News, Noise, and Fluctuations: An Empirical Exploration

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Abstract

We explore empirically models of aggregate fluctuations with two basic ingredients: agents form anticipations about the future based on noisy sources of information; these anticipations affect spending and output in the short run. Our objective is to separate fluctuations due to actual changes in fundamentals (news) from those due to temporary errors in the private sector's estimates of these fundamentals (noise). Using a simple model where the consumption random walk hypothesis holds exactly, we address some basic methodological issues and take a first pass at the data. First, we show that if the econometrician has no informational advantage over the agents in the model, structural VARs cannot be used to identify news and noise shocks. Next, we develop a structural approach which allows us to identify the model's parameters and to evaluate the role of news and noise shocks. Applied to postwar U.S. data, this approach suggests that noise shocks play an important role in short-run fluctuations.

Keywords: Aggregate shocks, business cycles, vector autoregression, invertibility.

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Introduction

A common view of the business cycle gives a central role to anticipations. Consumers and firms continuously receive information about the future, which sometimes is news, sometimes just noise. Based on this information, consumers and firms choose spending and, because of nominal rigidities, spending affects output in the short run. If ex post the information turns out to be news, the economy adjusts gradually to a new level of activity. If it turns out to be just noise, the economy returns to its initial state. Therefore, the dynamics of news and noise generate both short-run and long-run changes in aggregate activity. In this paper, we ask how aggregate time series can be used to shed light on this view of the business cycle.

We are interested in this view for two reasons. The first is that it appears to capture many of the aspects often ascribed to fluctuations: the role of animal spirits in affecting demand—spirits coming here from a rational reaction to information about the future—, the role of demand in affecting output in the short run, together with the notion that in the long run output follows a natural path determined by fundamentals.

The second is that it appears to fit the data in a more formal way. More specifically, it offers an interpretation of structural VARs based on the assumption of two major types of shocks: shocks with permanent effects and shocks with transitory effects on activity. As characterized by Blanchard and Quah (1989), Galí (1999), Beaudry and Portier (2006), among others, "permanent shocks" appear to lead to an increase in activity in the short run, building up to a larger effect in the long run, while "transitory shocks"—by construction—lead to a transitory effect on activity in the short run. It is tempting to associate shocks with permanent effects to news and shocks with transitory effects to noise.

In this paper, we focus on a simple model which provides a useful laboratory to address two issues: a methodological one and a substantive one. First, can structural VARs indeed be used to recover news and noise shocks? Second, what is the role of news and noise shocks in short-run fluctuations?

On the first question, we reach a strong negative conclusion—one which came as an unhappy surprise for one of the coauthors. In models of expectation-driven fluctuations in which consumers solve a signal extraction problem, structural VARs can typically recover neither the shocks nor their propagation mechanisms. The reason is straightforward: If agents face a signal extraction problem, and are unable to separate news from noise, then the econometrician, faced with either the same data as the agents or a subset of these data, cannot do it either.

To address the second question, we then turn to structural estimation, first using a simple method of moments and then Maximum Likelihood. We find that our model fits the data well and gives a clear description of fluctuations as a result of three types of shocks: shocks with permanent effects on productivity, which build up slowly over time; shocks with temporary effects on productivity, which decay slowly; and shocks to consumers' signals about future productivity. All three shocks affect agents' expectations, and thus demand and output in the short run, and noise shocks are an important source of short-run volatility. In our baseline specification, noise shocks account for more than half of the forecast error variance at a yearly horizon, while permanent technology shocks account for less than one third. This result is somewhat surprising when compared with variance decompositions from structural VARs where transitory "demand shocks" often account for a smaller fraction of aggregate volatility at the same horizons and permanent technology shock capture a bigger share (e.g., Shapiro and Watson, 1989, and Galí, 1992). Our methodological analysis helps to explain the difference, showing why structural VARs may understate the contribution of noise/demand shocks to short-run volatility and overstate that of permanent productivity shocks.

Recent efforts to empirically estimate models of news-driven business cycles include Christiano, Ilut, Motto and Rostagno (2007) and Schmitt-Grohé and Uribe (2008). These papers follow the approach of Jaimovich and Rebelo (2006), modeling news as advanced, perfect information about shocks affecting future productivity. We share with those papers the emphasis on structural estimation. The main difference is that we model the private sector information as coming from a signal extraction problem and focus our attention on disentangling the separate effects of news and noise.

The problem with structural VARs emphasized in this paper is essentially an invertibility problem, also known as non-fundamentalness. There is a resurgence of interest in the methodological and practical implications of invertibility problems, see, e.g., Sims and Zha (2006) and Fernández-Villaverde, Rubio-Ramírez, Sargent and Watson (2007). Our paper shows that non-invertibility problem are endemic to models where the agents' uncertainty is represented as a signal extraction problem. This idea has also recently surfaced in models that try to identify the effects of fiscal policy when the private sector receives information on future policy changes (see Leeper, Walker and Yang, 2009).

The paper is organized as follows. Sections 1 and 2 present and solve the model. Section 3 looks at the use of structural VARs. Section 4 presents the results of our structural estimation. Section 5 presents a explores a number of extensions and Section 6 concludes.

1 The model

1.1 Productivity and information

We want to capture the notion that, behind productivity movements, there are two types of shocks: shocks with permanent effects and shocks with transitory effects. In particular, we assume that the effects of the first shock build up gradually over time, possibly reflecting the gradual adoption of technological innovations. The effects of the second shocks instead decay gradually over time. One can think of the transitory component as either true or reflecting measurement error. This does not matter for our purposes.

Productivity (in logs) is given by the sum of two components:

$$a_t = x_t + z_t. \tag{1}$$

The permanent component, x_t, follows a unit root process given by

$$\Delta x_t = \rho_x \Delta x_{t-1} + \epsilon_t. \tag{2}$$

The transitory component, z_t, follows a stationary process given by

$$z_t = \rho_z z_{t-1} + \eta_t. \tag{3}$$

The coefficients ρ_x and ρ_z are in $[0, 1)$, and ϵ_t and η_t are i.i.d. normal shocks with variances σ_ϵ^2 and σ_η^2. Agents observe productivity, but not the two components separately. [1]

Consumers face a signal extraction problem, as they observe productivity a_t each period, but not its individual components. To capture the idea that they have additional sources of information, we allow them to observe an additional signal about the permanent component of productivity

$$s_t = x_t + \nu_t, \tag{4}$$

where ν_t is i.i.d. normal with variance σ_ν^2. The consumers know the structure of the model, i.e., know ρ and the variances of the shocks.

In this model, consumers' expectations about future productivity are driven by three types of shocks, the two productivity shocks ϵ and η and the noise ν in the additional signal. For short, we shall refer to them as the "permanent shock", the "transitory shock", and the

[1] A similar productivity process, combining growth rate shocks and level shocks, was recently used by Aguiar and Gopinath (2007) in an open economy calibration exercise. Boz, Daude and Durdu (2008) explore the role of different informational assumptions in that context.

"noise shock". Permanent shock is a slight (and common) misnomer, as it refers to a shock whose effects build up gradually.

For most of the paper, we assume that the univariate representation of a_t is the random walk

$$a_t = a_{t-1} + u_t, \tag{5}$$

and restrict attention to the family of processes (1)-(3) that are consistent with this assumption. We do this for two reasons. The first is analytical convenience, as it makes our arguments more transparent. The second is that, as we shall see, this assumption provides a good starting point when looking at postwar U.S. data. As will be clear, however, none of our central results depends on this assumption.

There is a one-parameter family of processes (1)-(3) which is consistent with the univariate random walk representation (5). These are the processes that satisfy:

$$\rho_x = \rho_z = \rho,$$

$$\sigma_\epsilon^2 = (1-\rho)^2 \sigma_u^2, \qquad \sigma_\eta^2 = \rho \sigma_u^2,$$

for some $\rho \in [0, 1)$, where σ_u^2 is the variance of u_t.[2]

Productivity may be the sum of a permanent process with small shocks that build up slowly and a transitory process with large shocks that decay slowly (high ρ, small σ_ϵ^2 and large σ_η^2), or it may be the sum of a permanent process which is itself close to a random walk and a transitory process close to white noise with small variance (low ρ, large σ_ϵ^2 and small σ_η^2). An econometrician who can only observe a_t cannot distinguish these cases. The sample variance of Δa_t gives an estimate of σ_u^2, but the parameter ρ, and thus ρ_x, ρ_z, σ_ϵ^2 and σ_η^2, are not identified. As we shall see, the fact that consumers have additional information on x_t will make it possible to identify ρ and the remaining parameters, using data on productivity and consumption.

[2] To prove this result, notice that (1)-(3) imply

$$Var[\Delta a_t] = \frac{1}{1-\rho_x^2}\sigma_\epsilon^2 - \frac{2}{1+\rho_z}\sigma_\eta^2,$$

and

$$Cov[\Delta a_t, \Delta a_{t-j}] = \rho_x^j \frac{1}{1-\rho_x^2}\sigma_\epsilon^2 - \rho_z^{j-1}\frac{1-\rho_z}{1+\rho_z}\sigma_\eta^2 \text{ for all } j > 0.$$

Under the assumed parameter restrictions these yield $Var[\Delta a_t] = \sigma_u^2$ and $Cov[\Delta a_t, \Delta a_{t-j}] = 0$ for all $j > 0$. Quah (1990, 1991) offers general results on the decomposition of a univariate process in permanent and transitory components with orthogonal innovations.

1.2 Consumption

The final step is to integrate this information structure in an economy in which spending decisions are based on agents' expectations of future productivity and in which spending determines output in the short run. Here we start with a very simple model of consumer spending. This model is both analytically convenient for thinking about identification issues and, as we shall see, provides a good starting point for looking at the data. In Section 3, we incorporate the same information structure in a richer DSGE model which incorporates both consumer and investment spending and more realistic assumptions on nominal rigidities.

Letting c_t denote the log of consumption, assume that it is a forward looking variable that satisfies the Euler equation

$$c_t = E_t[c_{t+1}], \tag{6}$$

where E_t denotes the expectation conditional on the consumers' information at date t, which is given by current and past observations of a_t and s_t.

We drastically simplify the supply side, by considering an economy with no capital, in which consumption is the only component of demand and output is fully determined by the demand side. Output (in logs) is given by

$$y_t = c_t, \tag{7}$$

and the labor input adjusts to produce y_t, given the current level of productivity. With a linear production function, the labor input is given (in logs) by $n_t = y_t - a_t$. We impose the restriction that output returns to its natural level in the long run and that natural output is is proportional to productivity, so

$$\lim_{j\to\infty} E[y_{t+j} - a_{t+j}] = 0. \tag{8}$$

In Appendix A, we show that this model can be derived as the limit case of a standard new Keynesian model with Calvo pricing, when the frequency of price adjustment goes to zero.

1.3 Solving the model

Combining equations (6) to (8), we obtain

$$c_t = \lim_{j\to\infty} E_t[a_{t+j}]. \tag{9}$$

Consumption, and by implication output, only depend on the consumers' expectations of productivity in the long run. Using equations (1)-(3), we can solve the long run productivity expectation in terms of expectations on x_t and x_{t-1} and obtain

$$c_t = x_{t|t} + \frac{\rho_x}{1-\rho_x}(x_{t|t} - x_{t-1|t}), \tag{10}$$

where we use the notation $x_{s|t}$ for $E_t[x_s]$.

To complete the solution of the model, we want to express consumption and productivity as functions of current and lagged values of the shocks $(\epsilon_t, \eta_t, \nu_t)$. To do so, we need to solve the consumers' signal extraction problem and derive the dynamics of consumers' expectations of the unobservable state variables of the productivity process.

The productivity process in state space form is given by

$$X_t = AX_{t-1} + BV_t,$$

where $X_t \equiv (x_t, x_{t-1}, z_t)'$ is the vector of state variables and $V_t \equiv (\epsilon_t, \eta_t, \nu_t)'$ is the vector of shocks. As consumers observe a_t and s_t each period, the observation equation is

$$S_t = CX_t + DV_t,$$

where $S_t \equiv (a_t, s_t)$.[3] The Kalman filter can then be used to express the consumers' expectations on X_t in recursive form as

$$\begin{aligned} X_{t|t} &= AX_{t-1|t-1} + K(S_t - S_{t|t-1}) \\ &= A(I-KC)X_{t-1|t-1} + KS_t, \end{aligned} \tag{11}$$

where the matrix of Kalman gains K depends on the parameters of the productivity process.

Equations (10)-(11) together with equations (1)-(3) fully characterize the dynamic responses of consumption and productivity to the shocks in V_t. Figure 1 shows these responses, using parameters in line with the estimates obtained later, in Section 2. The time unit is the quarter. The productivity process satisfy the random walk property, with $\rho_x = \rho_z = 0.89$,

[3]The matrices A, B, C, D are

$$A \equiv \begin{bmatrix} 1+\rho_x & -\rho_x & 0 \\ 1 & 0 & 0 \\ 0 & 0 & \rho_z \end{bmatrix}, B \equiv \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, C \equiv \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}, D \equiv \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

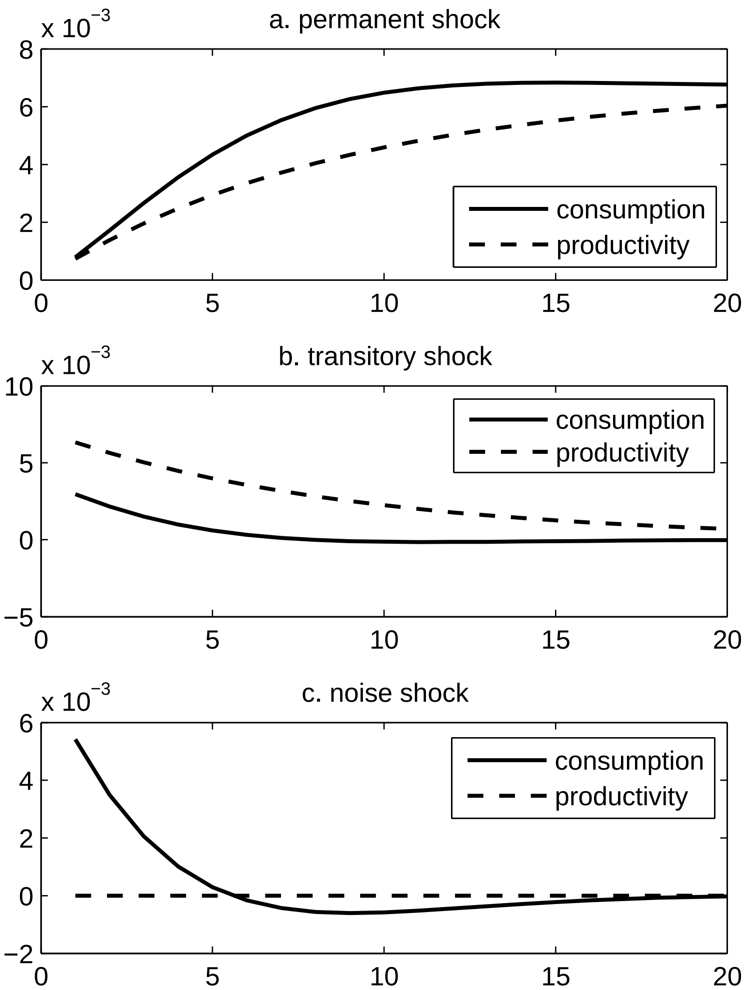

Figure 1: Impulse Responses to the Three Shocks

implying slowly building permanent shocks and slowly decaying transitory shocks. The standard deviation of productivity growth, σ_u, is set to 0.67% so the standard deviations of the two technology shocks, σ_ϵ and σ_η, are equal to 0.07% and 0.63%, respectively. The standard deviation of the noise shock, σ_ν, is set to 0.89%, implying a fairly noisy signal.

In response to a one-standard-deviation positive permanent shock ϵ_t productivity builds up slowly over time—the implication of a high value for ρ. Consumption also increases slowly. This reflects the fact that the standard deviations of the transitory shock η_t and of the noise shock ν_t are both large relative to the standard deviation of ϵ_t. Thus, it takes a long time for consumers to assess that the shock is permanent and to fully adjust consumption.

For our parameter values, consumption and output initially increase more than productivity, generating a transitory increase in employment. Smaller transitory shocks, or a more informative signal would lead to a larger initial increase in consumption, and thus a larger initial increase in employment. Larger transitory shocks, or a less informative signal, might lead instead to an initial decrease in employment.

In response to a one standard deviation increase in η_t, the transitory shock, productivity initially increases, and then slowly declines over time. As agents put some weight on it being a permanent shock, they initially increase consumption. As they learn that this was a transitory shock, consumption returns back to normal over time. For our parameter values, consumption increases less than productivity, leading to an initial decrease in employment. Again, for different parameters, the outcome may be an increase or a decrease in employment.

Finally, in response to a one standard deviation increase in ν_t, the noise shock, consumption increases, and then returns to normal over time. The response of consumption need not be monotonic; in the simulation presented here, the response turns briefly negative, before returning to normal. By assumption, productivity does not change, so employment initially increases, to return to normal over time.

2 Identification and estimation

We now turn to issues of identification and estimation. We will attack the problem from three sides. First, we derive the reduced form VAR representation of the process for consumption and productivity and show that, except in special cases, it is non-invertible. This means that it is not possible to use simple semi-structural identification assumptions to estimate the economy responses to our three shocks. Second, we show that, in our simple model, one can use three moments in the data to identify the model parameters. This exercise helps us understand what information in the data can be used to shed light on the role of news and noise shocks. Third, we show how to estimate the model using likelihood methods, which can be applied to our simple model, but also to richer models in which consumers solve a signal extraction model. The latter is the approach we will use for the DSGE estimations of Section 3.

In terms of observables for the econometrician, we will consider both the case where the signal s_t is observable, so the econometrician has time series for (a_t, c_t, s_t), and the case where only a_t and c_t are observed, as it will be the case in our empirical exercises.

2.1 VAR

2.1.1 Reduced form VAR

Our assumptions make it easy to derive the reduced form VAR representation of the processes for consumption and productivity. Rearranging (10), we obtain

$$(1-\rho)c_t = x_{t|t} - \rho x_{t-1|t}. \tag{12}$$

Writing the corresponding expression for c_{t-1} and taking differences side by side, we obtain

$$c_t = c_{t-1} + u_t^c, \tag{13}$$

with

$$u_t^c = \frac{1}{1-\rho}(x_{t|t} - x_{t-1|t-1}) - \frac{\rho}{1-\rho}(x_{t-1|t} - x_{t-2|t-1}).$$

Turning to productivity, equations (1) and (3) imply

$$\begin{aligned} a_t - \rho a_{t-1} &= x_t + z_t - \rho\left(x_{t-1} + z_{t-1}\right) \\ &= x_t - \rho x_{t-1} + \eta_t. \end{aligned}$$

Using (12), lagged one period, we then obtain

$$a_t = \rho a_{t-1} + (1-\rho)\, c_{t-1} + u_t^a, \tag{14}$$

with

$$u_t^a = x_t - x_{t-1|t-1} - \rho(x_{t-1} - x_{t-2|t-1}) + \eta_t.$$

To show that u_t^c and u_t^a in (13) and (14) are indeed innovations take expectations and use (2) to obtain

$$E_{t-1}[u_t^c] = \frac{1}{1-\rho} E_{t-1}[\epsilon_t] = 0,$$

$$E_{t-1}[u_t^a] = E_{t-1}[\epsilon_t + \eta_t] = 0.$$

This shows that u_t^c and u_t^a are innovations with respect to the consumers' information. Turning to the econometrician, we can let the econometrician observe either (c_t, a_t, s_t) or just (c_t, a_t). In either case the econometrician has (weakly) less information than the consumer and the law of iterated expectations implies $E_t^e[u_t^c] = 0$ and $E_t^e[u_t^a$, where E_t^e denotes the expectation conditional on the econometrician information at time t. Therefore, u_t^c and u_t^a represent innovations for consumption and productivity both in a reduced form tri-variate VAR in (c_t, a_t, s_t) and in a reduced form bi-variate VAR in (c_t, a_t).

Note that, under our assumptions, the univariate representations of both productivity and consumption are random walks. For productivity this follows from our assumptions on the productivity process, for consumption it follows from the behavioral assumption (6). When we move to multivariate representations including c_t and a_t, past productivity does not help predict consumption, but, as (14) shows, past consumption can help to predict productivity as it captures the consumers' information on the permanent component x_t.[4]

2.1.2 Structural VAR and invertibility problems

In two special cases, a structural VAR approach works. First, if the signal is perfectly informative, i.e., if $\sigma_\nu = 0$, consumers know exactly the value of the permanent component of productivity, x_t, and by implication, the value of the transitory component, $z_t = a_t - x_t$. In this case, equations (13) and (14) simplify to:

$$\begin{aligned} c_t &= c_{t-1} + \frac{1}{1-\rho}\epsilon_t, \\ a_t &= \rho a_{t-1} + (1-\rho)\, c_{t-1} + \epsilon_t + \eta_t. \end{aligned}$$

Consumption responds only to the permanent shock, productivity to both. In this case, a structural VAR approach works. Imposing the long-run restriction that only one of the shocks has a permanent effect on consumption and productivity, we can recover ϵ_t and η_t, and their dynamic effects.

[4]The special case in which consumption does not help to predict productivity is $\rho = 0$. As we shall see below, in this case a_t and c_t are perfectly collinear, so, given a_{t-1}, c_{t-1} provides no extra information on a_t. In this case, the innovations representation is not unique, as (14) can be replaced, for example, by $a_t = a_{t-1} + u_t^a$.

A second special case in which a structural VAR approach works is the case of an uninformative signal, i.e. the limit case where $\sigma_\nu \to \infty$, so consumers rely only on current and past productivity to forecast future productivity. Then, trivially, our random walk assumption for a_t leads to $c_t = a_t$. In this case, the two innovations u_t^c and u_t^a coincide and are identical to the innovation u_t in the univariate representation of a_t. The bivariate dynamics of consumption and productivity are given by

$$
\begin{aligned}
c_t &= a_{t-1} + u_t, \\
a_t &= a_{t-1} + u_t.
\end{aligned}
$$

It is not possible from the knowledge of u_t to recover the shocks ϵ_t and η_t. However, in this case the decomposition between temporary and permanent shocks is essentially irrelevant, given that no information is available to ever separate the two. We might as well take the random walk representation of productivity as our primitive productivity process and just interpret u_t as the single, permanent shock. With this approach, one can safely adopt a structural VAR approach. In other words, this is a case in which the VAR representation is non-invertible in terms of the original shocks but it is invertible in terms of an alternative representation. Notice that in this case, no information about the original shocks can be recovered either by structural VAR methods or by any other method.

In the two cases just considered, a structural VAR approach works for very different reasons. In the first, we can exploit the perfect information of the consumers to separate permanent and transitory shocks. In the second, we can ignore the "true" productivity process and just focus on the observable random walk for productivity. Unfortunately, once we move away from these special cases and have a partially informative signal, the reduced form VAR representation is non-invertible and a structural VAR approach cannot be pursued. In the general case, unlike in the first case, the consumers' information at time t is not sufficient to exactly recover the shocks. At the same time, unlike in the second case, consumption reflects some information on the transitory and permanent components of productivity. So the data contains information on the role of these shocks, but a structural VAR approach cannot be used to recover this information.

Consider an econometrician with time series for (c_t, a_t, s_t). The econometrician runs a trivariate reduced form VAR in (c_t, a_t, s_t) and obtains the reduced form innovations $U_t \equiv (u_t^c, u_t^a, u_t^s)$. He then tries to use some identification restriction to map the reduced form innovations into the economic shocks. No identification restriction can allow the recovery of the original shocks $(\epsilon_t, \eta_t, \nu_t)$. This is a corollary of the following general result, that shows that non-invertibility is an endemic problem in economic models where the consumers solve a non-trivial signal extraction problem.

Lemma 1 *Consider a model in which a vector of exogenous state variables X_t evolves according to $X_t = AX_{t-1} + BV_t$, where V_t is a vector of i.i.d. shocks. The agents in the model observe the signal vector $S_t = CX_t + DV_t$. Suppose the distributed lag representation of S_t is*

$$S_t = \Phi(L) V_t, \tag{15}$$

and is non-invertible. Suppose the vector of endogenous variables Y_t can be solved in terms of distributed lags of the agents' expectations $Y_t = \Xi(L) \begin{pmatrix} S_t & X_{t|t} \end{pmatrix}'$, where $X_{t|t} = E[X_t|S^t]$. Then the VAR representation of (Y_t, S_t) is non-invertible.

In our simple model the only endogenous variable in the vector Y_t is c_t. The representation of Y_t in terms of the expectation vector $X_{t|t}$ is given by equation (10). The non-invertibility of (15) follows immediately from the fact that S_t is bidimensional and the shock vector is tridimensional, except in the special cases seen above.[5] So the proposition applies.

The intuition behind this proposition is simply that all agents' decisions are function of their expectations, so if the agents' signals S_t are not sufficient to recover the underlying shocks, then the joint observation of these signals and of the agents' choice variables cannot generate additional information on these shocks. In the proposition we allow the endogenous variables Y_t to be a function not just of the current expectations $X_{t|t}$ but of all its past values. This implies that the proposition also applies to economic models with additional endogenous state variables, like capital, and, in particular, it applies to the models in Section 3.

Despite the non-invertibility problem, one could still hope that long run identification restrictions on a VAR could be used to separate the effect of the permanent shock ϵ_t from the combined effect of the other two shocks η_t and ν_t. The following proposition shows that also this partial identification fails.

Proposition 1 *Suppose that the econometrician observes (c_t, a_t, s_t) or (c_t, a_t). The estimated impulse response of c_t to any identified shock from a structural VAR will be, asymptotically, either permanent and flat or zero.*

Comparing this result with the impulse responses obtained in Figure 1 immediately shows that a structural VAR will be, in general, unable to recover the model's responses to any of

[5] The non-invertibility of (15) is easy to establish in our signal extraction model. For other models, one can use the conditions in Fernández-Villaverde, Rubio-Ramírez, Sargent and Watson (2007), just reinterpreting the state space model of the econometrician in that paper as the consumers' state space model in this paper.

our three shocks, given that none of them leads to a flat consumption response. The basic intuition here is this: if consumption is a random walk given the consumers' information sets, then an econometrician with access to the same information, or less, cannot identify any shock that has a transitory effect on consumption based on the reduced form VAR innovations at time t. If the econometrician could, so would the agents. But then they would optimally choose a consumption path that does not respond to these identified shocks.

Notice, that this is not a problem in the two special cases considered above, in which the responses of consumption are indeed flat. By continuity, the identification problem is less dramatic when σ_ν is either very small or very large. However, as we shall see, estimated values of σ_ν lie in an intermediate range in which the identification problem is serious. The quantitative relevance of the identification problem can be gauged from some simple simulations. Figure 2 shows the estimated impulse responses to the shocks with permanent and transitory effects obtained from a structural VAR estimation on (c_t, a_t) that uses a long-run restriction à la Blanchard and Quah (1989) to identify the permanent shock. In the same figure, we plot the true impulse responses to the three underlying shocks. The underlying parameters are the same as for Figure 1. The estimated impulse responses are obtained by generating a 10,000-period time series using the true model and running a structural VAR on it.

Look first at the true and estimated responses of productivity to a shock with permanent effects. The solid line in the top left quadrant plots the true response to a permanent technology shock, which replicates that in Figure 1, namely a small initial effect, followed by a steady buildup over time. The dashed line gives the estimated response from the structural VAR estimation: The initial effect is much larger, the later buildup much smaller. Indeed, simulations show that the less informative the signal, the larger the estimated initial effect, the smaller the later build up. (Remember that, when the signal is fully uninformative, the estimated response shows a one-time increase, with no further build up over time).

Turn to the true and estimated responses of consumption to a permanent shock in the bottom left quadrant. The solid line again replicates the corresponding response in Figure 1, showing a slow build-up of consumption over time. The dashed line shows the estimated response, namely a one-time response of consumption with no further build up over time.

The right quadrants show the true and estimated responses to shocks with transitory effects on output. The solid lines show the true responses to a transitory technology shock (thick line) and to a noise shock (thin line). The dashed lines give the estimated response to the single transitory shock from the structural VAR. They show that the estimated response of productivity to a transitory shock is close to the true response to a transitory technology

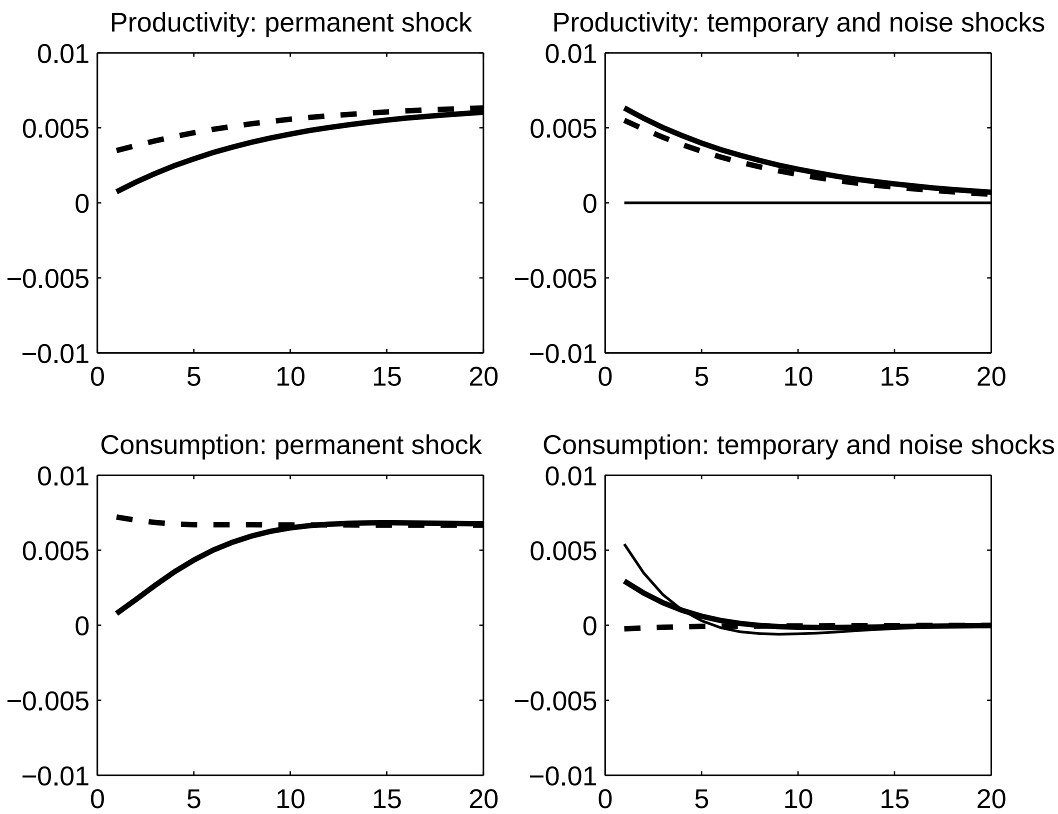

Figure 2: True and SVAR-based estimated impulse responses

shock, but the estimated response of consumption is equal to zero.

In short, the responses from the structural VAR overstate the initial response of productivity and consumption to permanent shocks, and thus give too much weight to these shocks in accounting for fluctuations. For productivity, the less informative the signal, the larger the overstatement. For consumption, the overstatement is independent of the informativeness of the signal.

2.1.3 What does a structural VAR deliver?

A different way of looking at the problem is to understand what is the correct interpretation of the identified shocks that the structural VAR delivers. It turns out that the structural VAR allows us to recover the process for a_t in its innovations representation. The process for a_t can be equivalently represented by the state space system:

$$\hat{x}_t = \hat{x}_{t-1} + u_t^c \tag{16}$$

$$a_t = \rho a_{t-1} + (1-\rho)\hat{x}_{t-1} + u_t^a. \tag{17}$$

To prove the equivalence it is sufficient to define $\hat{x}_t \equiv c_t$, and use the results in Section 2.1. A long-run identifying restriction on a bivariate VAR leads us to identify u_t^c as the permanent technology shock and will give a linear combination of u_t^c and u_t^a as the temporary shock. For some purposes, this representation may be all we are interested in. Clearly, that is not the case if we are trying to analyze the role of noise shocks in fluctuations.

But then why not start directly from (16)-(17) as our model for productivity dynamics and give consumers full information on the state $\hat{x}_t$? One reason why (16)-(17) is not particularly appealing as a primitive model is that the disturbances v_t^1 and v_t^2 in the innovation representation above are not mutually independent, and thus are hard to interpret as primitive shocks. In particular, our signal extraction model implies that v_t^1 and v_t^2 are positively correlated and their correlation is higher the higher the underlying value of σ_ν. As we shall see in the next section, this positive correlation is indeed observed in the data. Our informational assumptions provide a rationale for it.

2.2 Matching moments

We now turn to structural estimation. For our baseline model, structural estimation is particularly easy, and all parameters can be obtained matching a few moments of the model to the data.

In general, structural estimation allows us to exploit the cross-equation restrictions implied by the model to achieve identification. Equation (14), our reduced form equation for productivity, provides a good example of this principle: estimating this equation by OLS, allows us immediately to recover the parameter ρ. Next, σ_u^2 can be estimated by the sample variance of Δa_t. Having estimates for ρ and σ_u^2, we immediately get estimates for σ_ϵ^2 and σ_η^2.

Although identification is particularly simple here, the point holds more generally. In the class of models considered here, identification can be achieved exploiting two crucial assumptions: some behavioral assumption which links consumption (or some other endogenous variables) to the agents' expectations about the future, here equation (9), and an assumption of rational expectations.[6]

How well does our reduced form benchmark model (13)-(14) fits the time series facts for productivity and consumption? The answer is: fairly well. Although it clearly misses some of the dynamics in the data, it provides a good starting point.

Throughout this section, we only use time series for a_t and c_t. We construct the productivity variable as the logarithm of the ratio of GDP to employment and the consumption variable as the logarithm of the ratio of NIPA consumption to population. We use quarterly data, from 1970:1 to 2008:1. An issue we have to confront is that, in contradiction to our model, and indeed to any balanced growth model, productivity and consumption have different growth rates over the sample (0.34% per quarter for productivity, versus 0.46% for consumption). This difference reflects factors we have left out of the model, from changes in participation, to changes in the saving rate, to changes in the capital-output ratio. For this reason, in what follows, we allow for a secular drift in the consumption-to-productivity ratio (equal to 0.46%-0.34%) and remove it from the consumption series.[7]

The basic characteristics of the time series for productivity and consumption are presented in Table 1. Lines 1 and 2 show the results of estimated AR(1) for the first differences of the two variables. Recall that our model implies that both productivity and consumption should follow random walks, so the AR(1) term should be equal to zero. In both cases, the AR(1) term is indeed small, insignificant in the case of productivity, significant in the case

[6]The use of behavioral assumptions as identification assumptions to estimate an underlying exogenous process, connects our paper to a large body of work on household income dynamics. See, for example, Blundell and Preston (1998), who use the permanent income hypothesis as an identification assumption to decompose the household income process into transitory and permanent components.

[7]We are aware that, in the context of our approach, where we are trying to isolate potentially low frequency movements in productivity, this is a rough and dangerous approximation. But, given our purposes, it seems to be a reasonable first pass assumption. The reason why we concentrate on the sample 1970:1 to 2008:1 is precisely because, with longer samples, we are less confident that this approach does a satisfactory job at accounting for low frequency changes in the consumption-to-productivity ratio. When we turn to the variance decomposition, we will show that our results are robust to extending the sample.

of consumption.

Our model further implies a simple dynamic relation between productivity and consumption, equation (14), which can be rewritten as the cointegrating regression:

$$\Delta a_t = (1-\rho)(c_{t-1} - a_{t-1}) + u_t^a$$

Line 3 shows the results of estimating this equation. Line 4 allows for lagged rates of change of consumption and productivity, and shows the presence of richer dynamics than implied by our specification, with significant coefficients on the lagged rates of change of both variables.

Line	Dependent variable:	$\Delta a(-1)$	$\Delta c(-1)$	$(c-a)(-1)$
1	Δa	-0.06 (0.09)		
2	Δc		0.24 (0.08)	
3	Δa			0.05 (0.03)
4	Δa	-0.21 (0.10)	0.32 (0.12)	0.03 (0.02)
5	$\Delta(8)a$			0.03 (0.15)
6	$\Delta(20)a$			0.31 (0.30)
7	$\Delta(40)a$			0.98 (0.43)

Table 1: Consumption and Productivity Regressions.
Note: Sample: 1970:1 to 2008:1. $\Delta(j)a \equiv a(+j-1) - a(-1)$. In parenthesis: robust standard errors computed using the Newey-West window and 10 lags.

Our model's dynamic implications on the relation between consumption and productivity can be extended to longer horizons. Specifically, (14) can be extended to obtain the following cointegrating regression, which holds for all $j \geq 0$,[8]

$$a_{t+j} - a_t = (1-\rho^j)(c_{t-1} - a_{t-1}) + u_t^{a,j},$$

where $u_t^{a,j}$ is a disturbance uncorrelated to the econometrician's information at date t. Thus, according to the model, a larger consumption-productivity ratio should forecast higher future

[8]This is obtained by induction. Suppose it is true for j, that is, $E_t[a_{t+j}] = \left(1-\rho^j\right)c_t + \rho^j a_t$. Taking expectations at time $t-1$ on both sides yields

$$\begin{aligned} E_{t-1}[a_{t+j}] &= \left(1-\rho^j\right)E_{t-1}[c_t] + \rho^j E_{t-1}[a_t] \\ &= \left(1-\rho^j\right)c_{t-1} + \rho^j\left((1-\rho)\,c_{t-1} + \rho a_{t-1}\right) \\ &= \left(1-\rho^{j+1}\right)c_{t-1} + \rho^{j+1}a_{t-1}, \end{aligned}$$

the second equality follows from (6) and (14), the third from rearranging.

productivity growth at all horizons and the coefficient in this regression should increase with the horizon. Lines 5 to 7 explore this implication. We correct for the presence of autocorrelation due to overlapping intervals by using Newey-West standard errors. These results are roughly consistent with the model predictions, and all point to relatively high values for ρ: the point estimates implicit in lines 3, 5, 6 and 7 are, respectively, 0.95, 0.996, 0.98 and 0.91. The maximum likelihood approach below will use all the model restrictions to produce a single estimate of ρ, for now we just take the estimate from line 3, $\rho = 0.95$.

The idea that the forecasting power of the consumption-to-productivity ratio tells us something about consumers' information is closely related to a similar observation made by Cochrane (1994) using the consumption-to-output ratio. Indeed this observation was the motivating reason for Cochrane (1994) early proposal to introduce news shocks in business cycle models.

The standard deviation σ_u can be estimated directly from the univariate representation of a_t as the sample mean squared deviation of Δa_t, giving a point estimate $\sigma_u = 0.67\%$. Together with $\rho = 0.95$, this implies $\sigma_\epsilon = 0.03\%$ and $\sigma_\eta = 0.65\%$. In words, these results imply a very smooth permanent component, in which small shocks steadily build up over time, and a large transitory component, which decays slowly over time.

Recovering the variance of the noise shock is less straightforward, but it can be done matching another moment: the correlation coefficient between the reduced form innovations u_t^c and u_t^a. In particular, numerical results show that, given the remaining parameters, this moment is an increasing function of σ_ν. Therefore, we recover this parameter by matching the correlation in the data. The coefficient of correlation between Δc and the residual of the regression on line 3 (corresponding, respectively to u_t^c and u_t^a) is equal to 0.52. If the signal was perfectly informative this correlation would be equal to 0.05, while if the signal had infinite variance it would be 1.[9] Therefore, the observed correlation is consistent with the model and yields a fairly large standard deviation of the noise shock, $\sigma_\nu = 2.1\%$.

The fact that we are able in our benchmark model to recover all the model parameters by matching a few moments from the data, is clearly a special case. It is thus useful to develop a general approach, which can be applied to any specification of productivity or consumption behavior. We now discuss this approach, and then return to the data.

[9]These bounds can be derived from the analysis in Section 2.1. To obtain the first, some algebra shows that under full information $Cov[u_t^c, u_t^a]/\sqrt{Var[u_t^c]Var[u_t^a]} = (1-\rho)/\sqrt{(1-\rho)^2+\rho}$. The second bound is immediate.

2.3 Maximum Likelihood

To estimate a model where consumers face a non trivial signal extraction problem, one can, generally, proceed in two steps.[10]

- Take the point of view of the consumers. Write down the dynamics of the unobserved state vector X_t in state space representation and solve the consumers' filtering problem as we did in Section 1.3.

- Take the point of view of the econometrician, write down the model dynamics in state space representation and write the appropriate observation equation, which depends on the data available. The relevant state for the econometrician is given by $X_t^E \equiv (X_t, X_{t|t})$. The consumers' expectations become part of the unobservable states and the consumers' updating equation (11) becomes part of the description of the state's dynamics. If the econometrician observes c_t and a_t, the observation equations for the econometrician are (1) and (12). The econometrician's Kalman filter is then used to construct the likelihood function and estimate the model's parameters.

Table 2 shows the results of estimation of the benchmark model presented as a grid over values of ρ from 0 to 0.99.[11] For each value of ρ, we find the values of the remaining parameters that maximize the likelihood function and in the last column we report the corresponding likelihood value. The table shows that the likelihood function has a well-behaved maximum at $\rho = 0.89$, on line 6. The corresponding values of σ_ϵ and σ_η are 0.07% and 0.63%, respectively. The standard deviation of the noise shock σ_ν is 0.89%.

Relative to the moment matching approach in Section 2.2, the Maximum Likelihood approach uses all the implicit restrictions imposed by the model on the data generating process. This explains the difference between the estimates on line 6 of Table 2 and those obtained in Section 2.2. In particular, the Maximum Likelihood approach favors smaller values of ρ and σ_ν. However, if we look at line 8 of Table 2, we see parameters much closer to those in Section 2.2 and the likelihood gain from line 8 to line 6 is not too large. In other words, the data are consistent with a range of different combinations of ρ and σ_ν. When we look at the model's implications in terms of variance decomposition, we will consider different values in this range.

[10] More detailed derivations are provided in Appendix C.

[11] For all our Maximum Likelihood estimates we used Dynare. Our observables are first differences of labor productivity and consumption, so we use a diffuse Kalman Filter to initialize the variance covariance matrix of the estimator (a variance-covariance matrix with a diagonal of 10).

Line	ρ	σ_u	σ_ϵ	σ_η	σ_ν	ML
1	0.00	0.0067	0.0067	0.0000	0.0089	$-3 * 10^{12}$
2	0.25	0.0183	0.0137	0.0092	0.0000	859.2
3	0.50	0.0102	0.0051	0.0072	0.0000	980.5
4	0.70	0.0077	0.0023	0.0065	0.0026	1042.6
5	0.80	0.0071	0.0014	0.0064	0.0056	1064.5
6	0.89	0.0067	0.0007	0.0063	0.0089	1073.2
7	0.90	0.0067	0.0007	0.0064	0.0099	1073.1
8	0.95	0.0068	0.0003	0.0066	0.0234	1072.2
9	0.99	0.0063	0.0001	0.0063	0.0753	1068.5

Table 2: Maximum Likelihood Estimation: Benchmark Model

A simple exercise, using this approach, is to relax the random walk assumption for productivity, allowing ρ_x to differ from ρ_z, and allowing the variances of the shocks to be freely estimated. The estimation results are reported in Table 3 and are quite close to those obtained under the random walk assumption.

	Estimate	Standard error
ρ_x	0.8879	0.0478
ρ_z	0.8878	0.0474
σ_η	0.0065	0.0004
σ_ϵ	0.0007	0.0003
σ_ν	0.0090	0.0052
ML	1073.3	

Table 3: Maximum Likelihood Estimation: Unconstrained Model

2.4 Variance decomposition

What do our results imply in terms of the dynamic effects of the shocks and of variance decomposition? If we use the estimated parameters from the benchmark model (line 6 in Table 2), the dynamic effects of each shock are given in Figure 1 and were discussed in Section 1.3: A slow and steady build up of permanent shocks on productivity and consumption; a slowly decreasing effect of transitory shocks on productivity and consumption; and a slowly decreasing effect of noise shocks on consumption.

Figure 3 presents the variance decomposition, plotting the contribution of the three shocks to forecast error variance, from 1 to 20 quarters ahead. The main result is that noise

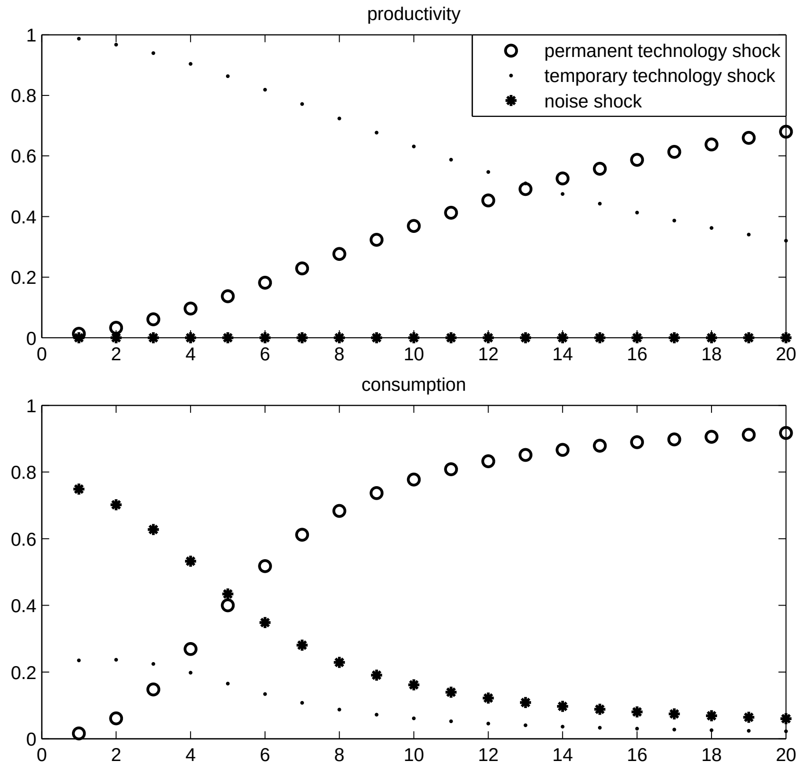

Figure 3: Variance Decomposition: Benchmark Model

shocks are an important source of short run volatility, accounting for more than 70% of consumption volatility at a 1-quarter horizon and more than 50% at a 4-quarter horizon, while permanent technology shocks play a smaller role, having almost no effect on quarterly volatility and explaining less than 30% at a 4-quarter horizon. It is interesting to compare this result to traditional SVAR exercises, such as Shapiro and Watson (1989) and Gali (1992), where demand shocks typically explain a smaller fraction of aggregate volatility and permanent technology shocks play a bigger role. The analysis in Section 2.1 helps to explain these differences, by showing that, asymptotically, a SVAR is biased towards assigning 100% of consumption volatility to the permanent shock.

In Table 4, we report the results of some robustness checks. On each line, we report the fraction of consumption variance due to the noise shock at a 1, 4 and 8-quarter horizon, for different parameter values. Line 1 corresponds to our benchmark estimation. Line 2 reports the results obtained by setting ρ at a higher level and choosing the remaining parameters by maximum likelihood (line 8 of Table 2). The variance decomposition at short horizons is not very different, but noise shocks turn out to be more persistent under this parametrization and explain a much bigger fraction of variance at a 8-quarter horizon. On line 3 we report the parameters obtained when estimating our model on a longer sample, 1948:1 to 2008:1. With this data set the estimate of ρ is larger and we obtain results analogous to the ones on line 2.

Finally, in lines 4 and 5 we experiment with changing only the volatility of noise shocks, keeping the other parameters fixed. In particular, relative to the benchmark, we first decrease and then increase σ_ν by one standard deviation (which is 0.0034 in our maximum likelihood estimate). Interestingly, it is the lower value of σ_ν that leads to the largest amount of noise-driven volatility. A lower σ_ν makes the signal s_t more precise, so consumers rely on it more. In our range of parameters, this leads to greater short-run volatility.

Line		Parameters			Noise-driven variance (fraction)		
		ρ	σ_u	σ_ν	1 Quarter	4 Quarter	8 Quarter
1	benchmark	0.89	0.0067	0.0089	0.75	0.53	0.23
2	high ρ	0.95	0.0068	0.0234	0.71	0.68	0.58
3	sample 1948:1-2008:1	0.96	0.0099	0.0382	0.73	0.71	0.64
5	low σ_ν	0.89	0.0067	0.0055	0.82	0.46	0.17
4	high σ_ν	0.89	0.0067	0.0123	0.68	0.53	0.26

Table 4: Variance Decomposition: Robustness Checks

2.5 Recovering the states: retrospective history

So far we have focused on using structural estimation to estimate the model's parameters. Now we turn to the question: what information on the unobservable states and on the shocks can be recovered from structural estimation? We begin with the states.

Using the Kalman smoother it is possible to form Bayesian estimates of the state vector X_t^E using the full time series available and obtain a retrospective history of the U.S. business cycle. The top panel of Figure 4 plots estimates for the permanent component of productivity x_t obtained from our benchmark model. The solid line correspond to x_t, the dashed line to the consumers' real time estimate of the same variable $x_{t|t}$. Notice that both x_t and $x_{t|t}$ are unobservable states for the econometrician, so the two lines correspond to the Bayesian estimates of the respective state (see Appendix C).

Looking first at medium-run movements, the model identifies a gradual adjustment of consumers' expectations to the productivity slowdown in the 70s and a symmetric gradual adjustment in the opposite direction during the faster productivity growth after the mid 90s. Around these medium-run trends, temporary fluctuations in consumers' expectations produce short-run volatility.

To gauge the short-run effects of expectational errors, however, the consumers' expectations of x_t are not sufficient, given that consumers project future growth based on their expectations of both x_t and x_{t-1}. For this reason, in the bottom panel of Figure 4, we plot the smoothed series for the consumers' real time expectations regarding long-run productivity, $x_{t+\infty|t} = (x_{t|t} - \rho x_{t-1|t})/(1-\rho)$, and compare it to the smoothed series for $x_{t+\infty}$. The model generates large short-run consumption volatility out of temporary changes in consumers' expectations of future productivity. Sometimes these changes occur when consumers' overstate current x_t (e.g., at the end of the 80s), other times when consumers slowly catch up to an underlying productivity acceleration and understate x_{t-1} (e.g., at the end of the 90s). Obviously, the model is too stylized to give a credible account of all cyclical episodes. For example, given the absence of monetary policy shocks the recession of 1981-82 is fully attributed to animal spirits.

The Kalman smoother also tells us how much information on the unobservable states is contained in past and future data. In particular, in Figure 5 we plot the root mean squared errors (RMSE) of the smoothed estimates of x_t and z_t, when data up to $t+j$ are available, for $j = 0, 1, 2, ...$ Formally, these RMSE correspond to the square root of $E_{t+j}[(x_t - E_{t+j}[x_t])^2]$, and can be computed using two different information sets: the econometrician's, which only includes observations of c_t and a_t, and the consumer's, which also includes s_t. For simplicity, we compute RMSE at the steady state of the Kalman filter, that is, assuming the forecaster has access to an infinite series of data, from $-\infty$ to $t+j$. In this case, the econometrician's information set coincides with the consumer's, that is, the econometrician can back up the current value of s_t perfectly from current and past observations of c_t and a_t. Although we have not established this result analytically, it holds numerically in all our examples: the computed RMSE of the econometrician's estimate of s_t goes immediately to zero at $j = 0$. This implies that, in our model, with a sufficiently long data set, the direct observation of s_t does not add much to the econometrician's ability to recover the unobservable states (or the shocks).

Figure 5 shows that the contemporaneous estimate of the current state x_t has a standard deviation of 0.44%. By using future data, this standard deviation almost halves, to 0.28%. However, most of the relevant information arrives in the first six quarters, after that, there are minimal gains in the precision of the estimate.

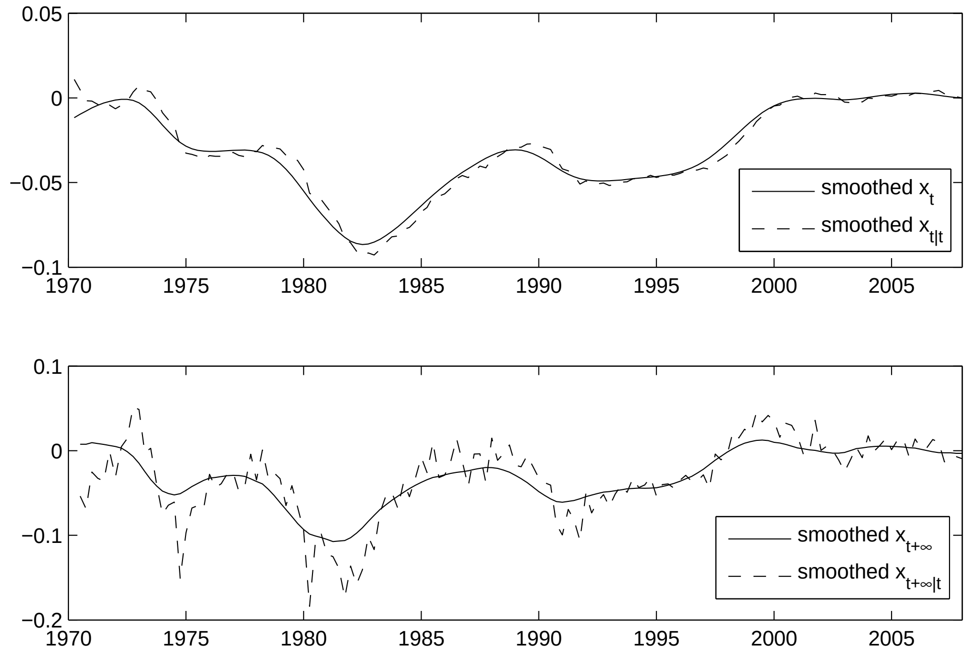

Figure 4: Smoothed estimates of the permanent component of productivity, of long-run productivity, and of consumers' real time expectations
Top panel: smoothed estimate of x_t (solid line) and of $x_{t|t}$ (dashed line)
Bottom panel: smoothed estimate of $x_{t+\infty}$ (solid line) and of $x_{t+\infty|t}$ (dashed line)

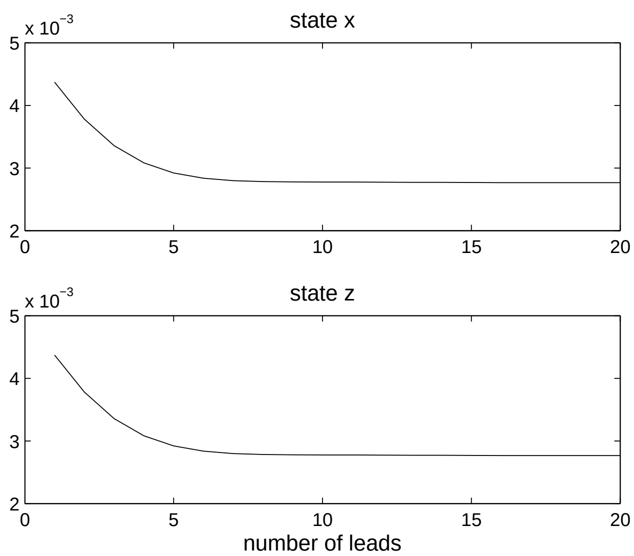

Figure 5: RMSE of the estimated states at time t using data up to $t + j$

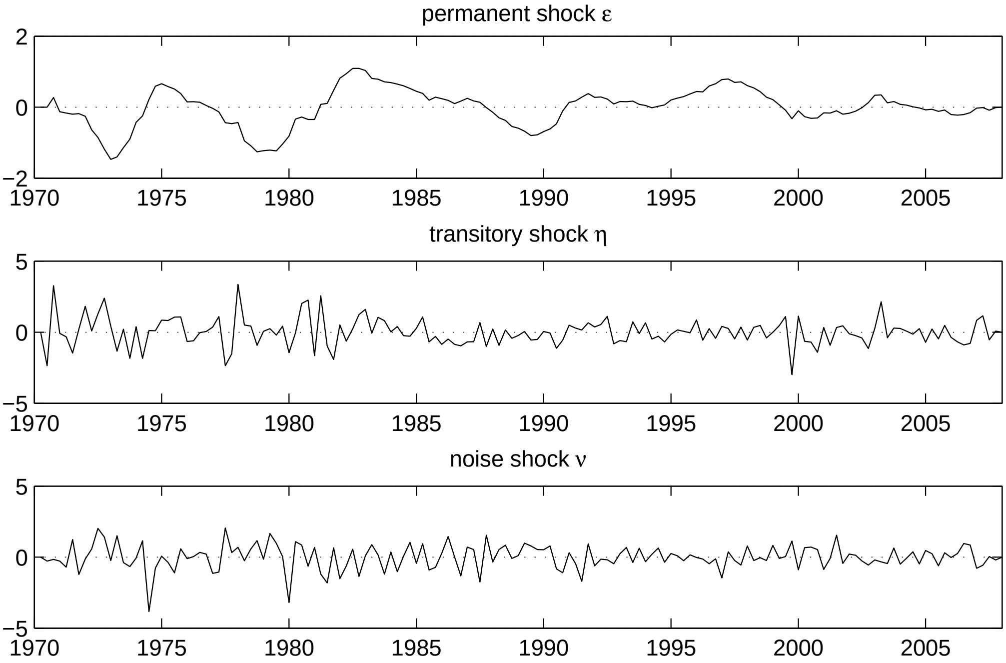

Figure 6: Smoothed estimates of the shocks

2.6 Recovering the shocks: more on invertibility

Turning to the shocks, we know from our discussion of structural VARs that the information in current and past values of c_t and a_t is not sufficient to derive the values of the current shocks. However, this does not mean that the data contain no information on the shocks. In particular, using the Kalman smoother the econometrician can form Bayesian estimates on ϵ_t, η_t, and ν_t using the entire time series available. Figure 6 plots these estimates for our benchmark model. As for the states, in Figure 7 we report the RMSE of the estimated shocks as a function of the number of leads available. To help the interpretation, each RMSE is normalized dividing it by the ex ante standard deviation of the respective shock (σ_ϵ, σ_η, and σ_ν).

Notice that if the model was invertible, the RMSE would be zero at $j = 0$. The fact that all RMSE remain bounded from zero at all horizons shows that even an infinite data set would not allow us to recover the shocks exactly.

The transitory shock η_t is estimated with considerable precision already on impact and the precision of its estimate almost doubles in the long run. The noise shock ν_t is less precisely estimated, but the data still tell us a lot about it, giving us an RMSE which is

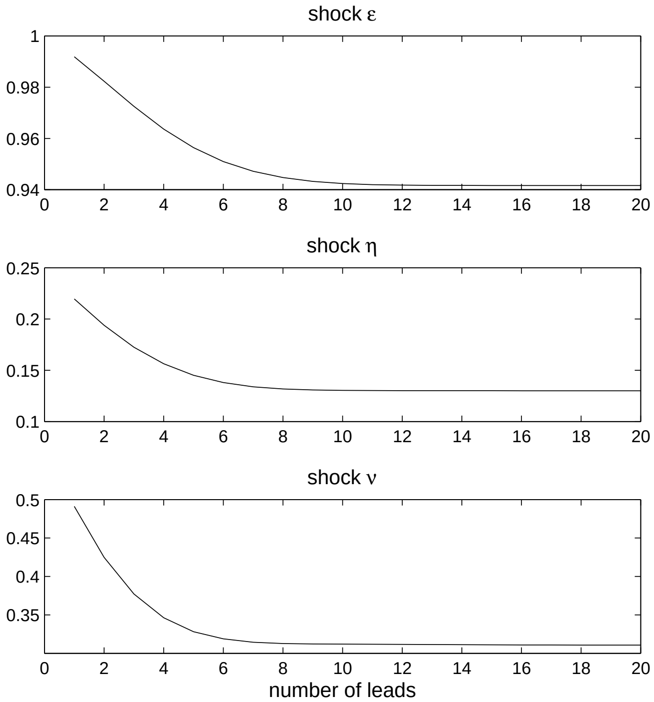

Figure 7: Normalized RMSE of the estimated shocks at time t using data up to $t + j$

about 1/3 of the prior uncertainty in the long run. The shock that is least precisely estimated is the permanent shock ϵ_t. Even with an infinite series of future data, the residual variance is about 94% of the prior uncertainty on the shock.

How do we reconcile the imprecision of the estimate of ϵ_t with the fact that we have relatively precise estimates of the state x_t, as seen in Figure 5? The explanation is that the econometrician can estimate the cumulated effect of permanent productivity changes by looking at productivity growth over longer horizons, but cannot pinpoint the precise quarter in which the change occurred. Therefore, it is possible to have imprecise estimates of past ϵ_t's, while having a relatively precise estimate of their cumulated effect on x_t. This also helps to explain the high degree of autocorrelation of the estimated permanent shocks in Figure 6. The smoothed estimates of ϵ_t in consecutive quarters tend to be highly correlated, as the econometrician does not know to which quarter to attribute an observed permanent change in productivity. Notice that the autocorrelation of the estimated shocks is not a rejection of the assumption of i.i.d. shocks, but purely a reflection of the econometrician's information. In fact, performing the same estimation exercise on simulated data delivers a similar degree of autocorrelation as the one obtained from actual data.

3 A DSGE exercise

So far, we focused on a very simple model of consumption. Now we extend our model to include investment, an explicit treatment of price setting by monopolistic firms, and monetary policy. We will introduce our additional ingredients in steps, starting from a simple new Keynesian model and eventually developing a small scale DSGE model that includes most of the ingredients proposed in the literature to better account for the empirical dynamics of output, inflation and the interest rate.

The first step is to estimate a simple new Keynesian model with Calvo pricing. The model is described in Appendix A and can be still be solved by hand, yielding the following process for consumption

$$c_t = d_1 a_t + d_2 x_{t|t} + d_3 x_{t|t-1} + d_4 z_{t|t} \tag{18}$$

where the coefficients d are non-linear functions of the following parameters: the discount factor β, a parameter ϕ, reflecting the response of the nominal interest rate to inflation in the monetary policy rule, and a parameter κ, capturing the degree of nominal and real rigidities in price setting. We set β at 0.99 and estimate the remaining parameters by Maximum

Likelihood, following the same steps laid out in 2.3. The results are reported in Table 5.

	Estimate	Standard error
κ	0.0011	0.0004
ϕ	1.4436	0.1403
ρ	0.8780	0.0225
σ_u	0.0067	0.0004
σ_ν	0.0065	0.0019
ML	1073.8	

Table 5: Maximum Likelihood Estimation: standard new Keynesian model

The implications of the new Keynesian model are very close to those of the benchmark model. In particular, the implied values of the coefficients in (18) are $d_1 = 0.0016$, $d_2 = 7.9250$, $d_3 = -6.9266$, $d_4 = 0.0359$, while, in our benchmark model, given $\rho = 0.878$, the corresponding values would be $d_1 = 0$, $d_2 = 1/(1-\rho) = 8.1967$, $d_3 = -\rho/(1-\rho) = -7.1967$, and $d_4 = 0$. The implied impulse responses are thus close to the ones in Section 1.3. The reason for this similarity is that the estimation delivers a very low value for κ, i.e., very sticky prices. However, adding more ingredients will allow us to obtain more realistic values for stickiness.

(...)

Finally, we present the results of a full Bayesian estimation of a small scale DSGE model. The main model ingredients are described here. The full model derivations are in Appendix D. Preferences feature habit formation and the utility function is

$$E\left[\sum \beta^t \left(\ln\left(C_t - hC_{t-1}\right) - \frac{1}{1+\phi} N_{jt}^{1+\phi}\right)\right].$$

There is variable capacity utilization, as the capital services supplied by the capital stock $\bar{K}$ are given by

$$K_t = U_t \bar{K}_{t-1},$$

where U_t is the degree of capital utilization and the cost of setting U_t is given by $Q(U_t)\bar{K}_{t-1}$ in terms of current goods. Investment is subject to adjustment cost according to

$$\bar{K}_t = (1-\delta)\,\bar{K}_{t-1} + D_t\left[1 - G\left(\frac{I_t}{I_{t-1}}\right)\right] I_t$$

where $d_t = \log(D_t)$ is investment specific technology shock. There is a continuum of mo-

Table 6: Full DSGE: estimated parameters

Parameter	Description	Prior	Posterior	Conf. bands		Distribution	Prior st. dev.
ρ	persistence x	0.6	0.9399	0.9158	0.9545	Beta	0.2
ρ_d	persistence d	0.6	0.4581	0.3663	0.5521	Beta	0.2
σ_u	variance a	0.5	1.1612	1.0808	1.2591	Inv. Gamma	0.15
σ_ν	variance noise	1	1.0316	0.7908	1.2095	Inv. Gamma	0.15
σ_d	variance d	5	11.402	8.3596	13.1803	Inv. Gamma	1.5
h	habit	0.5	0.5162	0.4774	0.5646	Beta	0.1
α	capital coeff.	0.3	0.1837	0.1739	0.1919	Normal	0.05
ϕ	inverse Frisch	2	2.129	1.0045	3.4899	Gamma	0.75
χ	capacity	4	4.4894	3.6394	5.1243	Normal	1
ξ	adjust. cost	5	2.9615	2.3779	3.0234	Gamma	1
θ	Calvo	0.66	0.8767	0.8599	0.901	Beta	0.1
θ_w	Calvo wage	0.66	0.8741	0.8335	0.9241	Beta	0.1

nopolistic competitive producers in the goods, with sticky prices à la Calvo. Similarly, labor services are supplies under monopolistic competition, with sticky nominal wages. The monetary authority follows a standard Taylor rule for the nominal interest rate.

The parameter estimates are reported in Table 6. Figure 8 shows the impulse responses for consumption, investment and output following a permanent shock and a noise shock. Table 7 reports the variance decomposition. (To be completed.)

4 Conclusions

On the methodological side, we have explored the problem of estimating models with news and noise, which we think provide an appealing description of the cycle. We have shown the limits of SVAR estimation and shown how these models can be estimated with structural methods. This implies that to identify the role of news and noise in fluctuations one must rely more heavily on the model's structure. Our simple model shows that a central role

Table 7: Variance decomposition

Quarter	Permanent	Transitory	Noise	Investment	Price markup	Wage markup	Monetary
Consumption							
1	0.010	0.187	0.509	0.005	0.030	0.113	0.112
4	0.160	0.181	0.321	0.009	0.020	0.185	0.064
8	0.543	0.083	0.123	0.008	0.009	0.145	0.026
12	0.731	0.041	0.064	0.005	0.010	0.087	0.013
Investment							
Quarter	Permanent	Transitory	Noise	Investment	Price markup	Wage markup	Monetary
1	0.000	0.011	0.020	0.942	0.010	0.010	0.006
4	0.014	0.027	0.032	0.855	0.026	0.037	0.009
8	0.119	0.033	0.028	0.696	0.036	0.079	0.009
12	0.319	0.026	0.020	0.509	0.030	0.090	0.007
Output							
Quarter	Permanent	Transitory	Noise	Investment	Price markup	Wage markup	Monetary
1	0.007	0.219	0.224	0.331	0.029	0.066	0.051
4	0.110	0.220	0.186	0.263	0.035	0.117	0.039
8	0.453	0.135	0.086	0.149	0.020	0.120	0.020
12	0.691	0.071	0.047	0.077	0.011	0.083	0.011

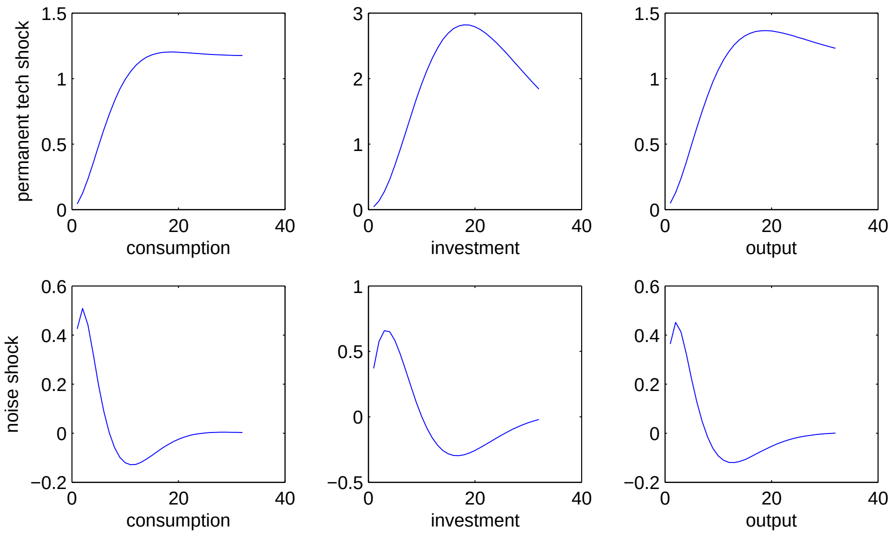

Figure 8: Impulse Responses, Bayesian DSGE

for identification is played by the consumer's Euler equation, that is, by the assumption that current movements in consumption are primarily driven by changes in the consumers' expectations on the economy's lung run potential.

On the empirical side, the data appear consistent with a view of fluctuations where the pattern of technological change is smooth, subject to random shocks which only build up slowly, while a sizable fraction of short-run volatility in consumption and output comes from noisy information on these long-run trends.

A useful extension for future work is to add variables to the empirical exercise, to better capture consumers' expectations about the future. For example, one could include financial market prices, following Beaudry and Portier (2006), or survey measures of consumer confidence, as Barsky and Sims (2008). However, the analysis in Section 2.1, where we allow the econometrician to directly observe all the signals observed by the consumers, shows that adding these variables will not solve the identification problems of SVARs.

Finally, it is useful to notice that the applicability of SVAR methods depends crucially on the way in which one models the information structure. In models where the consumer exactly observes shocks which will affect productivity in the future, invertibility problems may be less damning (see our comments in Section 2.1.3 and the analysis in Sims (2009)). However, we think that, in many instances, signal extraction models provide a more realistic and flexible description of the informational environment. When dealing with these models, the researcher can choose, depending on the question at hand, either to limit attention to the innovation representation of the consumers' forecasting problem or to take the structural approaches adopted here.

Appendix A. Derivations for the new Keynesian model

Consider a standard new Keynesian model, as laid out, e.g., in Gali (2008). Preferences are given by

$$E\sum_{t=0}^{\infty}\beta^{t}U\left(C_{t},N_{t}\right),$$

with

$$U\left(C_{t},N_{t}\right)=\log C_{t}-\frac{1}{1+\zeta}N_{t}^{1+\zeta},$$

where N_t are hours worked and C_t is a composite consumption good given by

$$C_{t}=\left(\int_{0}^{1}C_{j,t}^{\frac{\gamma-1}{\gamma}}dj\right)^{\frac{\gamma}{\gamma-1}},$$

$C_{j,t}$ is the consumption of good j in period t, and $\gamma>1$ is the elasticity of substitution among goods. Each good $j\in[0,1]$ is produced by a single monopolistic firm with access to the linear production function

$$Y_{j,t}=A_{t}N_{j,t}. \tag{19}$$

Productivity is given by $A_t=\exp a_t$ and a_t follows the process (1)-(3). Firms are allowed to reset prices only at random time intervals. Each period, a firm is allowed to reset its price with probability $1-\theta$ and must keep the price unchanged with probability θ. Firms hire labor on a competitive labor market at the wage W_t, which is fully flexible.

Consumers have access to a nominal one-period bond which trades at the price Q_t. The consumer's budget constraint is

$$Q_{t}B_{t+1}+\int_{0}^{1}P_{j,t}C_{j,t}dj=B_{t}+W_{t}N_{t}+\int_{0}^{1}\Pi_{j,t}dj, \tag{20}$$

where B_t are nominal bonds' holdings, $P_{j,t}$ is the price of good j, W_t is the nominal wage rate, and $\Pi_{j,t}$ are the profits of firm j. In equilibrium consumers choose consumption, hours worked, and bond holdings, so as to maximize their expected utility subject to (20) and a standard no-Ponzi-game condition. Nominal bonds are in zero net supply, so market clearing in the bonds market requires $B_t=0$. The central bank sets the short-term nominal interest rate, that is, the price of the one-period nominal bond, Q_t. Letting $i_t=-\log Q_t$, monetary policy follows the simple rule

$$i_t = i^* + \phi\pi_t, \tag{21}$$

where $i^* = -\log\beta$ and ϕ is a constant coefficient greater than 1.

Following standard steps, consumers' and firms' optimality conditions and market clearing can be log-linearized and transformed so as to obtain two stochastic difference equations which characterize the joint behavior of output and inflation in equilibrium. After substituting the policy rule we obtain:

$$\begin{aligned} y_t &= E_t[y_{t+1}] - \phi\pi_t + E_t[\pi_{t+1}], \\ \pi_t &= \kappa(y_t - a_t) + \beta E_t[\pi_{t+1}], \end{aligned}$$

where $\kappa \equiv (1+\zeta)(1-\theta)(1-\beta\theta)/\theta$ and where constant terms are omitted. As long as $\phi > 1$ this system has a unique locally stable solution where y_t and π_t are linear functions of the four exogenous state variables $a_t, x_{t|t}, x_{t-1|t}, z_{t|t}$,

$$\begin{pmatrix} y_t \\ \pi_t \end{pmatrix} = D_\kappa \begin{pmatrix} a_t \\ x_{t|t} \\ x_{t-1|t} \\ z_{t|t} \end{pmatrix}.$$

The matrix D_κ can be found using the method of undetermined coefficient as the solution to

$$\begin{bmatrix} 1 & \phi \\ -\kappa & 1 \end{bmatrix} D_\kappa = \begin{bmatrix} 0 & 0 & 0 & 0 \\ -\kappa & 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 0 & \beta \end{bmatrix} D_\kappa \begin{bmatrix} 0 & 1+\rho & -\rho & \rho \\ 0 & 1+\rho & -\rho & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & \rho \end{bmatrix}.$$

The elements of D_κ are a continuous non-linear function of κ and some lengthy algebra (available on request) shows that

$$\lim_{\kappa\to 0} D_\kappa = \frac{1}{1-\rho}\begin{bmatrix} 0 & 1 & -\rho & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

Since $\kappa \to 0$ when $\theta \to 1$, this completes the argument.

Appendix B. Proofs of Propositions 1 and 2

Proof of Proposition 2

Let w_t be an identified shock, corresponding to a linear combination of current and past observables. Applying the law of iterated expectations we get

$$E\left[c_{t+k}|w_t,\mathcal{I}^e_{t-1}\right] = E[\lim_{j\to\infty} E\left[a_{t+k+j}|\mathcal{I}_{t+k}\right]|w_t,\mathcal{I}^e_{t-1}] = \lim_{j\to\infty} E\left[a_{t+j}|w_t,\mathcal{I}^e_{t-1}\right],$$

for all $k \geq 0$ and, similarly,

$$E\left[c_{t+k}|\mathcal{I}^e_{t-1}\right] = \lim_{j\to\infty} E\left[a_{t+j}|\mathcal{I}^e_{t-1}\right].$$

It follows that the response of consumption to w_t is constant and equal to

$$E\left[c_{t+k}|w_t,\mathcal{I}^e_{t-1}\right] - E\left[c_t|\mathcal{I}^e_{t-1}\right] = \lim_{j\to\infty} E\left[a_{t+j}|w_t,\mathcal{I}^e_{t-1}\right] - \lim_{j\to\infty} E\left[a_{t+j}|\mathcal{I}^e_{t-1}\right],$$

for all $k \geq 0$.

Appendix C. Econometrician's Kalman Filter

The econometrician's state vector is given by

$$\xi^E_t \equiv \left(x_t, x_{t-1}, z_t, x_{t|t}, x_{t-1|t}, z_{t|t}\right)'.$$

Rewrite the dynamics of the vector of consumer expectations $(x_{t|t}, x_{t-1|t}, z_{t|t})$, from (11), as follows:

$$\begin{aligned} \begin{bmatrix} x_{t|t} \\ x_{t-1|t} \\ z_{t|t} \end{bmatrix} &= A \begin{bmatrix} x_{t-1|t-1} \\ x_{t-2|t-1} \\ z_{t-1|t-1} \end{bmatrix} + B \begin{bmatrix} 1+\rho_x & -\rho_x & \rho_z \\ 1+\rho_x & -\rho_x & 0 \end{bmatrix} \begin{bmatrix} x_{t-1} \\ x_{t-2} \\ z_{t-1} \end{bmatrix} + \\ &+ B \begin{bmatrix} 1 \\ 1 \end{bmatrix} \epsilon_t + B \begin{bmatrix} 1 \\ 0 \end{bmatrix} \eta_t + B \begin{bmatrix} 0 \\ 1 \end{bmatrix} \nu_t. \end{aligned}$$

Then the state ξ^E_t evolves according to:

$$\xi^E_t = Q\xi^E_{t-1} + R\left(\epsilon_t, \eta_t, \nu_t\right)'. \tag{22}$$

where the matrices Q and R are given by

$$Q = \begin{bmatrix} \begin{matrix} 1+\rho_x & -\rho_x & 0 \\ 1 & 0 & 0 \\ 0 & 0 & \rho_z \end{matrix} & \mathbf{0} \\ B \begin{bmatrix} 1+\rho_x & -\rho_x & \rho_z \\ 1+\rho_x & -\rho_x & 0 \end{bmatrix} & A \end{bmatrix},$$

$$R = \begin{bmatrix} \begin{matrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{matrix} \\ B \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} + B \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} + B \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{bmatrix}.$$

When the econometrician can observe (a_t, c_t), the observation equation is, in matrix form,

$$(a_t, c_t)' = TX_t, \tag{23}$$

where

$$T = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1+\frac{\rho_x}{1-\rho_x} & -\frac{\rho_x}{1-\rho_x} & 0 \end{bmatrix}.$$

The econometrician's filtering problem can then be solved from (22)-(23). The case in which the econometrician can also observe s_t is treated in a similar way. This filter can be used both to compute recursively the likelihood function and to derive smoothed estimates of the unobservable states in ξ_t^E, as in Section 2.5. Expanding the state space to include the shocks $(\epsilon_t, \eta_t, \nu_t)$, it is easy to compute their smoothed estimates, as in Section 2.6.

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