \( 2(v - c - 2t) > \phi(v - c - 2t) \). Because \( t > \frac{v - c}{2} \) when differentiation is high, \( v - c - 2t < 0 \). Therefore \( 2(v - c - 2t) \geq \phi(v - c - 2t) \Rightarrow \phi > 2 \) which is impossible. This shows that \( |\varepsilon_{\text{partial}}| > |\varepsilon_{\text{full}}| \) for all \( \phi \).
References
